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# On Prices' Cyclical Behaviour in Oligopolistic Markets\*

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## Abstract

We revisit the discussion about the relationship between price's cyclical features, implicit collusion and the demand level in an oligopoly supergame where a positive shock may hit demand and disrupt collusion. The novel feature of our model consists in characterising the post-shock noncooperative price and comparing it against the cartel price played in the last period of the collusive path, to single out the conditions for procyclicality to arise both in the short and in the long-run. This poses an issue in terms of an antitrust agency's ability to draw well defined conclusions on the firms' behaviour after the occurrence of the shock, with particular reference for the litigation phase after a cartel breakdown.

**JEL Codes:** C73, E60, L13

**Keywords:** demand shocks, cyclical pricing, tacit collusion

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# 1 Introduction

Rotemberg and Saloner (1986, R&S henceforth) argue that oligopolies are likely to behave more competitively when demand rises, especially under price competition and product homogeneity. Hence, pricing behaviour exhibits a countercyclical pattern.<sup>1</sup> We modify their framework in three directions: (i) the presence of product differentiation;<sup>2</sup> (ii) the possibility for firms to collude, after the occurrence of the demand shock, on virtually any price between the monopoly price and the Nash equilibrium one; and (iii) the role of the state of demand before the shock hits the industry, and the size of the shock itself. We show that in this framework the traditional R&S result no longer holds, and procyclical pricing may emerge depending on the state of demand before the shock hits, and the size of the demand shock. We do so under Bertrand and Cournot behaviour, comparing the collusive price under two alternative demand states (“low” and “high”) with the Nash price charged after the occurrence of a positive demand shock affecting the high demand state and triggering the output’s cyclical movement. This specific aspect of our model relies on Tirole’s (1988) exposition of R&S and has the same flavour of an analogous assumption in Spiegel and Stahl (2014).

Our results show that in such a framework the traditional countercyclical result may indeed flip over depending on the interplay between the size of the shock, the demand level observed in the last collusive period and the market variable being set by firms. More precisely, the larger the shock, the higher the tendency towards the emergence of a procyclical pattern. This happens because the natural countercyclical tendency due to competitive behaviour

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<sup>1</sup>The nature of the shock has been shaped in different ways to produce a variety of results not entirely in line with those in R&S. See Haltiwanger and Harrington (1991), Bagwell and Staiger (1997), Fabra (2006) and Knittel and Lepore (2010).

<sup>2</sup>The impact of product differentiation on collusive stability has been intensively analysed, from Deneckere (1983) to Lambertini (1997), but systematically under certainty. The only notable exception is Raith (1996), where, however, a Hotelling model is used.

is offset by the larger market size, thereby increasing the Nash equilibrium price. At the same time, as the difference between the pre-shock demand states shrinks, countercyclicality arises, as the model takes an increasingly R&S-like flavour.

In this respect, a few additional remarks are in order. As noted by Tirole (1988, p. 250), if one compares monopoly price in the low demand state with the Nash price in the high state, it is indeed possible that the price is higher during booms, whereby what R&S indeed refer to is the underlying amount of collusion (or lack thereof). Consequently, the same is necessarily true for infinitely many partially collusive prices. Without explicitly modelling partial collusion, our results imply that, if post-shock noncooperative pricing is procyclical, then by continuity any post-shock collusive price which could be sustained by firms is necessarily procyclical as well. In such a case, the disruption of collusion gives rise to a new price path whose procyclical pattern may make it very difficult (if not altogether impossible) for an antitrust agency to detect whether firms are colluding anew in high demand states or not, as the increase in price might be simply due to the shock. This has some implications about the interpretation of price increases during litigation, as a way of inducing underestimation of damages (cf. Harrington, 2004).

The remainder of paper is structured as follows. Section 2 lays out the model. Sections 3 and 4 illustrate the Bertrand and Cournot supergames, respectively. Section 5 summarizes our results by carrying out a comprehensive analysis of price cyclicity in the post-shock period only, while section 6 looks at the long-run cyclical properties of pricing. Section 7 concludes.

## 2 The model

As in R&S, we consider an oligopoly with  $n \geq 2$  single-product firms, over discrete time  $t = 0, 1, 2, \dots \infty$ . Demand functions include product differentiation as in Spence (1976) and Singh and Vives (1984), and firms share the

same technology summarised by the cost function  $C_i(q_i) = cq_i$ , where  $c$  is the constant marginal cost and  $q_i$  is firm  $i$ 's output.

Firms aim at maximising collective profits. This requires a stability condition defined by a critical threshold of the common discount factor  $\delta \in (0, 1]$ , which may be identified through several versions of the folk theorem, e.g., Friedman (1971), based on grim trigger strategies, or Abreu (1986), relying on optimal punishments. As will become quickly clear, the exact nature of the punishment is immaterial to our argument.

As in Tirole (1988, pp. 248-50), the state of demand  $\sigma_t$  is stochastic, and in each period can take one of two values,  $a > b > c$ , with probabilities  $\mathbf{p}(a) = m$  and  $\mathbf{p}(b) = 1 - m$ ,  $m \in [0, 1]$ . Firms set the market variable after observing the demand state. If a favourable permanent shock  $\varepsilon > 0$  occurs at time  $\tau$  in state  $a$  (leaving  $b$  unaffected), the high demand state becomes  $\hat{a} = a + \varepsilon > a > b$  and full collusion is stable iff  $\delta > E\delta^*(\varepsilon) \in (0, 1)$ , this threshold being higher than  $E\delta^*(a) \forall \varepsilon > 0$ . As shown in R&S,  $\partial\delta^*(\varepsilon)/\partial\varepsilon > 0$ . This property illustrates the traditional interpretation of the R&S model, whereby we should observe countercyclical pricing.

This is where we depart from the R&S approach, as this property intuitively holds if products are homogeneous and firms are price-setters, the Nash equilibrium involving marginal cost pricing. If some degree of differentiation is present, the Nash equilibrium price will be above marginal cost and below the monopoly price firms would set were they able to sustain full collusion in state  $\hat{a}$ , but not necessarily below the monopoly price corresponding to state  $a$ , let alone  $b$ . Thus, the cyclical properties of price must be assessed comparing the price charged after the occurrence of the shock with the price charged before the occurrence of the demand shock disrupting collusion.

We will outline the conditions for the noncooperative price pattern to be procyclical, both in Bertrand and in Cournot, under the condition  $\delta \in (0, E\delta^*(\varepsilon))$ , whereby firms are cannot collude in the best state. In doing so, we will single out (i) the *impact effect* of the shock, which requires evaluating

the Nash price in the period in which collusion is disrupted against the collusive price in the last period of the cartel path, and (ii) the *trend effect*, by assessing the average Nash price in the continuation of the supergame up to doomsday against the average price along the collusive path, both calculated using probabilities  $m$  and  $1 - m$ .

### 3 Bertrand behaviour

If firms are price setters, the relevant direct demand function for firm  $i$  is

$$q_i = \frac{\hat{a}}{1 + s(n-1)} - \frac{p_i [1 + s(n-2)] - s \sum_{j \neq i} p_j}{(1-s)[1 + s(n-1)]} \quad (1)$$

where  $s \in (0, 1]$  measures the degree of substitutability between any two varieties (i.e., is an inverse measure of differentiation). If firms are unable to collude on the monopoly frontier, they play the non cooperative Nash equilibrium price of the stage game:

$$p^{BN}(\varepsilon) = c + \frac{(1-s)(a + \varepsilon - c)}{2 + s(n-3)} \quad (2)$$

This must be compared to the cartel prices the same firms have practiced along the collusive path up to  $\tau-1$ , i.e., either  $p^M(a) = (a + c)/2$  or  $p^M(b) = (b + c)/2$ . The minimalistic condition for a procyclical price behaviour to arise after the shock is the following. If the state at  $\tau-1$  has been  $b$ , reverting to the Nash equilibrium price yields nonetheless an increase in price provided  $p^{BN}(\varepsilon) > p^M(b)$ , i.e.,

$$\frac{2(a + \varepsilon)(1-s) + c(n-1)s - b[2 + s(n-3)]}{2[2 + s(n-3)]} > 0 \quad (3)$$

i.e., for all

$$\varepsilon > \max \left\{ 0, \frac{(b-c)[2 + s(n-3)] - 2(1-s)(a-c)}{2(1-s)} \right\} \quad (4)$$

with  $[(b - c)(2 + s(n - 3)) - 2(1 - s)(a - c)] / [2(1 - s)] > 0$  for all

$$b \in \left( b_B \equiv \frac{2a(1 - s) + c(n - 1)s}{2 + s(n - 3)}, a \right) \quad (5)$$

The foregoing analysis boils down to the following:

**Lemma 1** *If the state of demand was  $b$  at  $\tau - 1$  and becomes  $a + \varepsilon$  at  $\tau$ , the reversion to Bertrand-Nash pricing implies a procyclical price pattern for all*

$$\varepsilon > \max \left\{ 0, \frac{b[2 + s(n - 3)] - 2a(1 - s) - c(n - 1)s}{2(1 - s)} \right\}.$$

*In the remainder of the parameter space, the price pattern is countercyclical.*

It is worth noting that Lemma 1 entails that if  $b$  is sufficiently lower than  $a$ , then any positive shock affecting the high demand state produces a procyclical price behaviour, *the switch from collusive to Bertrand-Nash pricing notwithstanding*.

Now we may look at the case in which at  $\tau - 1$  firms were colluding in correspondence of the high demand state  $a$ . The relevant comparison is therefore between the same Bertrand-Nash equilibrium price  $p^{BN}(\varepsilon)$  and the cartel price  $p^M(a)$ , with

$$p^{BN}(\varepsilon) - p^M(a) = \frac{2\varepsilon(1 - s) - (a - c)(n - 1)s}{2[2 + s(n - 3)]} > 0 \quad (6)$$

for all

$$\varepsilon > \frac{(a - c)(n - 1)s}{2(1 - s)} > 0 \forall s \in (0, 1) \quad (7)$$

This implies:

**Lemma 2** *If the state of demand was  $a$  at  $\tau - 1$  and becomes  $a + \varepsilon$  at  $\tau$ , the reversion to Bertrand-Nash pricing implies a procyclical price pattern for all  $\varepsilon > (a - c)(n - 1)s / [2(1 - s)]$ . The opposite holds for all  $\varepsilon \in (0, (a - c)(n - 1)s / [2(1 - s)])$ .*



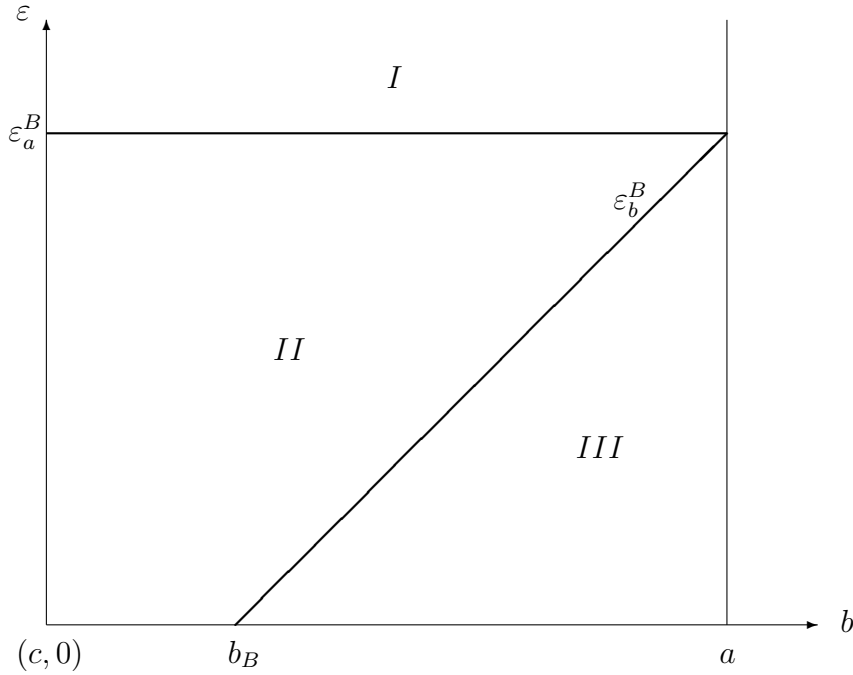
It is easily checked that the condition appearing in Lemma 2 is more demanding than that identified by Lemma 1. To do so, define

$$\varepsilon_b^B \equiv \frac{b[2 + s(n-3)] - 2a(1-s) - c(n-1)s}{2(1-s)}; \varepsilon_a^B \equiv \frac{(a-c)(n-1)s}{2(1-s)} \quad (8)$$

where superscript  $B$  stands for Bertrand, while the meaning of subscript  $a, b$  refers to the demand state in the last stage of the collusive path. Then, observe that  $\varepsilon_a^B - \varepsilon_b^B > 0$  everywhere. Accordingly, we may claim:

**Proposition 3** *The condition  $\varepsilon > \varepsilon_a^B$  suffices to ensure that the reversion to Bertrand-Nash behaviour involves a procyclical price pattern irrespective of the demand state realised in the last period of the cartel path.*

**Figure 1** Critical shock levels in the space  $(b, \varepsilon)$ .



Obviously,  $\varepsilon_a^B$  is independent of  $b$ . The graph of  $\{\varepsilon_a^B, \varepsilon_b^B\}$  in the space  $(b, \varepsilon)$  is drawn in Figure 1. In area *I*, in which  $\varepsilon > \varepsilon_a^B$ , procyclical Bertrand-Nash pricing obtains at  $\tau$  for all  $b \in (c, a)$ , irrespective of whether the state at  $\tau - 1$  was  $a$  or  $b$ . In area *II*, in which  $\varepsilon \in (\varepsilon_b^B, \varepsilon_a^B)$  procyclicality is observed at  $\tau$  only if the state of demand at  $\tau - 1$  was  $b$ . Finally, in area *III*, the pricing behaviour at the Bertrand-Nash equilibrium is countercyclical.

The intuition behind Proposition 3 is straightforward, as the necessary and sufficient condition for observing a price increase when collusion breaks down is more demanding if the state of demand was high in the last period of the collusive path.

## 4 Cournot behaviour

Here, the inverse demand for each product variety  $i$  as soon as the shock takes place is

$$p_i = \hat{a} - q_i - s \sum_{j \neq i} q_j \quad (9)$$

so that the resulting FOC writes as follows:

$$\frac{\partial \pi_i}{\partial p_i} = a + \varepsilon - c - 2q_i - s \sum_{j \neq i} q_j = 0 \quad (10)$$

The symmetric Cournot-Nash output  $q^{CN}(\varepsilon) = (a + \varepsilon - c) / [2 + (n - 1)s]$  gives rise to the following equilibrium price:

$$p^{CN}(\varepsilon) = c + \frac{a + \varepsilon - c}{2 + (n - 1)s} \quad (11)$$

As in the Bertrand case, one has to compare (11) with both collusive prices prevailing either in state  $b$  or  $a$  before the occurrence of the shock. Taking  $p^M(b)$ , one finds

$$p^{CN}(\varepsilon) > p^M(b) \Leftrightarrow \frac{2(a + \varepsilon - b) - (b - c)(n - 1)s}{2[2 + s(n - 1)]} > 0, \quad (12)$$

satisfied by all

$$\varepsilon > \max \left\{ 0, \frac{b[2 + s(n-1)] - 2a - c(n-1)s}{2} \right\} \quad (13)$$

with  $[b(2 + s(n-1)) - 2a - c(n-1)s]/2 > 0$  for all

$$b \in \left( b_C \equiv \frac{2a + c(n-1)s}{2 + s(n-1)}, a \right) \quad (14)$$

Accordingly, we may formulate:

**Lemma 4** *If the state of demand was  $b$  at  $\tau - 1$  and becomes  $a + \varepsilon$  at  $\tau$ , the reversion to the Cournot-Nash equilibrium implies a procyclical price pattern for all*

$$\varepsilon > \max \left\{ 0, \frac{b[2 + s(n-1)] - 2a - c(n-1)s}{2} \right\}.$$

*In the remainder of the parameter space, the price pattern is countercyclical.*

Now suppose firms were colluding in state  $a$  in the last period of the collusive path. In this case,  $p^{CN}(\varepsilon) > p^M(a)$  for all  $\varepsilon > (a - c)(n - 1)s/2$ . This implies:

**Lemma 5** *If the state of demand was  $a$  at  $\tau - 1$  and becomes  $a + \varepsilon$  at  $\tau$ , the reversion to the Cournot-Nash equilibrium implies a procyclical price pattern for all  $\varepsilon > (a - c)(n - 1)s/2$ . The opposite holds for all  $\varepsilon \in (0, (a - c)(n - 1)s/2)$ .*

Now, after defining

$$\varepsilon_b^C \equiv \frac{b[2 + s(n-1)] - 2a - c(n-1)s}{2}; \varepsilon_a^C \equiv \frac{(a - c)(n - 1)s}{2} \quad (15)$$

it is easily checked that  $\varepsilon_a^C - \varepsilon_b^C > 0$  everywhere. The resulting picture is qualitatively equivalent to Figure 1. Consequently, we may formulate the following:

**Proposition 6** *The condition  $\varepsilon > \varepsilon_a^C$  suffices to ensure that the reversion to Cournot-Nash behaviour involves a procyclical price pattern irrespective of the demand state realised in the last period of the cartel path.*

The intuition here is analogous to the Bertrand case. We may now put together the two scenarios to take a general look at the cyclical pricing behaviour after the occurrence of the shock.

## 5 A comprehensive look at price cyclicity

From (8) and (15), there emerges that  $\varepsilon_b^B > \varepsilon_b^C$  and  $\varepsilon_a^B > \varepsilon_a^C$  in the whole parameter space, while

$$\text{sign} \{ \varepsilon_b^B - \varepsilon_a^C \} = \text{sign} \left\{ \frac{(a-b)[2+s(n-3)] - (a-c)(n-1)s^2}{2(1-s)} \right\} \quad (16)$$

so that  $\varepsilon_b^B > \varepsilon_a^C$  for all

$$b \in \left( b_\varepsilon \equiv \frac{a(1-s)[2+s(n-1)] + c(n-1)s^2}{2+s(n-3)}, a \right) \quad (17)$$

The foregoing argument produces:

**Proposition 7**  $\varepsilon_a^B > \varepsilon_b^B > \varepsilon_a^C > \varepsilon_b^C$  for all  $b \in (b_\varepsilon, a)$ , while  $\varepsilon_a^B > \varepsilon_a^C > \varepsilon_b^B > \varepsilon_b^C$  for all  $b \in (c, b_\varepsilon)$ .

Then,  $b_\varepsilon$  can be compared with  $b_B$  and  $b_C$  appearing, respectively, in (5) and (14), to check that  $b_C \geq b_B$  and  $b_\varepsilon \geq b_B$  for all  $s \in [0, 1]$ , while

$$b_\varepsilon - b_C = \frac{(a-c)(n-1)s[(n-1)s^2 - (n-5)s - 2]}{[2+s(n-1)][2+s(n-3)]} \quad (18)$$

which is positive for all  $s \in \left( (n-5 + \sqrt{n^2 - 2n + 17}) / [2(n-1)], 1 \right]$  and negative otherwise. Hence, we have proved

**Proposition 8**  $b_\varepsilon \in (b_C, a)$  for all  $s \in \left( (n-5 + \sqrt{n^2 - 2n + 17}) / [2(n-1)], 1 \right]$ , while  $b_\varepsilon \in (c, b_C]$  for all  $s \in \left( 0, (n-5 + \sqrt{n^2 - 2n + 17}) / [2(n-1)] \right)$ .

The critical expressions of the shock,  $\{\varepsilon_a^B, \varepsilon_b^B, \varepsilon_a^C, \varepsilon_b^C\}$ , can be drawn in the space  $(b, \varepsilon)$ , as in Figure 2, in which  $b_\varepsilon > b_C > b_B$  since we have posed  $s \in ((n - 5 + \sqrt{n^2 - 2n + 17}) / [2(n - 1)], 1]$ .

The properties we are about to spell out in the following theorem are independent of the range of  $s$  being considered, as well as the exact sequence of  $\{b_B, b_C, b_\varepsilon\}$ .

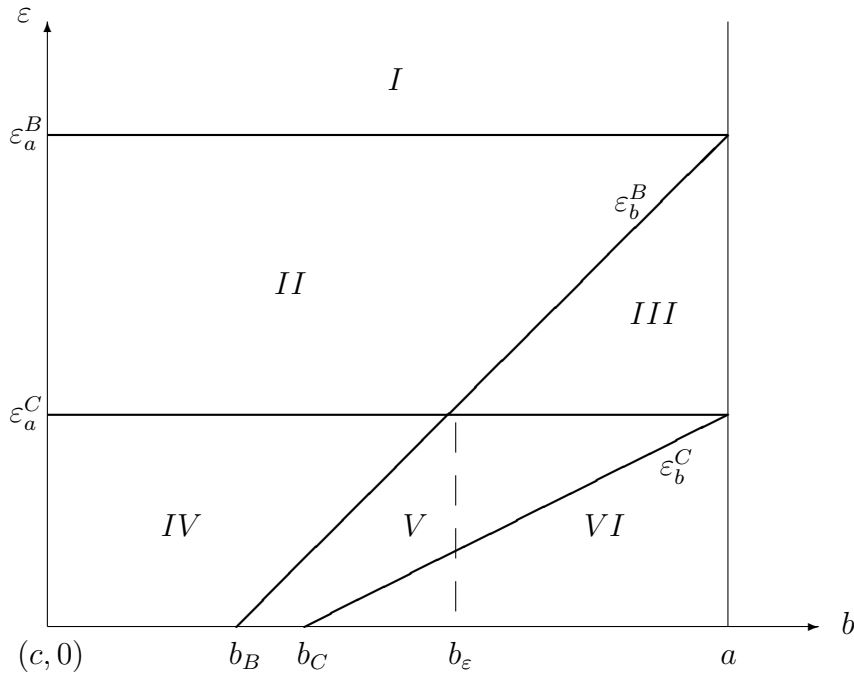
**Theorem 9** *The cyclical properties of non-cooperative Nash pricing at the Bertrand or Cournot equilibrium emerging after the abandonment of the collusive path in period  $\tau$  are the following:*

- *In area I, procyclicality is observed for all  $b \in (c, a)$ , under both Bertrand and Cournot behaviour.*
- *In area II, countercyclicality emerges under Bertrand behaviour if the state of demand was  $a$  at  $\tau - 1$ ; otherwise, pricing is procyclical.*
- *In area III, pricing is procyclical (resp., countercyclical) under Cournot (resp., Bertrand) behaviour, for any given state of demand at  $\tau$ .*
- *In area IV, pricing is procyclical (resp., countercyclical) under both Cournot and Bertrand behaviour, if the state of demand at  $\tau$  was  $b$  (resp.,  $a$ ).*
- *In area V, pricing is procyclical under Cournot behaviour if the state of demand at  $\tau$  was  $b$ ; otherwise, it is countercyclical.*
- *In area VI, countercyclicality is observed for all  $b \in (b_C, a)$ , under both Bertrand and Cournot behaviour.*

The above Theorem summarizes our main results. We may provide a solid intuition by looking at Figure 2. For any given state of demand in the collusive stage, as the size of the shock breaking down the cartel path increases (that is, we move along an imaginary vertical line starting at any

point on the horizontal axis) the likelihood of observing procyclical pricing behaviour increases. This is crystal clear if, for instance, we draw the imaginary vertical line close to  $a$ , along the upper limit of the demand space: in such a case, as  $\varepsilon$  increases, we move from complete countercyclicality (area  $VI$ ) to countercyclicality only under price competition (area  $III$ ) and then to complete procyclicality (area  $I$ ).<sup>3</sup>

**Figure 2** Critical shock levels in the space  $(b, \varepsilon)$ .



The same pattern can be observed if we start from any point on the segment  $(0, a)$ . In other words, as the size of the positive demand shock disrupting collusion increases, the probability of observing procyclicality increases,

<sup>3</sup>The fact that in area  $III$  countercyclicality is preserved only under Bertrand behaviour is easily understood, as price competition is intrinsically more intense than quantity competition, *all else equal*.

as the tendency towards price reduction brought about by non-cooperative behaviour is increasingly offset by the higher post-shock state of demand firms face. By the same token, if we draw an imaginary horizontal line starting from any point on the vertical axis lower than  $\varepsilon_a^B$  (that is, we increase the pre-shock state of demand for any given shock size), we observe a strengthening of the countercyclical pattern. Also in this case the intuition is solid: as the pre-shock low state of demand tends to coincide with the high one, we lose one of the departures from the traditional Rotemberg and Saloner framework (i.e. the alternative demand states before the shock) and therefore we are back to their standard countercyclical result.

## 6 The price trend

The foregoing discussion has focussed on the impact of the demand shock on the noncooperative price prevailing in the single period following the end of the cartel path, as compared to the collusive (monopoly) price characterising the last period of such path. Here we propose an analysis of the noncooperative price trend over  $[\tau, \infty)$  against the cartel price over  $[0, \tau - 1]$ . To this aim, we rely on probabilities  $m$  and  $1 - m$ , associated with states  $a$  (as well as  $a + \varepsilon$ ) and  $b$ , respectively, to construct the following expressions:

$$p_m^C = mp^M(a) + (1 - m)p^M(b); p_m^{KN} = mp^{KN}(\varepsilon) + (1 - m)p^{KN}(b) \quad (19)$$

which measure the trend prices under collusion and fully noncooperative behaviour, respectively. The subscript indicates that these prices are weighted averages calculated using probability  $m$ . In (19),  $p^{KN}(\varepsilon)$  coincides, alternatively, with (2) or (11) and  $K = B, C$  depending on the market variable being used by firms. Moreover,

$$p^{BN}(b) = \frac{b(1 - s) + c[1 + s(n - 2)]}{2 + s(n - 3)}; p^{CN}(b) = \frac{b + c[1 + s(n - 1)]}{2 + s(n - 1)} \quad (20)$$

Our exercise is based on the comparison between the average prices  $p_m^{KN}$  and  $p_m^C$  under price- and quantity-setting behaviour:

$$p_m^{BN} - p_m^C = \frac{2m(1-s)\varepsilon - s(n-1)[ma + (1-m)b - c]}{2[2 + s(n-3)]} \equiv \tau_B \quad (21)$$

$$p_m^{CN} - p_m^C = \frac{2m\varepsilon - s(n-1)[ma + (1-m)b - c]}{2[2 + s(n-1)]} \equiv \tau_C \quad (22)$$

with

$$\tau_B > 0 \Leftrightarrow \varepsilon > \frac{s(n-1)[ma + (1-m)b - c]}{2m(1-s)} \equiv \varepsilon_{\tau B} \quad (23)$$

$$\tau_C > 0 \Leftrightarrow \varepsilon > \frac{s(n-1)[ma + (1-m)b - c]}{2m} \equiv \varepsilon_{\tau C} \quad (24)$$

and obviously  $\varepsilon_{\tau B} > \varepsilon_{\tau C}$  for all  $s \in (0, 1]$ . It is also easily ascertained that  $\varepsilon_{\tau B}$  and  $\varepsilon_{\tau C}$  are both increasing in  $b$ , and so is their difference  $\varepsilon_{\tau B} - \varepsilon_{\tau C}$ . These two thresholds can be inserted in the graph appearing in Figure 2, to generate Figure 3, which has been drawn assuming, as before,  $s \in ((n-5 + \sqrt{n^2 - 2n + 17}) / [2(n-1)], 1]$ .

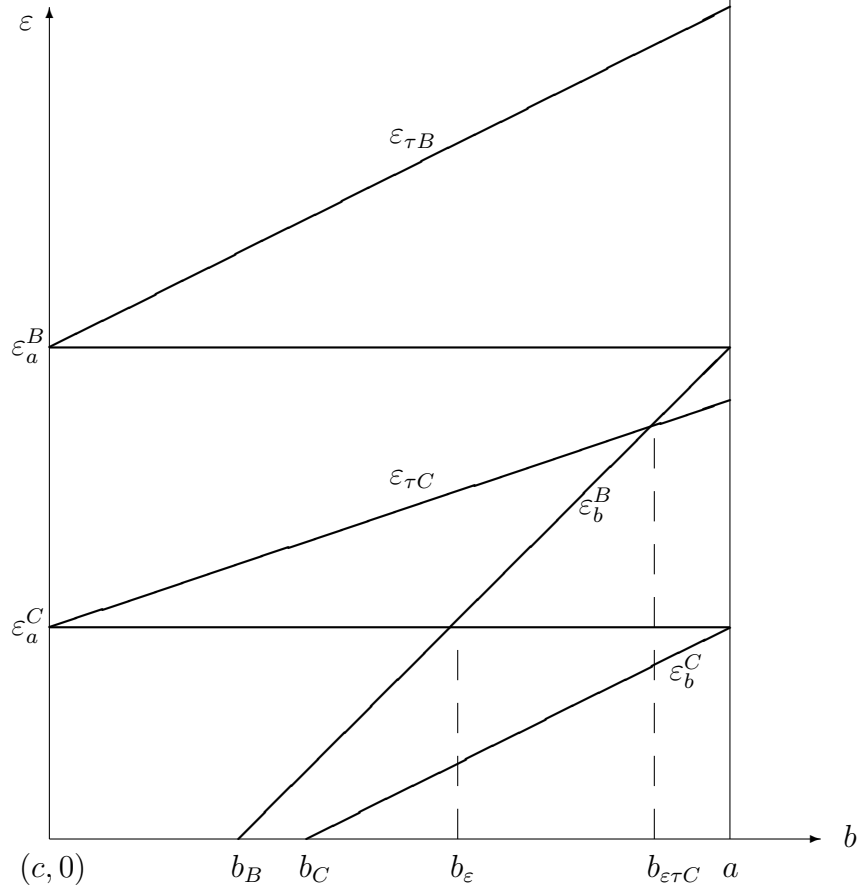
Above  $\varepsilon_{\tau C}$  (resp.,  $\varepsilon_{\tau B}$ ), the price trend arising after firms' reversion to Nash pricing is procyclical under quantity-setting (resp., price-setting) behaviour. Since  $\varepsilon_{\tau B} > \varepsilon_{\tau C}$  for all  $s \in (0, 1]$ , we may formulate our final result:

**Proposition 10** *For all degrees of product substitutability, the region of the space  $(b, \varepsilon)$  wherein the average Nash price after the shock is procyclical under Bertrand behaviour is a subset of the region where it is procyclical under Cournot behaviour.*

That is, the necessary and sufficient condition for the procyclicity of Bertrand-Nash pricing is sufficient to ensure the arising of procyclicity under Cournot-Nash pricing (but not vice versa).



**Figure 3** Impact effect and price trend in the space  $(b, \varepsilon)$ .



Proposition 10 has an additional relevant implication: if the trend of the Nash equilibrium pricing pattern is procyclical, then there will exist infinitely many degrees of partial collusion in prices or quantities which will generate procyclical collusive price trends even if full collusion along the frontier of industry profits cannot be sustained. This consideration alone, which intuitively holds without requiring a formal proof, reveals:

**Corollary 11** *If the Nash equilibrium price trend is procyclical, then any  $\delta \in$*

*$(0, E\delta^*(\varepsilon))$  allows firms to achieve a degree of partial collusion whose price trend is procyclical as well, as it lies everywhere above the Nash equilibrium trend itself.*

This also conveys a message of some interest to antitrust agencies. Consider the case where, after the shock disrupting a collusive path on the frontier of monopoly profits, firms generate the price trend  $p_m^{KN}$  described above (or an even higher one, due to the presence of some degree of partial collusion), using either prices or quantities. If  $\varepsilon > \varepsilon_{\tau B}$ , the observed price path is procyclical irrespective of the market variable being set. Now, if an antitrust authority observes that path, what shall it conclude? One of two things: either firms are playing noncooperatively (in the favourable state), and the price increase is to be entirely imputed to the occurrence of a large shock; or, they are colluding to some extent but any proof of this may well be out of reach. The possibility of having prices above the noncooperative level after a cartel breakdown has already been highlighted in Harrington (2004) in connection with the incentive for firms to adopt such a strategy as they are aware that post-collusion prices are commonly used in the process of calculating damages, in the litigation phase (Finkelstein and Levenbach, 1983). In terms of our analysis, an increase in price after the dissolution of a cartel may not be a strategic reaction trying to affect the calculation of damages.

## 7 Concluding remarks

We have taken a new look at the pricing behaviour of oligopolistic firms when a positive demand shock occurs, breaking implicit collusion. Our analysis has shown that when the traditional R&S setting is extended to explicitly account for the effect of the shock itself on the post-shock noncooperative equilibrium price, the standard countercyclical price result does not necessarily hold in general. The size of the cyclical fluctuation and the pre-shock demand state, along with the degree of product differentiation and the size of the market,

play a key role in shaping prices' response to the cycle. If the demand shock is large enough, or if it hits a relatively low state of demand, a procyclical pattern is likely to emerge. As our analysis look at Bertrand and Cournot Nash equilibria after the shock, our procyclical results extend to any degree of partial collusion firms might be able to sustain on the basis of their time preferences, up to the new monopoly frontier.

In addition to the antitrust implications of procyclical pricing, a more comprehensive analysis of pricing cyclical properties can have relevant implications for other dimensions of economic analysis. Procyclical prices - with marginal cost being constant or not more procyclical than prices - imply procyclical mark-ups, with crucial consequences on the size of the government spending multipliers, as shown by Hall (2009), Christiano *et al.* (2011) and Woodford (2011). In this respect, further research on the topic is required.

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