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Feasibility of a finite-difference time-domain model in large-scale acoustic simulations

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:

Giulia Fratoni, B.H. (2022). Feasibility of a finite-difference time-domain model in large-scale acoustic simulations. THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA, 152(1), 330-341 [10.1121/10.0012218].

Availability:

This version is available at: <https://hdl.handle.net/11585/891164> since: 2024-09-12

Published:

DOI: <http://doi.org/10.1121/10.0012218>

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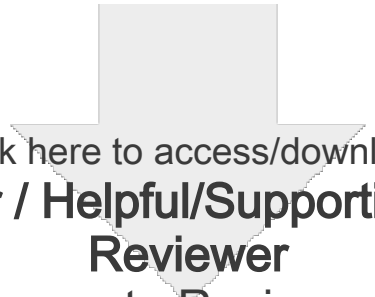
The Journal of the Acoustical Society of America

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--Manuscript Draft--

Manuscript Number:	JASA-07354R3
Full Title:	Feasibility of a finite-difference time-domain model in large-scale acoustic simulations
Article Type:	Regular Article
Corresponding Author:	Dario D'Orazio, Ph.D. University of Bologna Bologna, ITALY
First Author:	Giulia Fratoni, Ph.D.
Order of Authors:	Giulia Fratoni, Ph.D. Brian Hamilton Dario D'Orazio
Section/Category:	Architectural Acoustics
Keywords:	finite-difference time-domain simulations; Architectural acoustics; wave-based techniques; opera houses; multi-decay analysis.
Abstract:	Wave-based techniques for room acoustics simulations are commonly applied to low frequency analysis and small-sized simplified environments. The constraints are generally the inherent computational cost and the challenging implementation of proper complex boundary conditions. Nevertheless, the application field of wave-based simulation methods has been extended in the latest research decades. With the aim of testing this potential, the present work investigates the feasibility of a Finite-Difference Time-Domain (FDTD) code simulating large non-trivial geometries in wide frequency ranges. A representative sample of large coupled-volume opera houses allowed demonstration of the capability of the selected FDTD model to tackle such composite geometries up to 4 kHz. For such a demanding task, efficient calculation schemes and frequency-dependent boundary admittances are implemented in the simulation framework. The results of in situ acoustic measurements were used as benchmark during the calibration process of 3D virtual models. In parallel, acoustic simulations performed on the same halls through standard ray-tracing techniques enabled a systematic comparison between the two numerical approaches highlighting significant differences in terms of input data. The ability of the FDTD code to detect the typical acoustic scenarios occurring in coupled-volume halls is confirmed through multi-slope decay analysis and impulse responses' spectral content.
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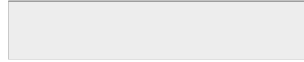
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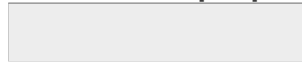




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Feasibility of a finite-difference time-domain model in large-scale acoustic simulations

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1 Wave-based techniques for room acoustics simulations are commonly applied to low
2 frequency analysis and small-sized simplified environments. The constraints are gen-
3 erally the inherent computational cost and the challenging implementation of proper
4 complex boundary conditions. Nevertheless, the application field of wave-based sim-
5 ulation methods has been extended in the latest research decades. With the aim
6 of testing this potential, the present work investigates the feasibility of a Finite-
7 Difference Time-Domain (FDTD) code simulating large non-trivial geometries in wide
8 frequency ranges. A representative sample of large coupled-volume opera houses al-
9 lowed demonstration of the capability of the selected FDTD model to tackle such
10 composite geometries up to 4 kHz. For such a demanding task, efficient calcula-
11 tion schemes and frequency-dependent boundary admittances are implemented in
12 the simulation framework. The results of *in situ* acoustic measurements were used as
13 benchmark during the calibration process of 3D virtual models. In parallel, acous-
14 tic simulations performed on the same halls through standard ray-tracing techniques
15 enabled a systematic comparison between the two numerical approaches highlighting
16 significant differences in terms of input data. The ability of the FDTD code to detect
17 the typical acoustic scenarios occurring in coupled-volume halls is confirmed through
18 multi-slope decay analysis and impulse responses' spectral content.

19 **I. INTRODUCTION**

20 Room acoustics simulation methods are traditionally classified in two distinct groups: the
21 first one based on the description of sound propagation through the wave equation, the sec-
22 ond one based on the approximation through rays (Savioja and Svensson, 2015; Vorländer,
23 2020). The wave-based approach approximates the resolution of the wave equation by em-
24 ploying spatial (Marburg and Nolte, 2008) or spatiotemporal discretization (Botteldooren,
25 1995; Pind *et al.*, 2019) of the system domains. The outcomes of this category remain
26 physically and theoretically valid in the whole frequency range, at the expense of high com-
27 putational cost and a certain degree of numerical dispersion (Bilbao, 2009). For this reason,
28 the application field is usually limited to low-medium frequencies and small rooms (Soares
29 *et al.*, 2022). The ray-based approach, commonly defined as Geometrical Acoustics (GA),
30 finds its roots in computer graphics principles, i.e. the assumption that sound propagates
31 through rays (Krokstad *et al.*, 1968). It is widely accepted in literature that the GA ap-
32 proach is sufficiently accurate above the Schroeder frequency and when the dimensions of
33 the hall are large enough compared to the wavelengths (Bork, 2005). Even though great
34 effort has been made to compensate the respective weaknesses of the two macro-categories
35 of simulation methods (Jeong *et al.*, 2008; Marbjerg *et al.*, 2015; Wareing and Hodgson,
36 2005), scholars generally agree with the use of hybrid models, that are expected to provide
37 accurate broadband results exploiting the benefits and minimizing the drawbacks of each
38 approach (Jeong, 2012; Savioja, 2010; Southern *et al.*, 2013).

39 The present study employs a set of large non-trivial geometries to investigate the per-
40 formance of a Finite-Difference Time-Domain (FDTD) model combined with a classical GA
41 algorithm. Four coupled-volume large halls (from 9000 m³ to 25000 m³) were selected as a
42 significant test sample to thoroughly explore the feasibility of the FDTD model in this un-
43 common application field. As assessed in previous studies (Luizard *et al.*, 2013; Wang *et al.*,
44 2019), the accuracy of the results is explored through Bayesian multi-decay analysis (Xiang
45 *et al.*, 2011) and frequency responses for particular source-receiver pairs (Lai and Hamilton,
46 2020; Soares *et al.*, 2022). During the present work, the impulse responses acquired through
47 in-situ measurements were used as a reference point while the same 3D virtual models cali-
48 brated with standard GA techniques were considered by way of comparison. With a special
49 focus on the input data involved (Jeong, 2009; Mondet *et al.*, 2020), the present work also
50 aims to enlarge the benchmark in computational acoustics with further datasets (Hornikx
51 *et al.*, 2015b).

52 This paper is organised as follows. A brief description of FDTD methods and their main
53 features is reported in Section II. In Section III the overall workflow is illustrated: the case
54 studies, the measurements, GA and FDTD simulations' setup. Section IV presents the
55 different input data employed in FDTD and GA calibration processes. Finally, results of
56 specific investigations on the FDTD code's accuracy to detect challenging acoustic scenarios
57 are provided in Section V.

58 **II. FINITE-DIFFERENCE TIME-DOMAIN METHODS**

59 The acoustic field within an enclosure can be described in terms of the scalar field velocity
60 potential $u = u(\mathbf{x}, t)$ through the partial differential equation (PDE):

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right)u = 0 \quad (1)$$

61 where ∇^2 is the 3D Laplacian operator, c is the sound speed in air, assumed as 343 m/s at
62 $T = 20^\circ C$ and relative humidity (RH) at = 50%. The relation between the velocity potential
63 $u = u(\mathbf{x}, t)$, the sound pressure $p = p(\mathbf{x}, t)$, and the vector particle velocity \mathbf{v} is expressed
64 by the equations:

$$p = -\rho \frac{\partial u}{\partial t}, \quad \mathbf{v} = -\nabla u \quad (2)$$

65 where ρ is the density of air, assumed as 1.213 kg/m³ at $T = 20^\circ C$ and RH = 50%. Given
66 the impossibility of finding the exact analytical solution of Equation 1 in large non-trivial
67 geometries, a common feature of all the wave-based methods is the replacement of continuous
68 domains with spatial or spatiotemporal discrete grids.

69 The FDTD methods - which are among the oldest methods to solve PDEs (Courant *et al.*,
70 1928) - approximate the spatial and the temporal derivatives of the differential equations,
71 calculating the numerical solution as a temporal recursion over a space grid (Botteldooren,
72 1995). The solution $u(\mathbf{x}, t)$ of the wave equation, with $\mathbf{x} \in \mathbb{R}^3$, may be approximated by a
73 grid function $u_{l,m,p}^n$ where

$$x = lh \quad y = mh, \quad z = ph, \quad t = nk \quad (3)$$

74 being l, m, n, p integer numbers, h the grid spacing, and k the time step. A group of explicit
 75 FDTD methods follows the same general scheme (Kowalczyk and Van Walstijn, 2010):

$$\delta_t^2 u_{l,m,p}^n = \lambda^2 [(\delta_x^2 + \delta_y^2 + \delta_z^2) + a(\delta_x^2 \delta_y^2 + \delta_x^2 \delta_z^2 + \delta_y^2 \delta_z^2) + b(\delta_x^2 \delta_y^2 \delta_z^2)] u_{l,m,p} \quad (4)$$

76 where λ is the dimensionless quantity defined as the Courant number $\lambda = ck/h$ and a and
 77 b are specific coefficients of each scheme. For instance, the choice of $a = b = 0$ provides the
 78 simplest Cartesian scheme, which is updated at each point $u_{l,m,p}^{n+1}$ with the equation:

$$u_{l,m,p}^{n+1} = (2 - 6\lambda^2)u_{l,m,p}^n + \lambda^2 S_{l,m,p} - u_{l,m,p}^{n-1} \quad (5)$$

79 where $S_{l,m,p} = u_{l+1,m,p}^n + u_{l-1,m,p}^n + u_{l,m+1,p}^n + u_{l,m-1,p}^n + u_{l,m,p+1}^n + u_{l,m,p-1}^n$ is a sum over nearest
 80 neighbours on the Cartesian grid. The fact that the update recursion is parallelisable over
 81 the spatial grid permits the use of parallel computing architectures, such as GPUs or multi-
 82 core CPUs (Webb and Bilbao, 2011).

83 A. Choosing grid spacing and time-step

84 Given the number of points per wavelength (PPW) corresponding to the desired dis-
 85 persion error (1%–2% in the present study (Hamilton, 2016)), the lower the h value, the
 86 higher the upper frequency f_{max} simulated by the FDTD model according to the expression
 87 (Bilbao, 2009):

$$h = \frac{c}{\text{PPW} f_{max}}. \quad (6)$$

88 As a first approximation the computational cost is proportional to h^{-3} , and thus to f^3 per
 89 time step, leading to much more simulation time required for small values of grid spacing

90 and for higher frequencies (Hamilton, 2016). Once h is set, the time-step is set according to
 91 stability considerations. A scheme is defined as *stable* when the solutions of the system do
 92 not grow exponentially and the Fourier transform uniformly converges (Strikwerda, 1989).
 93 The stability condition primarily depends on the Courant number λ , which is limited to:

$$\lambda \leq (\max [1, 2 - 4a, 3 - 12a + 16b])^{-1/2} \quad (7)$$

94 which is the so-called Courant-Friedrichs-Lewy (CLF) condition (Courant *et al.*, 1928) for
 95 this family of schemes. The consequent maximum value of time step in a 3D system is
 96 $k \leq \lambda h/c$, and generally the limit of stability is chosen for efficiency reasons.

97 **B. Boundary conditions**

98 Locally-reactive complex-admittances $Y(\mathbf{x}, s)$ are employed as boundary conditions. The
 99 general expression of the relation between the pressure $p(\mathbf{x}, s)$ and the normal velocity
 100 component at the boundary $\mathbf{n} \cdot \mathbf{v}(\mathbf{x}, s)$ is:

$$Y(\mathbf{x}, s)p(\mathbf{x}, s) = \mathbf{n} \cdot \mathbf{v}(\mathbf{x}, s) \quad (8)$$

101 where s is the usual transform variable (Bilbao *et al.*, 2015). The electrical-acoustical analogy
 102 with a parallel network of LRC circuits is employed as a one-port structure:

$$Y(\mathbf{x}, s) = \sum_{m=1}^M \frac{s}{L^{(m)}(\mathbf{x})s^2 + R^{(m)}(\mathbf{x})s + \frac{1}{C^{(m)}(\mathbf{x})}} \quad (9)$$

103 where $Y(\mathbf{x}, s)$ is the complex admittance, L , R , C are, respectively, the real-valued non-
 104 negative inductance, resistance and capacitance of the circuit, M is the number of different
 105 branches involved in the circuit.

106 **III. METHOD**107 **A. Case studies**

108 Since the aim of the present work is to test the potential of a FDTD model in appli-
 109 cations commonly considered as disadvantageous for wave-based methods, four large-scale
 110 opera houses have been selected as case studies. The significant size associated with those
 111 composite architectures is already a demanding task and time consuming for any 3D wave-
 112 based approach (Webb and Bilbao, 2011). A further challenge is the variety of acoustic
 113 characteristics throughout the audience areas, which is typical of such coupled-volume halls.
 114 Moreover, the fact that the volumetric proportions are completely different in the opera
 115 houses under study allows the analysis of the particular traits of each subcategory of halls
 116 (Hidaka and Beranek, 2000; Prodi *et al.*, 2015). A schematic geometrical representation of

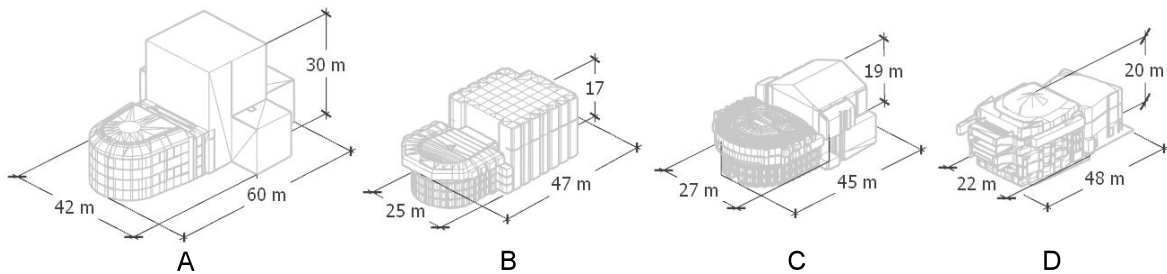


FIG. 1. (Color online) Geometrical representation and main dimensions of the coupled-volume halls under study.

117

118

119 the halls is provided in Figure 1 along with their main dimensions.

120 The first two halls show a relevant disproportion between the volume of the stage tower
 121 and the rest of the theatre affecting the sound energy decays at the listeners with significant

TABLE I. Details of the coupled-volume halls under study. Total volume (V), the main hall volume (V_{hall}), the maximum seating capacity (N), the reverberation time value averaged over 500 - 1000 Hz ($T_{30,M}$), the Schroeder frequency (f_c), the number of sound source (S) and receiver (R) locations, and the coupling factor (k_c) are provided for each hall.

Hall ID	V	V_{hall}	N	$T_{30,M}$	f_c	S	R	k_c
	(m ³)	(m ³)		(s)	(Hz)			
A	25400	5400	1030	1.53	16	4	25	0.35
B	12030	3000	800	1.77	24	2	23	0.25
C	10450	4000	835	1.37	23	2	23	0.13
D	8640	7400	1000	1.32	25	3	18	0.18

122 coupling effects (Garai *et al.*, 2016, 2015). The unusual large-sized fly-tower combined with
123 limited absorbing materials, such as draperies or opera sceneries, contribute to place hall
124 A and hall B in the first category of a specific taxonomy of performance spaces developed
125 for international opera houses by (Hidaka and Beranek, 2000) and adapted for Italian opera
126 houses by (Prodi *et al.*, 2015). Instead, the third hall shows regular proportions between
127 the stage house and the main hall, along with highly absorbing materials in the former
128 volume. These features contribute to rank hall C in the second category of the aforementioned
129 taxonomy. The fourth space, hall D, belongs to the “modern” category (Prodi *et al.*,
130 2015) due the specific design of the early reflection paths exploiting large balconies instead
132 of single boxes (D’Orazio *et al.*, 2020a). The main details about the coupled-volume halls

133 are provided in Table I: the total volume (V), the main hall volume (V_{hall}), the maximum
134 seating capacity (N), the reverberation time value averaged over 500 - 1000 Hz ($T_{30,M}$), the
135 Schroeder frequency (f_c), the number of sound source (S) and receiver (R) locations, and
136 the coupling factor (k_c). With reference to the classical coupled volumes theory (Cremer
137 and Müller, 1978), this last parameter refers to the sound source in the fly tower and the
138 receiver in the main hall.

139 B. Measurements

140 The four performance spaces have been acoustically assessed by means of several mea-
141 surement campaigns allowing the collection of a considerable amount of reference data in
142 terms of acquired impulse responses and objective room criteria (Garai *et al.*, 2016). During
143 the acoustic surveys the opera houses were in unoccupied conditions in compliance with the
144 reference standard ISO 3382-1. Each stage house was equipped with the usual opera scenery
145 and sound absorbing objects, i.e. drapes and curtains, while each orchestra pit was lacking
146 chairs and music stands. The impulse responses for each sound source-receiver pair were
147 acquired using a 5 second exponential sine sweep (ESS) signal. A custom high-SPL dodec-
148 ahedron was adopted as an omnidirectional source (D’Orazio *et al.*, 2016) and four Brüel
149 & Kjær 4190 microphones were employed as monoaural receivers. The locations of sound
150 sources and receivers were chosen following the Ferrara Charter procedure (Pompoli and
151 Prodi, 2000). As can be seen in Figure 2, at least two sound source locations were chosen
152 in each hall: the first below the proscenium arch and the second at the centre of the stage.
153 Where possible, two extra points for the sound source were considered in the orchestra pit, in
154

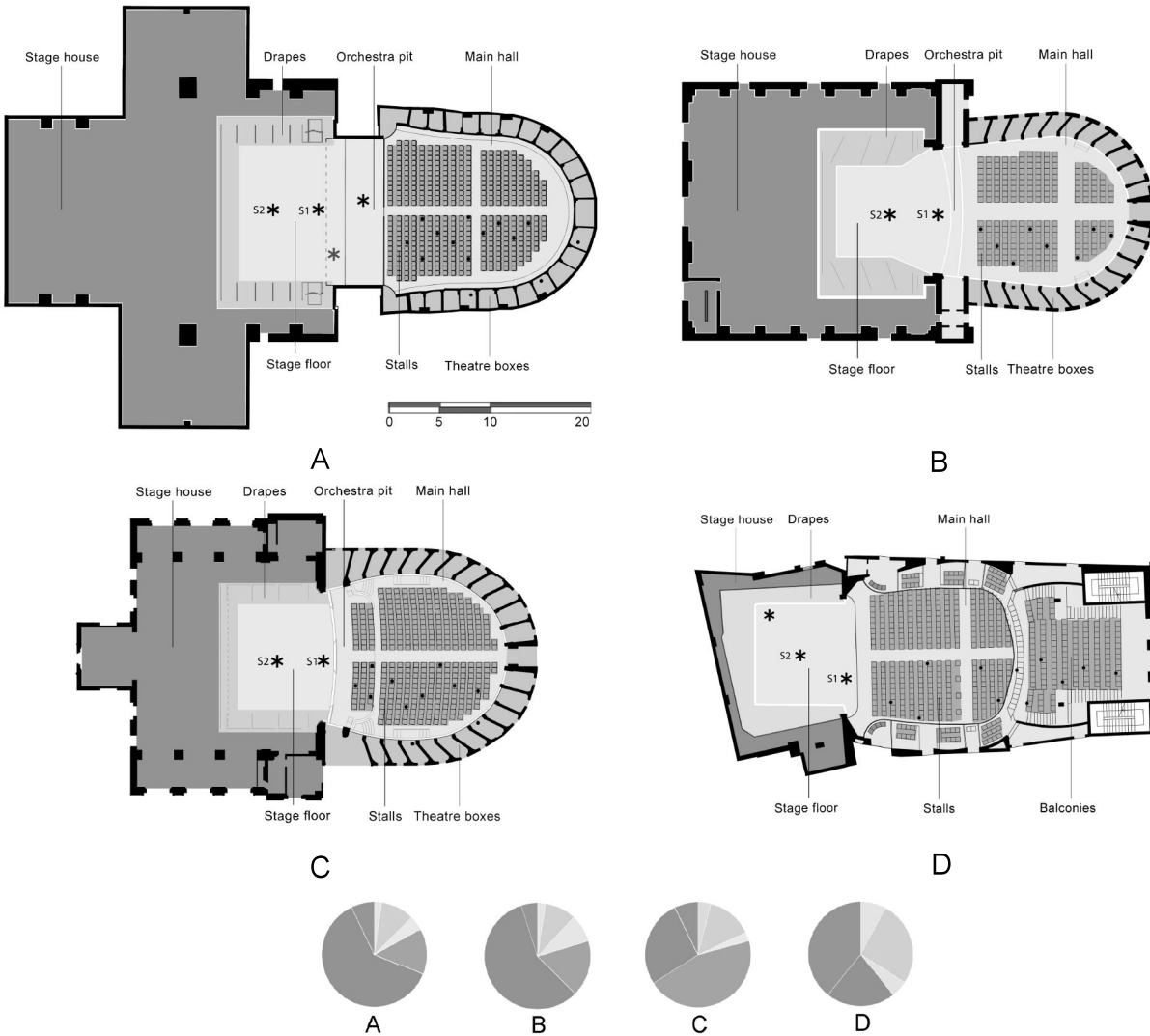


FIG. 2. (Color online) Sound source (*) and receiver (•) measurement plans of the surveyed coupled-volume halls. Pie charts show the proportion of materials labelled in the plans in terms of equivalent absorption area percentage, $\%A_{eq}$ (mean values are taken over 500 Hz and 1000 Hz.)

156 the covered and uncovered parts (see A in Figure 2). Receivers were organized following a
 157 dense mesh of points in one half of the audience areas, exploiting the symmetry of the main
 158 hall. A large number of receiver points were employed during the measurements to better
 159 detect the various acoustic characteristics in stalls, boxes, galleries (or balconies in D) (Sato

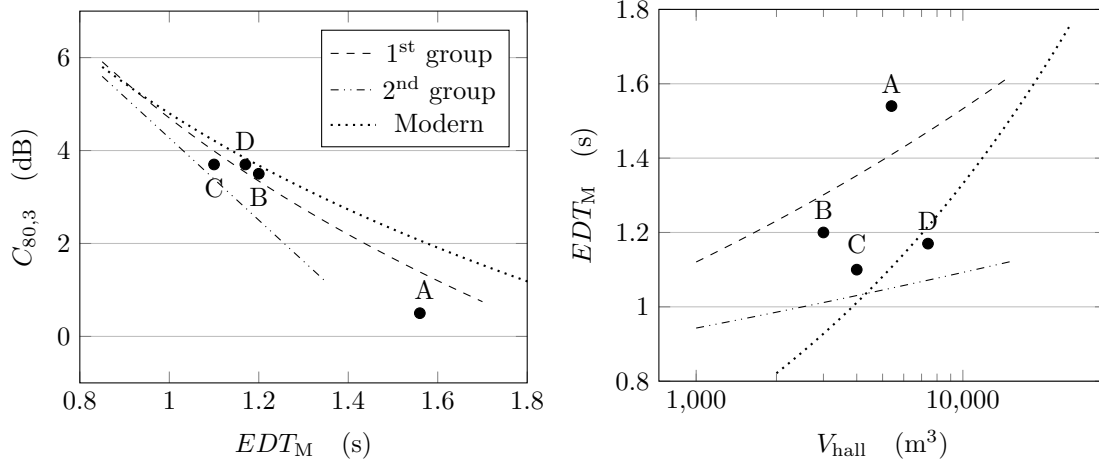


FIG. 3. Measured values of $C_{80,3}$ vs EDT_M (left) and EDT_M vs V_{hall} (right) of the coupled-volume halls under study. $C_{80,3}$ values are averaged over 500 Hz - 1000 Hz - 2000 Hz and EDT_M over 500 Hz - 1000 Hz. The three categories correspond to the taxonomy provided by (Hidaka and Beranek, 2000) for international opera houses and readapted by (Prodi *et al.*, 2015) for Italian opera houses with the additional “modern” category.

160 *et al.*, 2012). Also, in Figure 2 the main materials have been labelled and identified through
 161 different colours. Pie charts at the bottom of the same figure point out the different acoustic
 162 category of the halls according to the distribution of the materials’ equivalent absorption
 163 area, $\%A_{eq}$, whose values have been averaged over 500 Hz and 1000 Hz. Moreover, measured
 164 relationships between C_{80} vs EDT_M , and EDT_M vs V_{hall} confirm the aforementioned ranking
 165 of the halls, as can be seen in Figure 3 (Prodi *et al.*, 2015).

166 C. GA simulation

167 The GA simulation of the opera houses has been carried out with the hybrid ray-based
 168 commercial software Odeon Room Acoustics and according to the state-of-art procedure

169 (Brinkmann *et al.*, 2019; Vorländer, 2020). The 3D virtual models were built with the
170 proper geometry reduction required in room acoustics (Siltanen *et al.*, 2008) using Trimble
171 SketchUp. The transition order (TO) as defined in the Odeon system parameters is the
172 order of reflection threshold determining a switch between the use of the image source
173 method and stochastic ray-tracing for predicting an impulse response. It was set equal to
174 0, as recommended for non-trivial geometries with a large number of surfaces (Christensen,
175 1998; Rindel, 2000). The number of *late rays* was set equal to 100,000 and the maximum
176 reflection order was chosen equal to 2000 to obtain the highest level of accuracy (“Precision”
177 setup). Temperature and relative humidity were set equal to 20 C° and 50 % in all the GA
178 simulations of the present work, as these were the mean thermo-hygrometric conditions
179 measured *in situ* during the measurements.

180 During the modeling process the actual number of materials has been reduced to a smaller
181 group of CAD layers to limit the uncertainty of input data, i.e. the acoustic properties
182 assigned to each surface (Vorländer, 2013). This has been obtained by merging materials
183 with similar acoustic features into equivalent macro-layers, as in the case of the “boxes”
184 and “stage house” materials (D’Orazio *et al.*, 2020b, 2019). T_{30} , EDT , C_{80} , T_S have been
185 used as calibration metrics and, respectively, 10%, 10%, 1 dB, 10 ms have been considered
186 as tolerance ranges between measured and simulated values. The numerical models were
187 tuned on the measurements’ results by evaluating the mean values of each audience area in
188 each octave band from 125 Hz to 4000 Hz. All the virtual models were calibrated through a
189 first assignment of suitable sound absorption and scattering coefficients to the macro-layers
190 (Vorländer, 2020) and the successive iterative adjustments - within reliable ranges - till the

191 achievement of the calibration (Pilch, 2020). Particular sound absorbing characteristics of
192 some materials, such as the “stage grid” layer in the fly tower, were taken from a previous
193 survey (Garai *et al.*, 2015) and from specific studies (Kim *et al.*, 2010).

194 **D. FDTD simulation**

195 The FDTD scheme employed in the present study has been developed by (Bilbao *et al.*,
196 2015; Hamilton *et al.*, 2016). The procedure employed to tune the FDTD models keeping a
198 term of comparison with parallel GA calibrations is presented in Figure 4. The authors chose
199 to maintain the same approximation degree of the 3D geometries suggested for ray-tracing
200 techniques to obtain comparable FDTD and GA results.

201 For higher efficiency in terms of minimizing dispersion errors, the non-Cartesian 13-point
202 stencil cubic close-packed (CCP) scheme with $a = 1/4, b = 0$ and $\lambda = 1$ (see Eq. (4))
203 was used over a face-centered cubic (FCC) subgrid. Practical test on GPU have proven the
204 13-point scheme on the FCC grid to be more suited to large-scale room acoustic simulations
205 rather than, for instance, 27-point schemes on a cubic grid, for equal computational densities
206 (Hamilton and Webb, 2013). Concerning the stability requirement, it has been demonstrated
207 that the maximum Courant number allowed by stability condition (see Equation 7) grants
208 an efficient balance between the desired time cost and the minimization of the dispersion
209 error (Hamilton, 2016). The FDTD scheme was applied up to 4 kHz for the wave-based part
210 of simulated impulse responses. For upper frequencies a classical ray tracing at high-density
211 ($1e9$ rays) was employed, without the inclusion of scattering or diffraction. A value of PPW=
212 6.75 was chosen as points-per-wavelength returning numerical dispersion errors between 1%

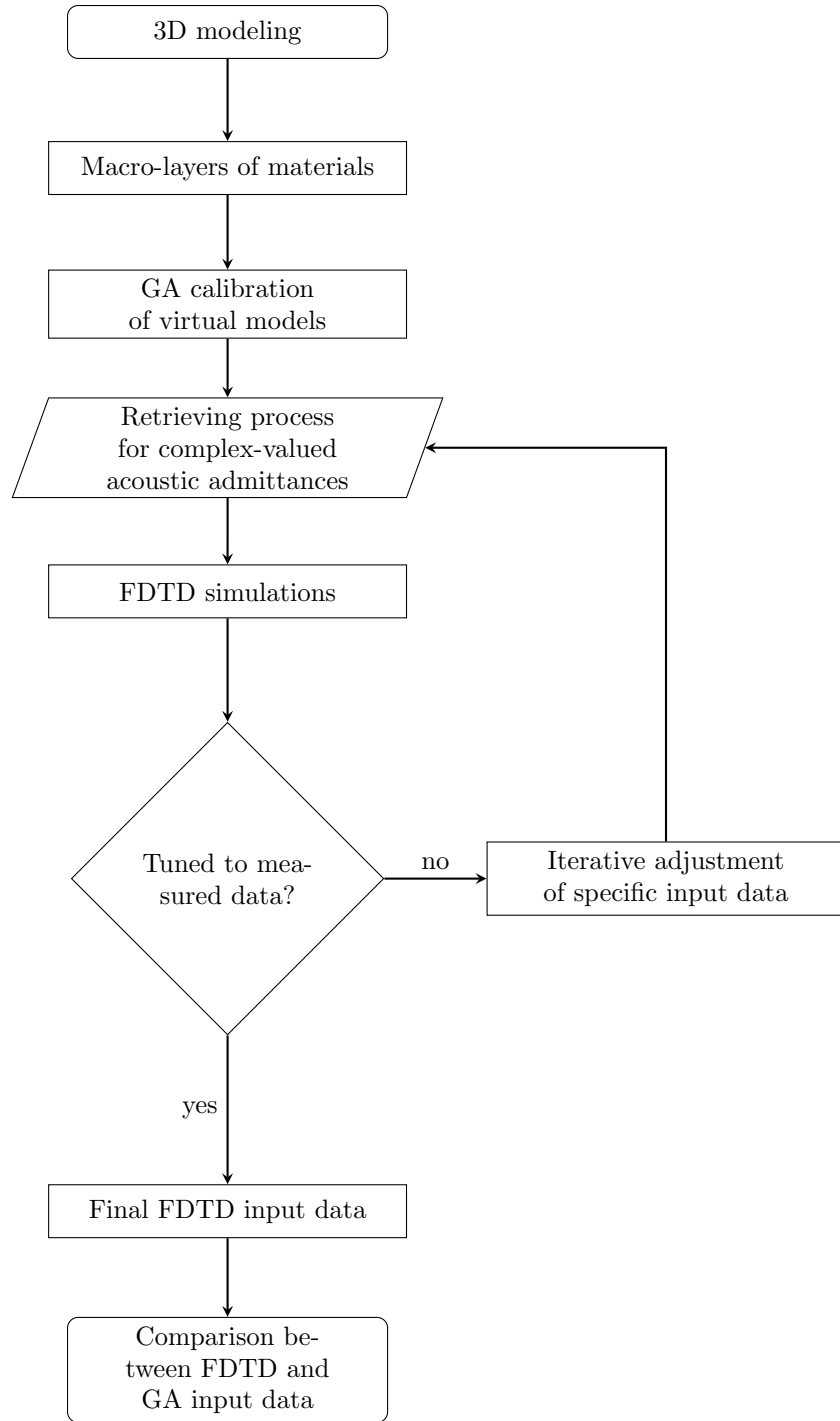


FIG. 4. FDTD calibration process carried out in each of the 3D virtual models, taking the measurements as a reference point and the parallel GA calibration as a term of comparison.

TABLE II. Simulation setup of FDTD simulations. The maximum frequency simulated with FDTD (f_{max}), the points per wavelength (PPW), the oversampling factor (σ), the grid spacing (h), the time step (k), the Courant number (λ), the time cost in terms of hours of runtime per second of computed impulse response (s_{output}), and the dispersion error (in percentage) are provided.

f_{max}	PPW	σ	h	$F_s = 1/k$	λ	Time cost	Dispersion error
(Hz)			(mm)	(Hz)		(hours/ s_{output})	%
4000	6.75	3.375	12.3	27500	$\simeq 1$	1	1%-2%

213 and 2%, with a grid spacing equal to $h \simeq 12.3$ mm and a consequent time step equal to
 215 $k = 0.03$ ms (see Table II). The impulse response length was set equal to 3 s to include even
 216 the higher values of reverberation time at lower frequencies. The computational task has
 217 been parallelized using CUDA over four Nvidia Titan X (2015 Maxwell architecture) GPUs,
 218 achieving 3 hours per simulated impulse response for each sound source, corresponding to 1
 219 hour per second of simulated impulse response.

220 Concerning the air absorption, an implementation has been added to the FDTD scheme
 221 used in the present work to consider viscothermal effects in air (Hamilton, 2021). In fact,
 222 while in small rooms and at low frequencies, that are the usual application fields of wave-
 223 based methods, the absorption of the air may be neglected, in large enclosed spaces it should
 224 be accurately calculated to obtain reliable simulations (Saarelma and Savioja, 2016). Energy
 225 losses caused by the viscosity of air have been introduced into the three-dimensional wave

226 equation, according to reasonable indoor conditions (relative humidity of the air, tempera-
 227 ture and pressure) (Hamilton, 2021).

228 At the boundaries, the well-known issue of staircase effects in regular-grid FDTD schemes
 229 over non-trivial geometries has been overcome through a finite-volume time-domain (FVTD)
 230 approach. Fitted cells allowed for a considerable reduction of effective surface area errors
 231 (from 50% to even 1%), leading to more accurate estimations of decay times and eventually
 232 more consistent simulations (Bilbao *et al.*, 2015).

TABLE III: Summary of FDTD calibrations against mea-
 surements: the sound source is placed at the centre of each
 stage and the receivers are spread throughout the audience
 areas: stalls, boxes and gallery in A, B, C; stalls and bal-
 conies in D. Measured and simulated $T_{30,M}$, EDT_M , $C_{80,M}$,
 $T_{S,M}$, $T_{30,125\text{Hz}}$ room criteria are provided along with the cor-
 responding differences. Subscript “M” indicates mean values
 across the octave bands centered on 500 and 1000 Hz.

Hall Criterion	Receivers											
	Overall			Stalls			Boxes/I balcony			Gallery/II balcony		
	Meas.	Sim.	Diff.	Meas.	Sim.	Diff.	Meas.	Sim.	Diff.	Meas.	Sim.	Diff.
$T_{30,M}$ (s)	1.57	1.56	0.6%	1.53	1.62	5.5%	1.53	1.56	8.1%	1.66	1.51	9.1%
EDT_M (s)	1.54	1.43	7.1%	1.56	1.59	1.7%	1.38	1.33	4.0%	1.69	1.38	18%

	$C_{80,M}$ (dB)	0.1	0.1	0.0	-0.2	-0.2	0.0	1.2	0.3	0.9	-0.8	0.2	1.0
	$T_{S,M}$ (ms)	119	110	9	122	118	4	106	108	2	130	103	27
	$T_{30,125\text{ Hz}}$ (s)	1.86	1.92	3.2%	1.82	1.98	8.8%	1.82	1.91	4.9%	1.92	1.87	2.6%
<hr/>													
	$T_{30,M}$ (s)	1.59	1.60	0.6%	1.65	1.68	1.7%	1.61	1.59	1.2%	1.51	1.53	3.1%
	EDT_M (s)	1.20	1.27	5.8%	1.25	1.34	7.1%	1.04	1.15	10.7%	1.30	1.31	1.4%
B	$C_{80,M}$ (dB)	3.5	3.0	0.5	3.4	2.9	0.5	4.3	3.5	0.8	2.7	2.7	0.0
	$T_{S,M}$ (ms)	75	81	6	75	84	9	70	77	7	79	81	2
	$T_{30,125\text{ Hz}}$ (s)	2.23	2.26	1.4%	2.29	2.31	0.9%	2.21	2.26	2.3%	2.19	2.20	1.4%
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	$T_{30,M}$ (s)	1.39	1.38	0.7%	1.27	1.34	5.1%	1.22	1.27	4.3%	1.68	1.53	8.8%
	EDT_M (s)	1.12	1.17	4.5%	1.15	1.18	2.8%	0.87	1.02	17%	1.35	1.31	2.6%
C	$C_{80,M}$ (dB)	3.7	3.4	0.3	4.9	4.6	0.3	5.5	4.8	0.8	0.8	0.9	0.1
	$T_{S,M}$ (ms)	73	78	5	58	65	7	59	69	10	103	99	4
	$T_{30,125\text{ Hz}}$ (s)	2.07	1.97	4.8%	2.12	2.16	1.9%	1.95	1.80	7.7%	2.15	1.95	9.3%
<hr/>													
	$T_{30,M}$ (s)	1.41	1.44	2.1%	1.40	1.47	4.9%	1.37	1.40	2.3%	1.45	1.45	0.0%
	EDT_M (s)	1.17	1.20	2.6%	1.36	1.42	4.1%	1.02	0.97	5.1%	1.13	1.21	6.6%
D	$C_{80,M}$ (dB)	3.6	3.8	0.2	3.6	3.0	0.6	3.8	4.9	1.1	3.3	3.6	0.3
	$T_{S,M}$ (ms)	77	76	1	75	83	8	73	64	9	82	81	1

$T_{30,125\text{ Hz}}$ (s)	1.96	2.05	4.6%	1.95	2.02	3.6%	1.92	2.03	5.7%	2.01	2.09	4.0%
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233 With reference to the process described in Figure 4, FDTD calibration was achieved when
 234 simulated room criteria converged to the measured ones in all the octave bands of interest
 235 (from 125 Hz to 4000 Hz) considering the sound source at the centre of the stage (Pilch,
 236 2020). Table III summarizes the main calibration results at mid frequencies (500 Hz - 1000
 237 Hz) along with a control index at low frequency ($T_{30,125\text{ Hz}}$) (Hidaka and Beranek, 2000;
 238 Prodi *et al.*, 2015). Measured, simulated and difference values are provided averaged over
 239 all the receiver positions and for each listener area: stalls, boxes and gallery in A, B, C;
 240 stalls, I balcony, II balcony in D. For C_{80} and T_S the Just Noticeable Differences (JNDs)
 241 provided by ISO 3382-1 were assumed as the tolerance ranges (Postma and Katz, 2015),
 242 whilst for T_{30} and EDT twice the JND value (10% of the measured value) were adopted due
 243 to the uncertainty of input data (Vorländer, 2013). At the end of the process, almost all
 244 the differences (94%) between simulated and measured values are smaller than the tolerance
 245 range chosen for each room criterion. Moreover, even considering 1 JND (5%) also for T_{30}
 246 and EDT values, the percentage remains high enough for calibration purposes (85% of the
 247 values) (Alvarez-Morales and Martellotta, 2015).

248 IV. FDTD VS GA DATASETS

249 The present section concerns the material properties employed as boundary conditions in
 250 the two simulation approaches. As already stated in Section III D, the calibration process
 251 was carried out on the same 3D virtual models of the halls to allow a comparison between

252 two distinct results obtained from procedures with a common starting point. However, one
 253 of the main issues in comparing such different input data is the way to convert the energy
 254 parameters employed in GA simulations into non-unique frequency dependent complex sur-
 255 face impedances (Jeong, 2012; Mondet *et al.*, 2020; Rindel, 2011). In the present work,
 256 boundary impedance conditions were derived from the energy parameters employed in GA
 257 calibrations using the electrical-acoustical analogy mentioned in Section II B and thoroughly
 258 described in (Bilbao *et al.*, 2015).

259 With reference to Figure 4, the iterative process of input data’s adjustment for FDTD
 260 calibrations was guided by specific reasons: modifications have been applied to those mate-
 261 rials and those octave bands whose α_{GA} are expected to be mostly affected by uncertainties
 262 (Jeong *et al.*, 2016; Savioja and Svensson, 2015; Vercammen, 2019). The approach of the
 263 authors is in line with the literature according to which prior information about the mate-
 264 rial should be used as a constraint during the non-unique retrieving process from real-valued
 265 absorption coefficients to complex-valued impedances (Mondet *et al.*, 2020). In detail, α_{GA}
 266 uncertainties are expected for the macro-layers corresponding to theatre boxes (D’Orazio
 267 *et al.*, 2019; Prodi *et al.*, 2015; Sato *et al.*, 2012), thin wooden parts with air cavity (Cox
 268 and d’Antonio, 2016; Vorländer, 2020), and seat rows (D’Orazio *et al.*, 2020b; Hidaka and
 269 Beranek, 2000).

271 Figure 5 shows the mean differences in percentage between FDTD and GA datasets at
 272 the end of the FDTD calibrations: $\Delta\alpha$ values refer to the difference $\alpha_{\text{GA}} - \alpha_{\text{FDTD}}$ where
 273 α_{GA} are the energy parameters actually assigned to the surfaces and α_{FDTD} have been re-
 274 converted from the acoustic admittances actually used in the simulation (see also Table IV

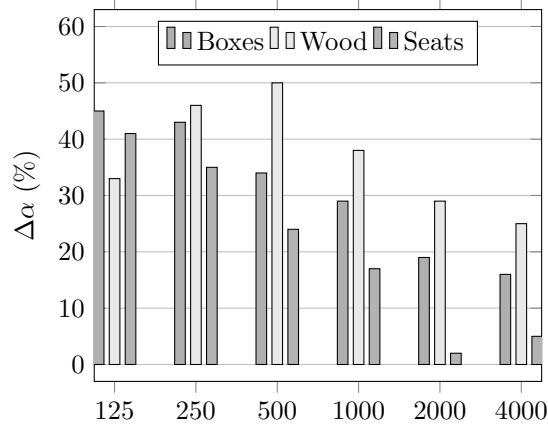


FIG. 5. (Color online) Mean values of percentage differences between FDTD and GA datasets employed as input data for the calibration processes. $\Delta\alpha = \alpha_{GA} - \alpha_{FDTD}$ where α_{GA} are the energy parameters actually assigned to the surfaces and α_{FDTD} have been re-converted from the acoustic admittances actually used in the simulation. Whilst “wood” and “seats” values refer to the halls A, B, C, D; “boxes” values refer only to halls A, B, C, since D has balconies instead of theatre boxes.

275 for a full overview). Values related to theatre “boxes” are with reference to halls A, B, C,
 276 while values related to “wood” and “seats” are with reference to halls A, B, C, D. Figure
 277 5 shows that α_{FDTD} values are actually lower than α_{GA} values for those groups of layers,
 278 with more accentuated differences at low frequencies. It is worth noting that a decrease up
 279 to 50 % may be required from α_{GA} values to obtain α_{FDTD} values. The mean difference
 280 in percentage is 39% in the octave bands centered on 125 Hz, 250 Hz, 500 Hz, whereas it
 281 drops to 20% for the upper octave bands assessed. The discrepancies of input datasets at
 282 low frequencies are consistent with literature (Mondet *et al.*, 2020; Sakamoto *et al.*, 2008),

283 even though the present work involves different methodological approaches and out of the
284 ordinary acoustic scenarios.

285 Since in room acoustic simulations the common datasets of surface materials typically
286 involve energy parameters, the outcomes presented in this section could be useful as a start-
287 ing point for a potential decrease of α_{GA} values for retrieving processes to obtain complex
288 impedances. It may be concluded that the experience of the user and the knowledge of
289 acoustic macro-effects of the rooms under study are still important in the management of
290 input data (Pilch, 2020).

291 V. RESULTS AND DISCUSSIONS

292 The present section illustrates the actual ability of the FDTD code to detect particular
293 acoustic features in distinct audience areas (Jeon *et al.*, 2008). The first set of outcomes
294 shows the Bayesian multi-decay analysis carried out on the simulated impulse responses.
295 The second part of the results presents the comparison of measured, GA and FDTD spectral
296 contents for particularly demanding scenarios: the lack of source-receiver direct sightline and
297 the overhang effects.

299 A. Multi-slope decay in coupled-volume halls

300 Coupled-volume geometries significantly affect the decay-curve shape. This is mainly
301 caused by spatial dependent factors, deriving from non-diffuse transfer of energy between
302 the subvolumes and the strong influence of relative source-receiver position (Summers *et al.*,
303 2004). In opera houses the decays generally show the so-called “cliff-type” characteristics,

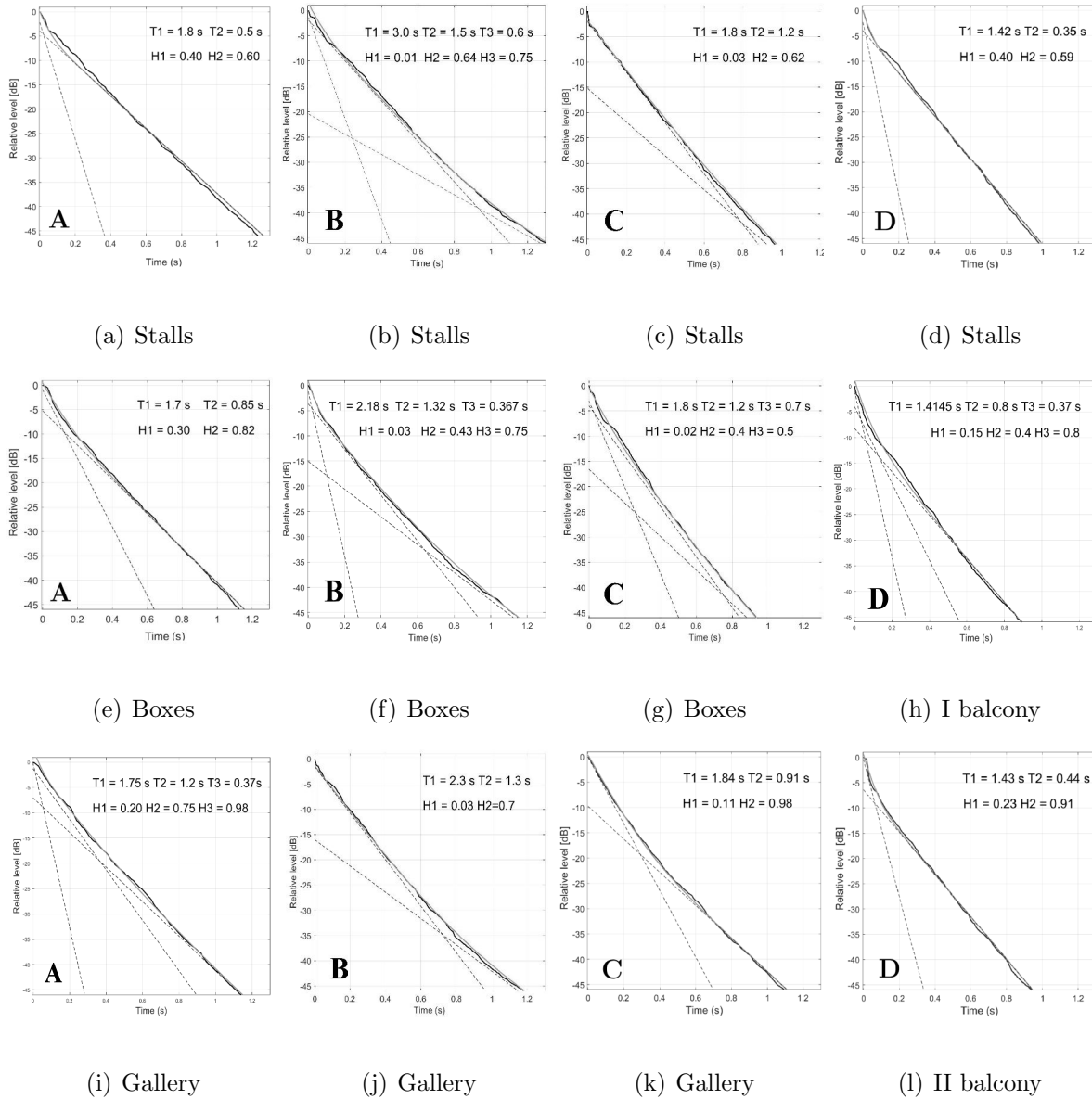


FIG. 6. (Color online) Multi-decay analysis of FDTD simulated IRs (Xiang *et al.*, 2011) at 1000 Hz in all the opera houses (A, B, C, D). The position of the sound source is at the centre of the stage (S2 in Figure 2) and each receiver's position belongs to a specific audience area. The black solid lines are the Schroeder decays derived from the simulated IRs, the dashed lines are the Bayesian decay components, the orange line is the resulting decay model curve. See text for the explanation of the quantities.

304 with EDT values smaller than T_{30} values (Barron, 1995c). Multi-decay analysis performed
 305 on the impulse responses generally allows these effects caused by coupled-volumes to be
 306 revealed (Xiang *et al.*, 2011).

307 In the present section the ability of FDTD to detect those complex acoustic phenomena
 308 depending on the location of the listeners is investigated. Main results are provided in Figure
 309 6 in terms of Bayesian analysis applied to the simulated IRs at 1000 Hz corresponding to
 310 three different source-receiver pairs in each hall. In this kind of analysis, the sound source
 311 is always located at the centre stage position (S2 points in Figure 2) and each receiver
 312 belongs to a distinct audience area (stalls, boxes and gallery in A, B, C; stalls, I balcony
 313 and II balcony in D). The decay parameters provided at top right of each subfigure should
 314 be considered with reference to the following expression:

$$H_S(\mathbf{H}, \mathbf{T}, t_k) = H_0(t_K - t_k) + \sum_{s=1}^S H_s e^{-13.8t_k/T_s} \quad (10)$$

315 where H_S are the Schroeder decay functions, $\mathbf{T} = T_1, T_2, \dots, T_S$ and $\mathbf{H} = H_1, H_2, \dots, H_S$
 316 are the decay parameters (Xiang *et al.*, 2011), S is the number of exponential decay terms
 317 that maximize the likelihood, K is a large number of data points ($1 \leq t_k \leq K$), and the
 318 linear term $H_0(t_K - t_k)$ is the background noise (not provided in Fig. 6). Figure 6 shows
 319 at least two slopes ($S = 2$) in all the source-receiver pairs because the volumes divided
 320 by the proscenium arch, i.e. the fly tower and the main hall, correspond to the two main
 321 sound energy contributions (D’Orazio *et al.*, 2020a; Garai *et al.*, 2016). The number of
 322 slopes may increase up to 3 when the receiver is in the theatre boxes (Figs. 6(f), 6(g)) or
 323 in the overhung seats (Fig. 6(h)), showing a considerably lower EDT compared to T_{30} . As
 324 EDT is more related to perceived reverberance, this effect is in line with the typical acoustic

325 perception in the boxes (Prodi *et al.*, 2015) and in the balconies (Barron, 1995a). In fact,
326 the first steep Bayesian decay component accounts for the weak early reflections caused by
327 the sound absorbing materials that are closer to the receiver (Sato *et al.*, 2012). However,
328 Figure 6(e) shows only two slopes, meaning that in the opera house with the highest value
329 of coupling factor ($k_c = 0.35$ in hall A) the main hall and the fly tower act more as a single
330 acoustic volume from the point of view of a listener in the boxes (D’Orazio *et al.*, 2020b).
331 The number of the slopes tend to decrease in the stalls due to the presence of reflective
332 sidewalls all around the seats (Garai *et al.*, 2015). With reference to Table I, it is important
333 to note that hall A and hall B are affected by stronger coupling effects between the fly tower
334 and the main hall ($k_c > 0.20$) compared to hall C and hall D ($k_c < 0.20$) (D’Orazio *et al.*,
335 2020b). In fact, three slopes are also detected for the receiver in the gallery in hall A (Fig.
336 6(i)) and for the receiver in the stalls in hall B (Fig. 6(b)), confirming the accentuated
337 coupling effects of these two opera houses compared, respectively, to Figs. 6(j), 6(k), 6(l)
338 and Figs. 6(a), 6(c), 6(d).

339 The FDTD model was here validated as an accurate simulation approach able to detect
340 complex coupling effects in different audience areas. In the present section these particular
341 acoustic phenomena are assessed from an energetic-statistic point of view, i.e. Bayesian
342 multi-slope analysis, whereas in the following section they are evaluated in terms of spectral
343 information.

B. Spectral content in challenging scenarios

With the goal to further investigate the performance of FDTD simulations, the frequency response of specific source-receiver pairs is here reported (Hornikx *et al.*, 2015a; Soares *et al.*, 2022; Wang *et al.*, 2019). The spectral analysis of measured and simulated impulse responses is provided to assess the ability of the FDTD simulation framework to return reliable results even in particular acoustic scenarios (Aretz *et al.*, 2009). Figure 7 shows the spectral contribution in two specific situations occurring in hall A and hall D: measured (black), FDTD simulated (orange) frequency responses are shown along with the corresponding condition simulated through GA techniques (green). Each frequency response has been third octave band filtered and reported up to 4000 Hz. All the frequency responses involved have been properly shifted to facilitate the comparison. Figure 7(a) refers to a configuration with no direct sightline in hall A: the sound source in the orchestra pit and the listener in the stalls. Such a source-receiver pair represents a demanding task for any simulation approach because most of the energy contribution at the receiver comes from the scattered sound energy from the edges of the orchestra pit (Barron, 2009; Meyer, 2009). At low frequencies it is possible to notice a considerable gap between the two simulation approaches (more than 10 dB at 125 Hz and 250 Hz). The handling of wave phenomena, such as the edge diffraction caused by the orchestra rail, accentuates the inherent and unavoidable discrepancies between wave-based and ray-based methods (Savioja and Svensson, 2015). Compared to the measures, in Figure 7(a) the overall average Normalized Root Mean Square Error (NRMSE) is equal to 8.6 for GA and 2.8 for FDTD. Figure 7(b) provides the results for the following

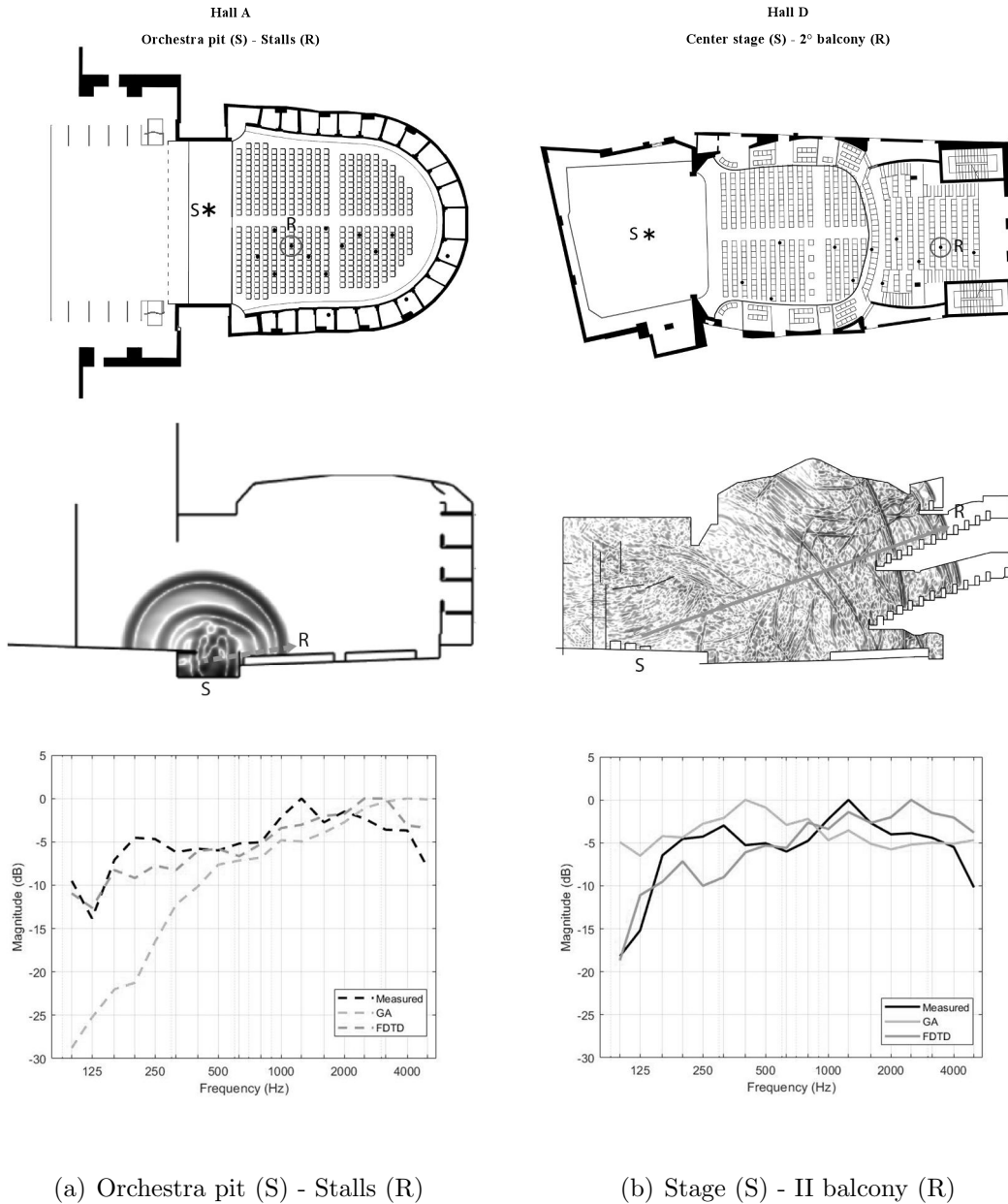


FIG. 7. (Color online) Spectral content of IRs corresponding to two challenging acoustic scenarios: on the left (hall A) no direct sightline (dashed line) between the sound source (S) in the orchestra pit and the receiver (R) in the stalls; on the right (hall D) balcony overhangs with the sound source (S) on the stage and the receiver (R) in the II balcony. Comparison among frequency responses derived from measured, FDTD simulated and GA simulated IRs is provided in third octave bands.

366 configuration in hall D: the sound source at the centre of the stage and the listener in the
367 overhangs of the second balcony. As hall D is a performance space with weaker coupling
368 effects, acoustic simulations are generally expected to provide more accurate results in terms
369 of acoustical parameters for each audience area (Summers *et al.*, 2004). In fact, it is possible
370 to observe that the similarity between the spectral information of the measured IRs and the
371 simulated IRs with FDTD and GA methods in Figure 7(b) is generally higher than Figure
372 7(a) due to the direct sightline between the sound source and the receiver ($\text{NRMSE} < 5$ for
373 both the methods). However, some discrepancies among the frequency responses are still
374 present. For instance, the gap between measured and FDTD at 250 Hz reflects the difficulty
375 of computing the unpredictable level variations in overhung seats at low frequencies, which
376 are strongly affected by grazing incidence (Barron, 1995b). Concerning measured and GA
377 values, greater deviations are visible at 125 Hz (more than 12 dB). In Figure 7(b) the overall
378 average NRMSE is 4.7 for GA and 3.3 for FDTD.

380 In order to further check and validate the reliability of the FDTD code up to 4000 Hz, the
381 match between the traditional IR room acoustic parameters derived from measurements and
382 from FDTD simulations are provided for the same two source-receiver pairs. Figure 8 shows
383 the comparison between measured (black markers) and FDTD simulated (white markers)
384 T_{30} , EDT , C_{80} , T_S from 125 Hz to 4000 Hz: 75% of the differences between measured and
385 simulated values lie within the tolerance ranges (error bars) chosen for each room criterion.

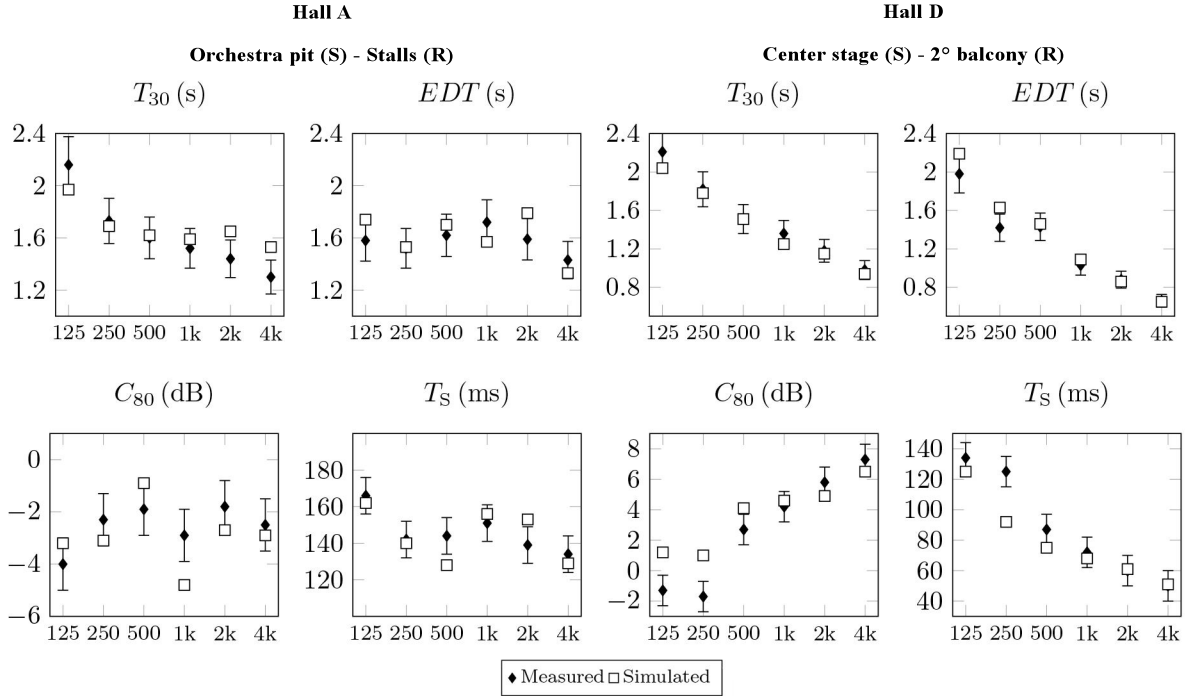


FIG. 8. Measured and FDTD simulated T_{30} , EDT , C_{80} , T_S values for the same source-receiver pairs assessed in Figure 7. The tolerance ranges selected for each parameter are also shown: 10 % for T_{30} , 10 % for EDT ; 1 dB for C_{80} , 10 ms for T_S .

386 CONCLUSIONS

387 In conclusion, the main concerns about using FDTD methods in non-trivial large envi-
 388 ronments have been primarily caused by high computational cost and the implementation
 389 of frequency-dependent wall impedances. Such issues, that represented significant obstacles
 390 up to the most recent decades, have been increasingly overcome in the last years. In the
 391 present work, current opportunities in FDTD large-scale room acoustics applications have
 392 been exploited, with a special focus on the input data assigned to the boundaries and the
 393 simulation of acoustic coupling phenomena.

394 The FDTD model chosen for this study was tested in four different large-scale coupled-
395 volume halls for wider frequency ranges compared to the usual wave-based applications. The
396 FDTD calibration of the 3D virtual models was developed keeping the parallel standard GA
397 procedure by way of comparison. At the end of the calibration processes, the analysis of
398 input datasets used in the two simulation approaches has been provided in term of differences
399 between α_{GA} and the equivalent α_{FDTD} derived from complex-valued impedances actually
400 employed in FDTD calibrations. The outcomes highlight a significant overestimation of α_{GA}
401 compared to α_{FDTD} with accentuated discrepancies at low frequencies, in line with previous
402 findings.

403 Moreover, the accuracy of the results and the reliability of the overall process have been
404 validated by means of Bayesian multi-decay analysis and spectral contents' assessment for
405 challenging source-receiver pairs. The former insight confirms the ability of the wave-based
406 approach to detect specific acoustic coupling phenomena, such as the presence of 3 decay
407 components in theatre boxes. These outcomes are consistent with the engaged literature on
408 opera houses and the typical acoustic traits in each audience area. The latter analysis proves
409 the spectral information to be reasonably well modeled by the FDTD model up to 4 kHz, even
410 in the case of source-receiver with no direct sightline or in case of strong grazing incidence
411 in overhung seats in the balconies. Materials employed for calibrating the coupled-volume
412 virtual opera houses - including 3D models, measurements' results, boundary conditions -
413 are freely available in online repositories (Fratoni, a,b).

414 **ACKNOWLEDGMENTS**

415 The authors would like to thank Anna Rovigatti for having patiently helped to calibrate
 416 GA models.

Material	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
A						
Marble	0.02	0.02	0.03	0.03	0.04	0.05
Plaster	0.09	0.09	0.09	0.10	0.10	0.10
Carpet	0.04	0.05	0.06	0.08	0.16	0.20
Both						
Curtains	0.10	0.16	0.50	0.65	0.73	0.73
Stage house	0.25	0.25	0.25	0.25	0.20	0.20
Stage grid	0.35	0.45	0.50	0.65	0.65	0.65
Seats	0.35	0.45	0.55	0.60	0.60	0.60
FDTD						
Boxes	0.08	0.12	0.14	0.15	0.15	0.15
Wooden stage	0.20	0.15	0.10	0.10	0.10	0.10
GA						
Boxes	0.16	0.25	0.29	0.29	0.30	0.30
Wooden stage	0.30	0.28	0.20	0.16	0.14	0.05

B

Marble	0.01	0.01	0.01	0.02	0.02	0.02
--------	------	------	------	------	------	------

	Plaster	0.03	0.03	0.04	0.04	0.04	0.04
	Wooden linings	0.22	0.12	0.10	0.08	0.06	0.07
	Curtains	0.05	0.10	0.11	0.18	0.30	0.35
	Stage house	0.05	0.03	0.03	0.03	0.03	0.03
	Stage grid	0.25	0.35	0.55	0.55	0.55	0.55
	Wooden stage	0.18	0.12	0.10	0.09	0.08	0.07
<hr/>							
FDTD	Boxes	0.15	0.14	0.14	0.12	0.16	0.18
	Seats	0.20	0.30	0.40	0.50	0.52	0.58
<hr/>							
GA	Boxes	0.22	0.30	0.30	0.25	0.22	0.22
	Seats	0.40	0.50	0.58	0.61	0.58	0.58
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C							
	Marble	0.01	0.01	0.02	0.02	0.02	0.02
	Plaster	0.02	0.02	0.03	0.03	0.03	0.03
	Curtains	0.15	0.35	0.55	0.77	0.70	0.60
	Stage house	0.12	0.14	0.15	0.15	0.16	0.16
	Stage grid	0.15	0.15	0.25	0.40	0.55	0.55
	Wooden ceiling	0.18	0.12	0.10	0.09	0.08	0.07
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FDTD	Boxes	0.09	0.18	0.30	0.35	0.38	0.38
	Seats	0.20	0.38	0.55	0.65	0.80	0.75
GA	Boxes	0.20	0.22	0.30	0.40	0.40	0.40
	Seats	0.40	0.55	0.70	0.80	0.80	0.70
D							
	Marble	0.01	0.01	0.02	0.02	0.03	0.03
	Plaster	0.01	0.03	0.03	0.04	0.06	0.06
	Curtains	0.20	0.35	0.35	0.45	0.50	0.50
	Stage house	0.12	0.15	0.15	0.18	0.18	0.18
	Stage grid	0.15	0.15	0.25	0.40	0.50	0.50
	Carpet	0.04	0.07	0.15	0.20	0.25	0.30
FDTD	Seats (stalls)	0.35	0.32	0.40	0.50	0.60	0.65
	Seats (balconies)	0.48	0.60	0.50	0.60	0.52	0.62
GA	Seats (stalls)	0.48	0.50	0.50	0.58	0.58	0.60
	Seats (balconies)	0.52	0.60	0.78	0.88	0.70	0.65

TABLE IV: Main material properties employed in the simulation of hall A, B, C, D. Energy parameters have been fitted through RLC circuit analogy. See text for details.

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