An Integer Linear Programming model for integrated train stop planning and timetabling with time-dependent passenger demand

Published Version:
Qi J., Cacchiani V., Yang L., Zhang C., Di Z. (2021). An Integer Linear Programming model for integrated train stop planning and timetabling with time-dependent passenger demand. COMPUTERS & OPERATIONS RESEARCH, 136, 1-19 [10.1016/j.cor.2021.105484].

Availability:
This version is available at: https://hdl.handle.net/11585/861144 since: 2024-02-23

Published:
DOI: http://doi.org/10.1016/j.cor.2021.105484

(Article begins on next page)

An Integer Linear Programming model for integrated train stop planning and timetabling with time-dependent passenger demand

Jianguo Qi[1], Valentina Cacchiani[2*], Lixing Yang[1*], Chuntian Zhang[1], Zhen Di[1,3]

[1] *State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing, 100044, China*

[2] *DEI, University of Bologna, Viale Risorgimento 2, I-40136 Bologna, Italy*

[3] *College of Transportation and Logistics, East China Jiaotong University, Nanchang 330013, China*

Abstract

We consider the problem of jointly optimizing train stop plan and timetable for a set of trains on a railway corridor, with focus on time-dependent passenger demand: these are very relevant elements in railway operations, and closely related to the service quality provided to the passengers. The train stop planning problem requires to determine the stopping pattern for each train, while the train timetabling problem calls for finding departure and arrival times of each train at each visited station. The solution of both problems determines the train services available to the passengers, with the corresponding stops, the departure and arrival times, and the total travel time. To define train services that are convenient for the passengers, it is important to satisfy the passenger demand: in particular, we take into consideration the *desired departure time intervals* of passengers from their origin station towards their destination station. Our goal is to determine a solution to the integrated problem that includes these three aspects (stop plan, timetable, time-dependent passenger demand). The integrated problem is solved at the tactical level, and includes constraints imposing that the demand is satisfied in the desired time intervals, and the train capacity is respected. The solution of the problem defines the train services, by determining the passenger flow on trains and the time intervals in which passengers have to be served.

We formulate an Integer Linear Programming (ILP) model for this integrated problem, and solve it with the commercial optimization solver CPLEX on a real-world instance of the Wuhan-Guangzhou high-speed railway line in China, by considering different time interval lengths in which the demand must be satisfied. In addition, we compare the obtained results with those determined by solving a model that neglects the time-dependency of the passenger demand. The computational results show that the train stop plan and timetable are deeply influenced by the desired departure time intervals of the passengers, and that neglecting time-dependency results in more than 30% of the passengers departing in a different time interval than the desired one.

Keywords: Joint optimization model; Train timetable; Train stop plan; Time-dependent passenger demand

1 Introduction

As one of the most important transportation modes on the land, railway transportation, especially the high-speed railway, has shown great increase tendency to provide transport services to the passengers.

* *Corresponding authors: E-mail addresses: valentina.cacchiani@unibo.it (V. Cacchiani); lxyang@bjtu.edu.cn (L. Yang)*

As an example, in China the total length of railway lines had already reached to 127,000 km by the end of 2017, 25,000 km of which are high-speed railway lines. With the expansion of the railway network and the increase of the number of passengers transported by trains, railway companies are faced with more challenges to provide higher quality of service to the passengers. One of the key elements for delivering a good service consists in determining a *train timetable* that is highly satisfactory for the passengers: this requires not only to respect all safety infrastructural constraints (such as minimum headway times between trains), but also to explicitly consider the *origin-destination (OD) passenger demand* in order to ensure higher frequency at the most popular stations, and adequate train capacity to perform all passenger trips. In addition, each passenger wants to travel in a *specific time interval* of the day according to her trip purpose, and might decide to choose an alternative transportation means if the train schedule does not meet her preference. Moreover, the travel time from origin to destination plays a very important role in the passenger choice. A way to keep the travel time short is to appropriately plan the *train stopping patterns* (i.e., to define at which stations each train should stop), by avoiding unnecessary stops: indeed, a minimum dwelling time is usually needed for the train to complete the necessary operations when it is scheduled to stop at a station (Yang et al., 2016), and this causes a travel time increase. Thus, there are three relevant issues that have to be considered to define train services that are convenient for the passengers: the choice of the train stopping patterns, the definition of the corresponding timetables, and the satisfaction of time-dependent passenger demand.

The first problem is known as Train Stop Planning. Clearly, this problem is closely related to passenger demand, since more trains should stop at the most attractive stations, while stops could be avoided where they are not necessary. Furthermore, the train stops should be planned according to the desired passenger departure times. Therefore, the second problem, called the Train Timetabling problem (that requires to define, for each train, departure and arrival times at each visited station) should be solved in combination with the former and by taking into consideration time-dependent passenger demand in order to determine better solutions.

In this paper, we focus on the problem of jointly optimizing these three problems in an integrated way, i.e., we solve the *Train Timetabling and Stop Planning problems with Time-dependent passenger Demand* (TTSP-TD) for a high-speed line in China. We observe that this problem is solved at the tactical level, i.e., several months before operations. In the considered real-world application, passengers generally make long distance trips without transfers, and aim at reaching their destination in the shortest possible time, thus our goal is the minimization of the total train travel time. In addition, the passenger desired departure time is particularly relevant for a long-distance service, in which the number of trains for each OD pair of stations is limited, hence we consider time-dependent demand. Note that we do not need to know the demand for each time instant (e.g., minute) of the time horizon, but rather for time intervals: indeed, for long-distance services, passengers have a desired departure time interval (e.g., between 8 and 9 in the morning) but usually they do not require a specific time instant (e.g. 8:05). Since the studied problem is solved at the tactical level, it can be hard to know the passenger desired departure times in a very precise way: more likely, if the passengers have to provide their preference some months in advance (e.g., through a survey), we can get information about a preferred interval during which they would like to travel (instead of a specific time instant). After the timetable is published, the passengers will select their departure times based on the available choices. Therefore, we do not require that the demand is known in a very precise way, but accounting for the preferred time intervals, instead of considering only the global daily demand between each origin-destination pair, allows to provide a better service for the passengers. In the TTSP-TD, we assume that the origin and destination stations of the each train line have been previously defined, while the train stops and timetables have to be determined, and solve the problem at tactical level. By solving the TTSP-TD, the stop plans and timetables will be determined by taking into account the passenger perspective through the time-dependent demand satisfaction. Then, the computed timetables will become public to the passengers that, clearly, will select their favorite travel options based on the available train services.

The interest on integration between stop planning, timetabling and passenger demand is evident from the many recent works appeared in the literature (see Section 1.1 for a detailed analysis) that combine these elements. Most of these works handle together two out of these three components. There are works that jointly optimize timetabling and stop planning but disregard demand or address it in an approximate way, articles that explicitly take demand into account together with the definition of the stopping patterns or with the computation of the timetables. Very recently, some works have successfully addressed the three problems at once (sometimes by introducing some simplifications such as daily demand instead of time-dependent demand or predefined stopping patterns). We aim at contributing in this research stream by tackling the fully integrated problem, for which we propose an Integer Linear Programming Model and investigate the impact of the length of the passenger desired time intervals on the solution quality and computing time.

By solving the TTSP-TD, we also directly determine how many tickets should be allocated to each train and OD pair. Indeed, based on the train timetable and stop plan that we obtain by solving TTSP-TD (that take passenger demand and flow on trains into account), we know between which OD stations and at what time each train service is available and the corresponding used train capacity. In other words, since trains can only provide service at the stations where they stop and the train timetable specifies the departure and arrival times of each train at each station, the distribution of tickets on trains is a byproduct of TTSP-TD. In this work, we do not consider different travel classes, i.e., we do not distinguish between different types of passengers (e.g., first class or economy class), but define the number of tickets (seats) allocated on each train for every OD pair. Ticket distribution clearly influences the quality of service to the passengers because they can only travel according to the tickets available from the railway company, that usually specify the train number, the seat number, and the specific departure time from the origin station and arrival time at the destination. Unreasonable distribution of the tickets may lead to many unsold tickets or, in contrast, to unsatisfied passenger demand (Qi et al., 2018c).

Aiming at generating a practical operation plan, we propose an Integer Linear Programming (ILP) model to optimize the TTSP-TD in which train timetable, stop plan and time-dependent passenger demand are determined in a fully integrated way. In Section 1.1, we give an overview of the related literature, and in Section 1.2 we list the contributions of this work. Section 2 presents the description of the TTSP-TD problem, that is formulated as an ILP model in Section 3. In Section 4 we report the obtained computational results for a small-scale example and for the Wuhan-Guangzhou high-speed railway corridor. Finally, we conclude with remarks and future work in Section 5.

1.1 Literature review

Due to the importance of the train timetabling and stop planning problems in the railway operations, a huge amount of works related to these topics have appeared in the literature during the past decades (Schöbel, 2012; Lusby et al., 2011; Borndörfer et al., 2018). In this section, we review only the most closely related works, namely those that integrate train timetabling with stop planning or consider one or both of these problems while taking passenger demand into account. As it will be evident from the literature analysis, most of the works dealing with passenger demand focus only on one of the two aspects (timetabling or stop planning), since the aspect of passenger demand significantly increases the problem complexity, especially if the demand is time-dependent.

1.1.1 Timetabling and stop planning

The integration of these two problems has received significant attention, due to the close connection between these problems, and the improvements that can be achieved by combining them (Cao et al., 2014; Niu et al., 2015; Yang et al., 2016; Yue et al., 2016; Jiang et al., 2017; Gao et al., 2018). Generally, due to the complicated connection between the train stop plan and timetable, most works did not consider a completely integrated problem, but rather combined the train timetabling problem with the choice of a set of potential stopping patterns (especially the skip-stopping patterns, that allow to skip some stops that reducing the travel time). Cao et al. (2014) proposed a 0-1 Integer Programming model for skip-stop operation strategy with the purpose of minimizing the waiting and trip times of all passengers and the train travel times. They developed a Tabu Search algorithm to find the optimized solution. Jiang et al. (2017) studied the train timetabling problem with the option for some trains to skip some stops on highly congested railway line: an ILP model was formulated by using a time-space graph representation of the problem, and a Lagrangian-relaxation based algorithm was developed for solving the formulated model on instances of the Beijing-Shanghai high-speed line. Gao et al. (2018) studied the problem of scheduling additional trains on a railway corridor, with the aim of minimizing the timetable deviation for the existing trains and the travel time for the additional trains, and by considering the stop plans of the exiting trains as not fixed. A bi-objective Mixed Integer Linear Programming (MILP) formulation and a three-stage optimization model were proposed and tested on instances of the Hangzhou-Ningbo-Wenzhou high-speed rail corridor.

Recently, some works have focused on finding the optimized stop plan and timetable with a more general set of stopping patterns, which usually allow for better performances on meeting heterogeneous passenger demand. In order to obtain a systematic optimized train stop plan and timetable on a tactical level, Yang et al. (2016) proposed a MILP model under the condition that the station-based passenger demand should be serviced. The commercial software GAMS with CPLEX solver was applied to solve the model on instances of the Beijing-Shanghai high-speed line. Yue et al. (2016) also investigated the problem of simultaneously optimizing the train timetable and stop plan on high-speed railway corridors. With the purpose of maximizing the sum of the train profits, an ILP model was formulated, in which the penalization of stopping times and number of stops was included, and a column-generation-based algorithm was applied to solve the proposed model. We also mention that, in the studies proposed by Yue et al. (2016) and Jiang et al. (2017), passenger demand is not explicitly considered. Instead, some related constraints are imposed, respectively, on the minimum number of trains performing trips between pairs of stations and on the maximum number of stops that can be skipped. Similarly, Yang et al. (2016) imposed the sum of the estimated capacity of trains stopping at each station to be larger than the corresponding passenger demand without tracking the detailed passenger distributions on each train, since they studied the collaborative optimization of train stop planning and timetabling problems on the tactical level.

1.1.2 Stop planning and/or Timetabling with passenger demand

Both train stop planning and timetabling problems are deeply influenced by passenger demand. Since passengers can only access trains at the stopping stations, the passenger demand and its distribution on trains is a key decision for developing an effective train stop plan. In addition, passenger demand and distribution on trains should be taken into consideration in the train timetabling problem with the purpose of minimizing the passenger desired departure times from their origin stations and their travel times. The interrelation of stop planning and timetabling with passenger demand is evident from the large amount of works that have recently appeared in the literature.

Stop planning with passenger demand Considering the close connection between the train stop plan and passenger travel choice, a bi-level programming model was proposed by Deng et al. (2009), with a passenger flow assignment model as the lower level model, and was solved by a simulated annealing algorithm. By considering different classifications of stations and trains, a Mixed-Integer Programming model and a two-stage optimization model were respectively formulated by Fu and Nie (2011) and Fu et al. (2012) for the train stop planning problem with the purpose of obtaining high-quality train service between the stations with higher classification and penalizing additional stops, while satisfying OD passenger demand requirements. Based on the formulation proposed by Fu et al. (2012), Fu et al. (2013) further improved the model by limiting the set of paths that passengers of each OD pair can choose, based on the ratio between the passenger travel distance and the train trip distance, and on allowing transfers only at a subset of stations. By establishing the linear connection constraints between passenger distribution on trains and train stop selection, Qi et al. (2018c) proposed a Mixed Integer Linear Programming (MILP) model to simultaneously optimize the choice, for each train, of its operation zone (defined by the origin and terminal stations), the train stop plan and passenger distribution to satisfy the given OD passenger demand. A bi-level optimization model was proposed by Parbo et al. (2018): the upper level problem consists in the skip-stop pattern choices, while the lower level is a route choice model in which passengers are routed according to the selected stopping patterns. A heuristic method was developed for solving this problem and tested on instances of the suburban railway network in the Copenhagen Region. In Shang et al. (2018), the skip-stop pattern was used as a way to derive an equity-oriented schedule in terms of passenger waiting times at stations. The problem was formulated as a multi-commodity flow problem on a space-time-state network, in which the state defines the equity level, and solved in a Lagrangian relaxation framework for instances of the Beijing subway network.

Timetabling with passenger demand Although the described studies take the passenger distribution into consideration in the process of optimizing train stop plan, the timetable optimization is usually omitted or considered in a relatively rough way. However, the interaction between timetabling and passenger requirements plays an important role in the process of making railway operation plans, in order to enhance the attractiveness of railway transportation and, consequently, many works have recently studied train timetabling combined with passenger demand. An interesting survey on the literature that considers the passengers perspective was published by Parbo et al. (2016). Most works that integrate timetabling and passenger demand neglect the train stop plan optimization (i.e., the all-stop mode is considered or predefined stopping patterns are employed), but often include the time-dependency of the demand. Niu and Zhou (2013) considered a time-dependent demand matrix representing the number of passengers for each OD pair in each time interval, and presented an Integer Programming model for passenger loading and train departure events in a setting of oversaturated conditions, aiming at minimizing passenger waiting times. A genetic algorithm was developed and tested on an instance of a subway line in China. Sun et al. (2014) formulated three MILP models to determine the departure time of each train in a mass rapid transit system, while considering passenger loading on trains, based on a set of alternative waiting profiles. These models were solved by using CPLEX solver on a case study of a mass rapid transit line in Singapore. Canca et al. (2014) proposed a Nonlinear Integer Programming model in which the dynamic behavior of the demand, expressed as a continuous function of time for each OD pair, is combined with timetable optimization, with the goal of minimizing the total average waiting time. The model was tested on an instance of Madrid rapid transit system. The same objective and case study were considered in Barrena et al. (2014a) and Barrena et al. (2014b), where the passenger demand is expressed as the number of passengers that want to travel between an OD pair in a given time interval. Three MILP formulations were proposed in Barrena et al. (2014a), and solved with a branch-and-cut algorithm, while an Adaptive Large Neighborhood Search algorithm, presented in Barrena et al. (2014b), was designed to solve the problem in a heuristic way. In order to minimize the total passenger waiting time, Niu et al. (2015) proposed a Non-Linear Programming model with a predefined set of skip-stop patterns, in which the time-dependent OD passenger demands were taken into account. The model with minute- or hour-dependent demand was implemented in GAMS and tested on the Shanghai-Hangzhou high-speed corridor. To account for passenger satisfaction in the design of timetable, Robenek et al. (2016) formulated a MILP model with the objective of maximizing the train operating company profit while maintaining a given level of passenger satisfaction, expressed as an ϵ-constraint, and based on the scheduled passenger delay with respect to the preferred arrival time at a passenger destination. The model consists in an additional step between line planning and traditional train timetabling, and accounts for the passenger routing in the network, while it does not include track safety constraints. Robenek et al. (2018) extended the model proposed in Robenek et al. (2016) by including a probabilistic demand forecasting model and a ticket pricing problem. They solve the obtained problem by using a simulated annealing heuristic on a case study of Israeli Railways. Zhu et al. (2017) defined a bi-level model, in which the upper level solves the timetable design problem, while the lower level determines passenger flows on paths based on the timetable computed in the upper level. A two-stage genetic algorithm was developed and applied to test an instance of a subway line in Beijing. Yin et al. (2017), Mo et al. (2019), Yang et al. (2020) considered bi-objective train timetabling problems, accounting for dynamic passenger demand in metro systems, in which one goal is energy minimization. Yin et al. (2017) proposed two MILP models, based on a space-time network representation, in which the second objective is to minimize passenger waiting times. In one model they also considered the utilization of regenerative barking energy among trains in the same power supply substation. A Lagrangian relaxation-based heuristic algorithm was developed and tested on an instance of Beijing metro. Mo et al. (2019) designed a modified Tabu Search algorithm with the aim of minimizing energy costs and passenger waiting times, while considering the choice of the train type. They tested it on an instance of Beijing metro line. Yang et al. (2020) studied a timetable optimization problem maximizing the use of regenerative energy and minimizing total travel time. They implemented a genetic algorithm to derive an efficient Pareto frontier, and tested it on an instance of Beijing metro line. We observe that all these works on timetabling problems do not include train stop planning, but instead the train stopping patterns are given as input to the considered problems.

Stop planning and Timetabling with passenger demand We conclude this section by presenting recent works that combine both stop planning and timetabling, and additionally include passenger demand. Qi et al. (2018a) integrated the two problems with passenger distributions on trains by considering constraints that require to satisfy passenger demand between OD pairs and respect train capacity: they proposed an ILP model in which passenger demand is simplified to a global aggregated daily value, i.e., neglected that passenger demand is time-dependent. In Qi et al. (2018b) and Cacchiani et al. (2020), studies on robustness against demand uncertainty in this integrated framework were conducted: four MILP models, based on Light Robustness, were proposed, dealing with different ways of achieving protection for increased demand, and various levels of detail on the required input. The aim was to find an integrated plan featuring a good trade-off between robustness and efficiency (expressed as travel time and number of train stops). Hao et al. (2019) address the integration of train timetabling and stop planning with passenger demand: also in this case, the demand is defined as the number of passengers between OD pairs in a global aggregated way. A bi-objective MILP model, based on a time-space network, was proposed for this problem, with the goal of minimizing total train travel time and maximizing the number of transported passengers. A weighted-sum method was applied to determine Pareto optimal solutions. The approach was tested on an instance of the Beijing-Shanghai corridor.

In contrast with the described works, Meng and Zhou (2019) considered that the passenger demand is given for each OD pair and departure/arrival time windows (as a function that relates demand volume with service interval times). In their work, the stop planning was represented as the selection of stopping patterns in a set of given candidates, which is a further step with respect to works that consider stopping patterns completely as given on input. The goal was to maximize the revenue obtained by transported passengers profit, that depends on the selected stopping patterns and train schedules, minus the travel cost. To model the problem, trains were grouped into teams, built based on dynamic passenger volume and corresponding OD stations, and on the given candidate stopping patterns. The problem was formulated through a state-space-time network, in which the state accounted for the passenger loading, and was solved by a Lagrangian-based heuristic algorithm. Experiments were conducted on an instance of the Beijing-Shanghai high-speed corridor with 24 stations and 20 train teams.

Dong et al. (2020) studied the problem of finding the set of stops and the timetable for a commuter railway, considering time-dependent passenger demand. The goal was the minimization of the sum of the passenger waiting time at their departure station, the additional travel time for passengers due to acceleration and deceleration at train stops, and the total train running time. The authors presented a very complex Non-linear model, and developed an Adaptive Large Neighborhood Search (ALNS) algorithm. Due to the complexity of the model, a simplified version of it was applied only to test a small scale instance with 4 trains and 6 stations, while the ALNS was employed to solve a real-world instance of the Jinshan railway in Shanghai with 8 stations and 36 trains.

1.2 Contribution

From the literature overview, we can see that the integration of stop planning and timetabling has received relevant interest in recent years: in some works (see Section 1.1.1) the two problems have been combined but passenger demand has been neglected, in other works (see Section 1.1.2) each problem has individually been studied with attention to time-dependent passenger demand. Very recently effective approaches (Qi et al. (2018a), Qi et al. (2018b), Cacchiani et al. (2020), Hao et al. (2019), Meng and Zhou (2019) and Dong et al. (2020)) that consider all three elements together, as in the TTSP-TD, have been proposed.

We aim at contributing along this research stream on problems that integrate the mentioned three elements, by overcoming some simplifications introduced in previous works (such as dealing with daily demand instead of time-dependent demand, or using predefined stopping patterns instead of incorporating the stop selection in the problem), and by developing an ILP model to solve the integrated problem. More specifically, the TTSP-TD includes the choice of the train stopping patterns, train departure and arrival times at the stations of the line, and passenger flow distribution on trains. As already mentioned, the passenger demand is given for each OD pair and time interval, and we impose that the demand is satisfied in every interval, so that we account for the passenger desired departure times. Additional constraints require to respect train capacities, and minimum headway times. The goal is to minimize the total train travel time: indeed, we consider long-distance train services without transfers, and the aim is to let the passengers reach their destination in the shortest possible time, while the desired departure times are taken into account through the satisfaction of the demand in every time interval.

In particular, this paper aims to provide the following contributions to the framework of railway operations:

(1) to formulate an ILP model for the joint optimization of train stop planning, timetabling and time-dependent demand (TTSP-TD). Previous works that studied problems similar to the TTSP-TD either considered daily demand or simplified the stop planning problem, or proposed highly non-linear models. As a byproduct of the ILP solution, we also derive ticket distribution on trains for different OD pairs and time intervals;

(2) to perform numerical experiments on the Wuhan-Guangzhou high-speed railway line aiming at showing the performance of the proposed ILP model. In particular, we consider different time interval lengths for the time-dependent passenger demand, and compute the corresponding stop plans and timetables, with the constraints that the passenger demand must be satisfied for each time interval.

(3) to compare the solution obtained by solving the proposed model with that computed by solving the ILP model presented in Qi et al. (2018a), that neglects the time-dependency of passenger demand, in order to show how relevant it is to deal with time-dependent demand.

2 Problem statement

Passenger demand plays a crucial role in the process of designing railway operation plans, since one of the main purposes of a railway company is to transport passengers from their origins to their destinations, ensuring high quality of service so that passengers choose this transport means and the company can increase its revenue. Several elements come into play in the way of evaluating the quality of service: in particular, we focus on the travel time and on the passengers desired departure times. These elements are particularly important for a long-distance railway line, in which passengers generally do not transfer to another train, and require short travel times to reach their destination and departure times as close as possible to their expectations. Due to various trip purposes, the desired departure times of the passengers traveling between the same OD pair are different. To provide appropriate service to the passengers, the railway company has to define effective stop plans and timetables, and determine the ticket distribution that should match the given time-dependent passenger demand.

In order to show the connection between time-dependent passenger demand, train stop plan, timetable and tickets distribution more clearly, a simple example is presented in Fig.1. We consider a railway corridor with four stations (S1, S2, S3 and S4), and two trains, A and B, that are operated on this corridor with maximum train capacity of 5. The time-dependent passenger demands are displayed in Table 1, where q_{sisj} represents the number of passengers that want to travel between stations i and j, and the last row reports the total passenger demand over the considered time horizon $[0, 14]$. In Fig.1, rectangles are employed to show the specific ticket distribution plan, in a similar way as done in Qi et al. (2018c): the width and height of each rectangle represent, respectively, the number of tickets and the distance of each service. In addition, Fig.1 shows the stop plan and timetable as time-space diagrams. In Fig.1(a), we report the case in which time-dependent passenger demand is taken into account, while Fig.1(b) shows what happens when the passenger demand is only considered as a global demand.

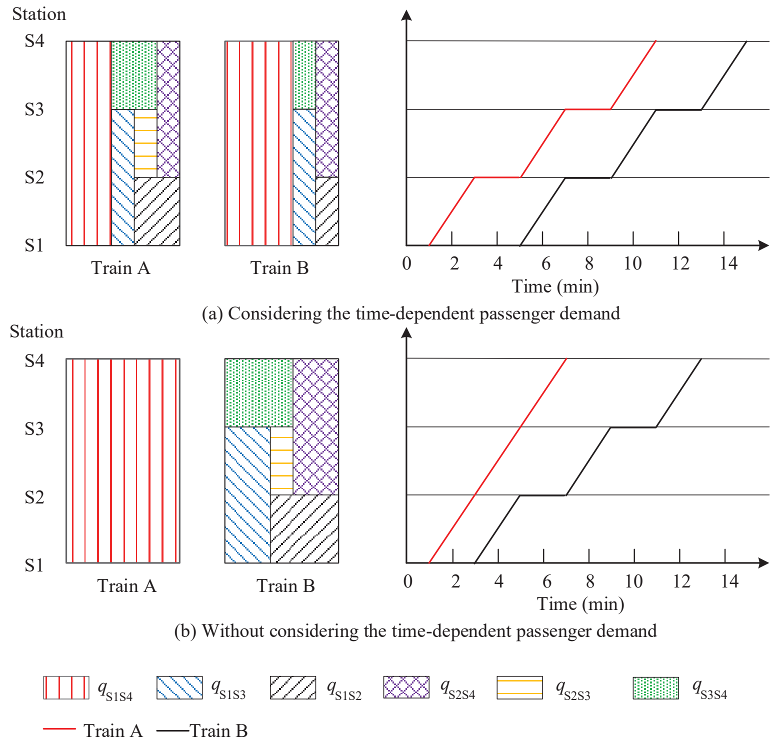

Fig. 1. An example of the TTSP-TD, showing the corresponding stop plan, timetable and passenger (ticket) distribution.

Table 1. The time-dependent passenger demand for the railway line with 4 stations

Time interval	Passenger demand
$[0, 2]$	$q_{S1S2} = 2, q_{S1S3} = 1, q_{S1S4} = 2;$
$[4, 6]$	$q_{S1S2} = 1, q_{S1S3} = 1, q_{S1S4} = 3, q_{S2S3} = 1, q_{S2S4} = 1;$
$[8, 10]$	$q_{S2S4} = 1, q_{S3S4} = 2;$
$[12, 14]$	$q_{S3S4} = 1;$
$[0, 14]$	$q_{S1S2} = 2, q_{S1S3} = 2, q_{S1S4} = 5, q_{S2S3} = 1, q_{S2S4} = 2, q_{S3S4} = 3.$

Clearly, due to the different desired departure times of the passengers, even for the passengers having the same OD stations, the stop plan, timetable and ticket distribution are quite different in the two cases. Specifically, in order to transport passengers as close as possible to their desired departure times, trains A and B are scheduled to stop at every station and the total travel time of each of these two trains is 20 min (see Fig.1(a)). As two passengers want to depart from station S1 to S4 in time interval $[0, 2]$ and the other three passengers with the same OD stations want to depart in time interval $[4, 6]$, two tickets are allocated to a trip from station S1 to S4 on train A and the other three tickets to a trip from station S1 to S4 on train B. In a similar way, tickets are assigned to the other trips. On the contrary (see Fig.1(b)), if the desired passengers departure times are not considered, train A becomes a non-stop train dedicated to transport passengers from station S1 to S4 in order to reduce the total travel time that becomes 16 min. As a consequence, five tickets from station S1 to station S4 are allocated on train A, since the different desired departure times are ignored. As a result, three passengers, who wanted to depart in time interval $[4, 6]$ from station S1 to S4, have to depart in a different time interval (in this small example, they depart at least 2 min before their desired departure time). The passengers who wanted to depart in time interval $[0, 2]$ from station S1 to S2 have to depart 2 min later since there is no train service in their preferred time interval. Similar situations can be seen for the other passengers OD pairs. Obviously, if we consider a real-life case, longer time deviations, with respect to the passengers desired departure times, will occur. Therefore, it is very important that the desired departure times are taken into consideration in the process of optimizing the train stop plans and timetables.

In the following, we formally describe the TTSP-TD problem. Let $[t_0, t_T]$ be the time horizon in which passengers request to be transported from their origin stations. We discretize the time horizon into a set T of time intervals $[t_0 + t * \delta, t_0 + (t + 1) * \delta]$ of time length δ, where t represents the index of the time interval in the set T. Time-dependent OD matrices are given on input for each OD pair of stations and time interval of the time horizon, and represent the number of passengers who want to travel between that OD pair of stations and want to depart from the origin station during that time interval. More precisely, we define Q^t_{ij} to be the number of passengers who want to travel from station i to station j departing from their origin station during time interval t (i.e., Q^t_{ij} is an entry of the given matrices). Different from what is usually done in metro or commuter railway systems, in which it is fundamental to deal with very short time intervals according to the high frequency of the service, we divide the time horizon into longer (e.g., one hour) time intervals. Indeed, in long distance railway lines, passengers departure times can be associated with time slots of relatively long duration: after the stop plans and timetables are determined, passengers will buy tickets for specific train services, and will not have to wait at their origin stations. However, it is important to meet the passengers desired departure times in these given longer time intervals, to guarantee an adequate service. Note that, as mentioned in the introduction, we do not require to know the demand in a very accurate way (e.g., for every minute of the day), but having information on preferred time intervals (e.g., every hour) instead of satisfying only the daily demand between each origin-destination pair allows to define train services that are more passenger oriented. We also mention that the model we propose is general, so that it can be applied to finer interval discretization, although the computing time required for solving it would clearly increase.

Beside the time-dependent OD matrices, we are given on input the description of the railway line as a set S of stations, and the set K of trains to be scheduled. For each train $k \in K$, the operation zone is specified, i.e., its origin O_k and terminal D_k stations are provided, the subset $S_k \subseteq S$ of stations it can visit is given (but the specific stopping pattern has to be determined), and a wide time window $[T^E, T^L]$ is defined to temporally limit the train trip from its origin station to its destination station. This time window represents the period during which all trains are operated. Note that $t_0 \leq T^E \leq t_0 + \delta$ so that it is possible to satisfy the passenger demand of the first time interval. In addition, $t_T \leq T^T$, i.e. the end of the time horizon in which passengers require to depart from their origin stations is earlier than the end of the train trip time window, so as to allow passengers to be transported to their destinations. The train trip time windows given on input are the same for all trains so that the most appropriate time windows will be selected through the solution of the ILP model. For each train $k \in K$, we also know its fixed travel time t^{trav}_{ki} between station i and the consecutive station $i + 1$ of the line in set S_k, its minimum dwelling time t^{dwel}_{ki} at station $i \in S_k$ (to be used if the train stops at i) and its maximum dwelling time $t^{maxdwel}_{ki}$ at station $i \in S_k$. Moreover, we are given the capacity C_k, defined as the number of passengers that the train can carry, and the maximum number N_k of stations at which train k can stop. The latter input is used to create stopping patterns that are more balanced (in terms of number of stops) between different trains.

To ensure an appropriate service for all stations, we must have, for each station i of the line, a minimum number R_i of trains that stop at station i. Finally, for safety requirements, minimum departure and arrival headway times h_d and h_a are imposed.

TTSP-TD calls for determining, for each train, a stop plan and a timetable, so that the time-dependent passenger demand is satisfied, while respecting train capacities for transporting passengers, constraints on train operation zones and trip time windows, train travel times, minimum and maximum dwelling times at stops, maximum number of stops for each train, minimum number of trains stopping at each station, and minimum headway times between consecutive train departures and arrivals. The goal is to minimize the total train travel time.

Some assumptions, adopted in this paper to formulate the ILP model for the TTSP-TD problem, are summarized below:

> **Assumption 1:** we assume that the number of trains is sufficient to transport the time-dependent passenger demand, i.e., the constraint on passenger demand satisfaction is considered as a hard constraint. We refer to Cacchiani et al. (2020); Qi et al. (2018b) for methods to derive robust stop plans and timetables in a setting of uncertain demand.

> **Assumption 2:** the travel time of each train between consecutive stations is assumed to be constant and the time required by acceleration and deceleration operations when the train is scheduled to stop at a station is not taken into consideration.

> **Assumption 3:** the time horizon is divided into time intervals, and we assume that passengers will board a train if it departs from their origin station (possibly after a stop) at a time that falls within their desired time interval, i.e., we do not explicitly include in the model the specific desired departure time of each passenger but rather consider his/her desired departure time interval. In the objective function, we do not include the time deviation with respect to a passenger desired departure time (often known as waiting time in the literature). By this assumption, we observe that this time deviation is at most the length of the discrete time interval.

3 Model formulation

In this section, we formulate an ILP model for the described TTSP-TD problem. We present, the parameters used in the model (Section 3.1), define its decision variables (Section 3.2), constraints (Section 3.3), and the objective function (Section 3.4). The complete model is reported in Section 3.5, where we also present a variant, that will be used to show the importance of taking into account the time-dependency of the demand: in particular, we will solve the model presented in Qi et al. (2018a) that neglects the time-dependency of the demand, and we will evaluate the obtained solution, in terms of satisfaction of the time-dependent passenger demand, by solving the model variant. In Section 3.6, we analyze the complexity, in terms of number of variables and constraints, of the proposed model.

3.1 Symbols and parameters

All the symbols and parameters used in the formulation are listed in Table 2.

Table 2. Symbols and parameters of the ILP model.

Symbols	Definition
S	set of stations.
K	set of trains.
T^E	the earliest departure time of any train from its origin station.
T^L	the latest arrival time of any train at its destination station.
t_0	beginning of the time horizon.
t_T	end of the time horizon.
T	set of time intervals covering the time horizon $[t_0, t_T]$,
	$T = \{t_0, t_0 + \delta, \dots, t_T - \delta, t_T\}$.
δ	length of each time interval.
t	index of time interval $[t_0 + t * \delta, t_0 + (t+1) * \delta]$.
i, i', j	indices of stations, $i, i', j \in S$.
k, l	indices of trains, $k, l \in K$.
S_k	set of stations visited by train k.
O_k	origin station of train k.
D_k	terminal station of train k.
t_{ki}^{trav}	travel time of train k from station i to station $i+1$.
t_{ki}^{dwel}	minimum dwelling time of train k at station i.
$t_{ki}^{maxdwel}$	maximum dwelling time of train k at station i.
h_d	minimum headway time between two consecutive trains departing from the same station.
h_a	minimum headway time between two consecutive trains arriving at the same station.
Q_{ij}^t	total passenger demand to be transported from station i to station j
	with desired departure time from i in time interval $[t_0 + t * \delta, t_0 + (t+1) * \delta]$.
C_k	maximum loading capacity of train k.
N_k	maximum number of stops of train k.
R_i	minimum number of trains that must stop at station i.
M	a sufficiently large number.

3.2 Decision variables

To model the TTSP-TD problem, we introduce decision variables used to determine the train stop plan, the timetable and the time-dependent passenger demand distribution on trains.

3.2.1 Variables related to train stop plan and timetable

To determine the stopping pattern of each train, we introduce binary variables that define the set of stations at which each train has to stop. This choice is determined based on the passenger demand that must be satisfied, on the constraints on the minimum number of stops required at each station of the line, and on maximum number of stops for each train. The following variables are introduced for every $k \in K$, $i \in S_k$:

- x_{ki}: binary variable equal to 1 if train k stops at station i (0 otherwise).

The train timetable specifies, for each train, its departure and arrival times at the visited stations. To determine the train timetable, we define the following variables for every $k \in K$, $i \in S_k$:

- t^d_{ki}: integer variable representing the departure time of train k from station i;
- t^a_{ki}: integer variable representing the arrival time of train k at station i.

These variables will be constrained to respect single train constraints, such as its travel time between consecutive stations, and its minimum and maximum dwelling times at stations where it stops, as well as constraints that involve more trains in order to guarantee minimum headway times between their departures and arrivals. Moreover, we introduce additional variables that specify the train ordering along each section: indeed, overtaking is not allowed along a section connecting two consecutive stations of the line. The following variables are introduced for every $k, l \in K$, $k \neq l$, $i \in S_k \cap S_l$:

- y_{kli}: binary variable used to determine the departure order of trains k and l from station i and their arrival order at station $i+1$; it is equal to 1 if train k departs from station i and arrives at station $i+1$ before train l (0 otherwise).

3.2.2 Variables related to time-dependent passenger demand distribution

To satisfy time-dependent passenger demand Q^t_{ij} ($i, j \in S$, $i < j$, $t \in T$) and respect train capacity C_k ($k \in K$), we must determine a feasible passenger flow distribution from each origin station i to each destination station j on (possibly several) trains $k \in K$, such that passengers willing to depart in time interval t from station i directed to station j can board a train departing in the same interval. We introduce the following variables, representing, respectively, the passenger flow distribution and the temporal train departure distribution in the time intervals:

- q^{kt}_{ij}: integer variable representing the number of passengers of OD pair (i, j) departing in time interval t (i.e., with desired departure time in $[t_0 + t * \delta, t_0 + (t+1) * \delta]$) that travel on train k;
- b^{kt}_{ij}: binary variable equal to 1 if train k departs in time interval t from station i towards station j.

We observe that q^{kt}_{ij} accounts for detailed passenger flow distribution in order to ensure that train capacities are respected, and to guarantee that the passenger demand is satisfied for each OD pair and time interval. Moreover, b^{kt}_{ij} is used to match passenger demand and train services under constraints on time-dependency, i.e., it ensures that passengers of OD pair (i, j) with desired departure time in time interval t can board a train k only if it departs from i in the same time interval and travels to station j.

Finally, we note that, through the values of variables q^{kt}_{ij} in the solution of TTSP-TD, we also define the ticket distribution corresponding to the stop plan and timetable we obtained. In particular, q^{kt}_{ij} can be interpreted as the number of tickets on train k that have to be sold to passengers departing in time interval t who want to travel between stations i and j. Based on the values of these variables in the computed solution, the railway company can decide how to effectively allocate tickets on each train, according to the expected passenger demand. As an example of the importance of ticket distribution, consider:

- a time horizon with two time intervals $t1$ and $t2$,
- a line with three stations $S1$, $S2$ and $S3$,
- three trains A, B and C all departing from $S1$ and arriving at $S3$ but with only train B stopping at $S2$, with A and B travelling during time interval $t1$ and C during $t2$,

- three OD pairs: $OD1$ and $OD2$ with passengers who want to travel, respectively, from $S1$ to $S2$ and from $S1$ to $S3$ in time interval $t1$, and $OD3$ with those who want to travel from $S1$ to $S3$ in time interval $t2$.

Based on the given time intervals and OD pairs, passengers of $OD1$ must board train B (the only train stopping at station $S2$), and those of $OD3$ must board train C (the only train available). However, passengers of $OD2$ can board train A or train B. Depending on the passengers volume in $OD1$ and $OD2$ (and on train capacities), it may be very important to allocate passengers of $OD2$ on train A, in order to keep seats available to passengers of $OD1$ on train B. An extension of tickets/seats distribution is to distinguish the number of seats for different travel classes, but this is not considered here in order to simplify the problem.

3.3 Constraints

This section is dedicated to the presentation of the constraints defined in the proposed ILP model. For the sake of clarity, we present the constraints by dividing them into categories based on the purpose they have. We first express constraints related to the construction of the train stop plan (Section 3.3.1) and of the timetable (Section 3.3.2), and constraints that link these two problems (Section 3.3.3). Then, we define constraints related to the time-dependent passenger demand satisfaction and flow distribution (Section 3.3.4), and constraints that connect the passenger flow variables with the decision variables of the other categories (Section 3.3.5).

3.3.1 Constraints on train stop planning

Three types of constraints belong to this category, and are used to determine the train stopping patterns. The first one is the set of operation zone constraints that define the fixed origin and destination stations of each train. In railway systems, in order to provide transportation service to the passengers, the whole railway line is usually decomposed into non-overlapping sections, each with an associated subset of trains that operate in that section. The following constraints (1) are imposed to ensure that each train $k \in K$ departs from its origin station O_k and arrives at its terminal station D_k:

$$x_{kO_k} = x_{kD_k} = 1, \ \forall \ k \in K \tag{1}$$

The second set of constraints limits the number of stops of each train k to N_k. If a train stops at many stations, a longer travel time is required by this train, since a minimum dwelling time is usually needed to complete the necessary operations (such as letting passengers board or alighting from the train). The objective function of the proposed model minimizes the total travel time, thus avoiding unnecessary stops. However, it accounts for the sum of the travel times of all trains, while constraints (2) aim at deriving solutions in which the number of stops is more balanced among different trains:

$$\sum_{i \in S_k} x_{ki} \leq N_k, \ \forall \ k \in K \tag{2}$$

The last set of constraints related to the train stop plan concerns the requirement on the minimum number R_i of stops for each station $i \in S$. Since passengers can only board and alight from a train at stations at which it stops, constraints (3) are imposed to ensure more flexible transportation service to the passengers:

$$\sum_{k \in K: i \in S_k} x_{ki} \geq R_i, \ \forall i \in S \tag{3}$$

We observe that the specific values of N_k and R_i usually depend on the type of train (e.g., its speed, priority, etc.) and on the considered station (e.g., the economic and political position, passenger demands, etc.).

3.3.2 Constraints on timetabling

These constraints are employed for defining the departure and arrival times of trains at every station they visit, by taking into account the train trip time window, the train travel time between consecutive stations, the maximum dwelling time at each station, and the minimum headway times required between consecutive train departures or arrivals. Through these constraints the train ordering along each rail segment is also determined, so that no overtaking occurs along a segment between two consecutive stations. Constraints (4) and (5) limit, respectively, the allowed departure time of the train from its origin station and its arrival time to its destination. Often in the traditional train timetabling problem, a time interval related to the departure time from the origin station is provided for each train to keep the balance of services over the whole considered time horizon. Since we take the time-dependent origin-destination passenger demand into consideration, instead of providing a time interval for each train from its origin station, we only provide a time window for the entire train trip. The appropriate departure time windows will then be determined by solving the ILP model.

$$\begin{cases} T^E \leq t^d_{kO_k}, \quad \forall k \in K & (4) \\ t^a_{kD_k} \leq T^L, \quad \forall k \in K & (5) \end{cases}$$

The departure and arrival times of each train must satisfy the fixed travel time along a rail segment between two consecutive stations. This requirement is imposed by constraints (6):

$$t^a_{ki+1} - t^d_{ki} = t^{trav}_{ki}, \ \forall k \in K, \ i \in S_k \setminus \{D_k\} \tag{6}$$

Similar constraints are defined for the train dwelling time at each station. However, the minimum dwelling time should only be respected at stations at which the train stops. These constraints will be presented in Section 3.3.3, as they link the train stop planning and the timetabling problems, since the stopping patterns are not predefined. In addition, we impose, through constraints (7), a maximum dwelling time for each train at every visited station (if the train does not stop, then the constraint is always satisfied), in order to avoid very long waiting times at stations along the trip, that negatively affect the passengers perception of trip duration:

$$t^d_{ki} - t^a_{ki} \leq t^{maxdwel}_{ki}, \ \forall k \in K, \ i \in S_k \setminus \{O_k, D_k\} \tag{7}$$

Although the objective function consists of the minimization of the total travel time, and thus naturally also minimizes the total dwelling time, it does not prevent the assignment of very long dwelling times to a few trains. Indeed, the objective function accounts for the total dwelling times and does not distinguish between having a very long dwelling time for a few trains or many short dwelling times (giving the same total quantity) assigned to more trains. Therefore, constraints (7) are imposed to limit the maximum dwelling time for each train at each visited station.

Finally, departure and arrival headway constraints (8)-(11) are imposed for every pair of trains k and l at the stations they both visit to keep the required minimum time distance between these trains. Note that, since the train ordering is not predefined but has to be determined by solving the model, two sets of constraints, (8)-(9), are imposed for the minimum departure headway times, and two sets, (10)-(11), for the minimum arrival headway times, by considering either train k departing from station i before train l, or vice versa. These constraints employ a big-M value, and in particular we consider $M = T^L$. In addition, we report the definition of the y variables domain in constraints (12).

$$\begin{cases} t^d_{ki} + h_d \leq t^d_{li} + M \cdot (1 - y_{kli}), \;\; \forall \;\; i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}), \;\; k,l \in K \;\; \text{with} \;\; k < l & (8) \\ t^d_{li} + h_d \leq t^d_{ki} + M \cdot y_{kli}, \;\; \forall \;\; i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}), \;\; k,l \in K \;\; \text{with} \;\; k < l & (9) \\ t^a_{ki+1} + h_a \leq t^a_{li+1} + M \cdot (1 - y_{kli}), \;\; \forall \;\; i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}), \;\; k,l \in K \;\; \text{with} \;\; k < l & (10) \\ t^a_{li+1} + h_a \leq t^a_{ki+1} + M \cdot y_{kli}, \;\; \forall \;\; i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}), \;\; k,l \in K \;\; \text{with} \;\; k < l & (11) \\ y_{kli} \in \{0,1\}, \;\; \forall \;\; i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}), \;\; k,l \in K \;\; \text{with} \;\; k < l & (12) \end{cases}$$

3.3.3 Constraints linking train stop planning and timetabling

When a train stops at a station, a minimum dwelling time is needed in order to complete the necessary boarding/alighting passengers or other operations. Constraints (13) are imposed to this aim, and they also express the connection between the decision variables of the stop planning and timetabling problems:

$$t^d_{ki} - t^a_{ki} \geq x_{ki} \cdot t^{dwel}_{ki}, \; \forall k \in K, \; i \in S_k \setminus \{O_k, D_k\} \tag{13}$$

3.3.4 Constraints on time-dependent passenger demand satisfaction and flow distribution

The central focus of the stop plan and timetable construction is the time-dependent passenger demand.In this work, constraints on passenger demand satisfaction are imposed as hard constraints, i.e., for each time interval and OD pair, the corresponding volume of passengers Q^t_{ij} must be transported:

$$\sum_{k \in K: i,j \in S_k} q^{kt}_{ij} \geq Q^t_{ij}, \; \forall \; t \in T, \; i,j \in S \text{ with } i < j \tag{14}$$

In addition, we also include constraints on train capacity for each train $k \in K$: by considering a station i, and, respectively, a preceding station i' and a following station j (all visited by the train), the total passenger volume on train k must be at most its capacity C_k no matter at which time interval the passengers departed from i':

$$\sum_{t \in T} \sum_{i' \in S_k, i' \leq i} \sum_{j \in S_k, j > i} q^{kt}_{i'j} \leq C_k, \; \forall \; k \in K, \; \forall \; i \in S_k \setminus \{D_k\} \tag{15}$$

Note that constraints (14) ensure that passenger are transported from their origin station to their destination station by departing in the desired time interval, and constraints (15) guarantee that train capacity is always respected at every station that the train visits. Recall that variables q^{kt}_{ij} represent the number of passengers travelling from station i to station j on train k and departing in time interval t: as already mentioned, the values of these variables in a solution of the proposed model can be used to allocate tickets to different trains, so as to satisfy the expected passenger demand and respect the train capacities.

3.3.5 Constraints linking passenger flow distribution with train stop planning and timetabling

The time-dependent passenger demand clearly influences the stop plan. In addition, the time-dependent passenger demand has a relevant impact on the timetable. In this section, we present the constraints that link the decision variables taking care of these aspects.

Constraints (16) and (17) connect stop planning with passenger flow distribution by imposing that no passenger can access train k at station i if k does not stop at i. Note that we need two sets of constraints because we impose that no passenger can access train k at station i either for departing from or for arriving at that station. In these constraints, we consider the train capacity C_k as big-M value. In addition, we define the domain of the x and q variables, respectively in constraints (18) and (19):

$$\begin{cases} \sum_{t\in T}\sum_{j\in S_k, j>i} q_{ij}^{kt} \leq M * x_{ki},\ \forall\ k \in K,\ \forall\ i \in S_k \setminus \{D_k\} & (16) \\ \sum_{t\in T}\sum_{j\in S_k, j<i} q_{ji}^{kt} \leq M * x_{ki},\ \forall\ k \in K,\ \forall\ i \in S_k \setminus \{O_k\} & (17) \\ x_{ki} \in \{0,1\},\ \forall\ k \in K,\ \forall\ i \in S_k & (18) \\ q_{ij}^{kt} \geq 0, integer,\ \forall\ t \in T,\ k \in K,\ \forall\ i,j \in S_k \text{ with } i<j & (19) \end{cases}$$

Variables b_{ij}^{kt} were introduced to define whether the time interval t is selected for the departure of train k from station i travelling towards station j, i.e., these variables define the time interval selected for each train. Therefore, these variables have to be linked with t_{ki}^{d} variables representing the departure time of each train k from each station i. Constraints (20) and (21) express this linking by specifying that if the departure time t_{ki}^{d} is in interval t, i.e., $t_{ki}^{d} \in [t_0 + t * \delta, t_0 + (t+1) * \delta]$, then train k departs from i towards j in that interval, i.e., b_{ij}^{kt} assumes value 1. Otherwise, train k departs from i in a different time interval, and consequently b_{ij}^{kt} equals 0. Note that these constraints are imposed for every station visited by train k, since it is important to know if passengers can board the train (in their desired time intervals) along its entire trip. These constraints make use of a big-M value that we set equal to T^L.

$$t_0 + t * \delta - t_{ki}^{d} \leq (1 - b_{ij}^{kt}) * M,\ \forall\ t \in T,\ k \in K,\ \forall\ i,j \in S_k,\ i<j \tag{20}$$

$$t_{ki}^{d} - t_0 - (t+1) * \delta \leq (1 - b_{ij}^{kt}) * M,\ \forall\ t \in T,\ k \in K,\ \forall\ i,j \in S_k,\ i<j \tag{21}$$

Obviously, variables b_{ij}^{kt} are very important to rule whether there can be passengers of OD pair (i,j) with desired departure time in interval t that can travel on train k. In other words, they are used to match passenger OD flow with trains in every time interval. Constraints (22) express the indirect linking between passenger flow distribution and the timetable definition, and adopt as big-M value $M = C_k$.

$$q_{ij}^{kt} \leq b_{ij}^{kt} * M,\ \forall\ t \in T,\ k \in K,\ \forall\ i,j \in S_k,\ i<j \tag{22}$$

In detail, constraints (22) guarantee that passengers departing from station i in time interval t to station j can travel on train k only if k departs (from its origin station or after a stop) in the same time interval from station i and travels to station j. This also implies that no tickets can be allocated on train k for passengers of OD pair (i,j) departing in interval t if k does not depart from i to j in the same interval.

3.4 Objective function

The goal of the TTSP-TD problem is the minimization of the total travel time, which has been widely adopted as evaluation index in the train scheduling problem (Szpigel, 1973; Higgins et al., 1997; Zhou and Zhong, 2005; Yang et al., 2016). This goal is particularly relevant in the context of long-distance rail line that we are studying, in which most passengers do not transfer to other trains. Hence it is not important to consider the passenger transfer time in the objective. In addition, we underline that the deviation time between the passenger desired departure time and the effective schedule time is not included in the objective function, since we impose the satisfaction of passenger demand in each desired time interval as constraints. The objective function is formulated as (23) by expressing, for each train, the time difference between its arrival time at its terminal station and its departure time from its origin station:

$$\min T_{train} = \sum_{k \in K} (t^a_{kD_k} - t^d_{kO_k}) \tag{23}$$

3.5 Mathematical model

The studied TTSP-TD can be formulated as the following ILP model:

$$\begin{cases} \text{objective function (23)} \\ s.t. \quad \text{constraints (1)} - (22) \end{cases} \tag{24}$$

3.5.1 Modified model for comparison with the model by Qi et al. (2018a)

In the computational experiments, we will perform a comparison with the model proposed by Qi et al. (2018a), that neglects the time-dependency of passenger demand (i.e., considers the global daily demand for each OD pair), in order to show the benefits of model (24), that takes it into account. In particular, we want to compute the number of passengers for which the desired departure time interval was not respected by the solution of the model proposed in Qi et al. (2018a). To this aim, we will first solve the model by Qi et al. (2018a), and then we will solve a variant of model (24) (described in the following) in order to evaluate the number of passengers departing in a different time interval than the desired one.

Since the model by Qi et al. (2018a) does not consider time dependency, a preferred departure time, for each train, from its origin station is predefined in order to have train services spread along the time horizon. Note that this preferred departure time is not based on passenger demand. In order to have a fair comparison between the two models, we assign, in the model by Qi et al. (2018a), as preferred departure times those computed by the best solution of model (24). Then, we solve the model by Qi et al. (2018a) with CPLEX and obtain stop plan and timetable.

Our purpose is then to evaluate how well/badly the passenger demand is served, in terms of desired departure time intervals, according to the stop plan and timetable computed by Qi et al. (2018a) in comparison to what is achieved with the solution of model (24): note that the entire passenger demand is completely satisfied for each OD pair in both models, thus the difference stands only in the time interval in which it is served. In order to compute the volume of passengers who have deviation time with respect to their desired time intervals, we formulate an evaluation model (EM) obtained by applying the following changes to model (24):

1. A new class of non-negative variables u^t_{ij} is introduced to represent the number of passengers travelling from station i to station j that would like to depart from i during time interval $[t, t+\delta]$ but cannot be satisfied (i.e, they have to depart in a different time interval);

2. Passenger demand constraints (14) are changed as follows to ensure the feasibility of the solution:

$$\sum_{k \in K: i,j \in S_k} q^{kt}_{ij} \geq Q^t_{ij} - u^t_{ij}, \; \forall\, t \in T, \; i,j \in S \text{ with } i < j \tag{25}$$

   Indeed, the solution of model by Qi et al. (2018a) might not be able to satisfy the demand in the corresponding time intervals, and the u^t_{ij} variables are used as slack variables.

3. New constraints, similar to those presented in (Qi et al., 2018a), are added for ensuring the satisfaction of the total passenger demand for each OD pair:

$$\sum_{t\in T}\sum_{k\in K:i,j\in S_k} q_{ij}^{kt} \geq \sum_{t\in T} Q_{ij}^t,\ \forall\, i,j \in S \text{ with } i<j \tag{26}$$

4. The objective function is modified to minimize the total number passengers who are subject to departure deviation time with respect to their desired departure time interval:

$$\min Z = \sum_{t\in T}\sum_{i,j\in S \text{ with } i<j} u_{ij}^t \tag{27}$$

5. The x_{ki}, t_{ki}^d, t_{ki}^a, and y_{kli} variables are fixed in EM according to the solution of the model by Qi et al. (2018a), because we want to evaluate the corresponding stop plan and timetable. The b_{ij}^{kt} variables are directly fixed by constraints (20)-(21), based on the computed timetable. Therefore, in EM, beside the new variables u_{ij}^t, only the passenger flow variables q_{ij}^{kt} are optimized.

The EM model reads as follows.

$$\begin{cases} \text{objective function (27)} \\ s.t. \quad \text{constraints } (15)-(17),(19)-(22) \text{ and } (25)-(26) \end{cases} \tag{28}$$

This model returns the minimum number of passengers that cannot be served in their desired departure time interval: the larger the value of objective function (27), the more evident the relevance of accounting for time-dependent demand is.

3.6 Complexity of the formulation (24)

We conclude Section 3 by a short discussion on the complexity of the proposed model (24). It is well-known that the train timetabling problem is an NP-hard problem (Cai and Goh, 1994). In the TTSP-TD, the train timetabling problem is integrated with the train stop planning, and with the passenger flow distribution based on the passenger desired departure time intervals, making the complete problem very hard. In particular, due to the close connection among all decision variables it is extremely difficult to find an optimal solution to the TTSP-TD problem. We report in Table 3 the list of sets of variables and constraints, and the corresponding maximum cardinality.

Clearly, the complexity of this model is mainly determined by the numbers of trains $|K|$, stations $|S|$ and time intervals $|T|$. As can be seen from Table 3, the most numerous variables are q_{ij}^{kt} and b_{ij}^{kt}, indexed over OD pairs, trains, and time intervals, and the largest set of constraints consists of the linking constraints (20)-(22), that involve variables q_{ij}^{kt} and b_{ij}^{kt}. Therefore, it is evident that the time-dependent component increases the model complexity.

4 Numerical experiments

In this section, we report the results of the experiments we performed on a small-scale example (Section 4.1) and on a real-world instance based on Wuhan-Guangzhou high-speed railway corridor (Section 4.2) to show the performances of the proposed model. We used IBM CPLEX 12.7 as solver of the proposed model. All the experiments were implemented in Visual Studio 2015 and run on a personal computer with Intel Core I5-7200M CPUs and 8 GB RAM.

Table 3. Number of variables and constraints in formulation (24)

Variables or constraints	Maximum cardinality
Variable t^d_{ki}	$\|K\| \cdot (\|S\| - 1)$
Variable t^a_{ki}	$\|K\| \cdot (\|S\| - 1)$
Variable q^{kt}_{ij}	$\|T\| \cdot \|K\| \cdot \|S\| \cdot (\|S\| - 1)/2$
Binary variable x_{ki}	$\|K\| \cdot \|S\|$
Binary variable y_{kli}	$\|K\| \cdot (\|K\| - 1) \cdot (\|S\| - 1)/2$
Binary variable b^{kt}_{ij}	$\|T\| \cdot \|K\| \cdot \|S\| \cdot (\|S\| - 1)/2$
Operation zone constraints (1)	$2 \cdot \|K\|$
Maximum stop constraints (2)	$\|K\|$
Minimum stop constraints (3)	$\|S\|$
Time window constraints (4)-(5)	$2 \cdot \|K\|$
Travel time constraints (6)	$\|K\| \cdot (\|S\| - 1)$
Maximum dwelling time constraints (7)	$\|K\| \cdot (\|S\| - 2)$
Headway constraints (8)-(11)	$2 \cdot (\|S\| - 1) \cdot \|K\| \cdot (\|K\| - 1)$
Minimum dwelling time constraints (13)	$\|K\| \cdot (\|S\| - 2)$
Passenger demand constraints (14)	$\|T\| \cdot \|S\| \cdot (\|S\| - 1)/2$
Loading capacity constraints (15)	$\|K\| \cdot (\|S\| - 1)$
Linking constraints (16)-(17)	$2 \cdot \|K\| \cdot (\|S\| - 1)$
Linking constraints (20)-(22)	$3 \cdot \|T\| \cdot \|K\| \cdot \|S\| \cdot (\|S\| - 1)/2$

4.1 A small-scale example

We first consider a railway line with 5 stations, as shown in Figure 2, and 4 trains (T1, T2, T3, T4) to be operated on this line: the first two trains are operated between stations S1 and S5 while the other two trains are operated between stations S2 and S5. These trains are characterized by different maximum speeds: T1 and T3 are high-speed trains, while T2 and T4 are low-speed trains. For simplicity, we consider the travel time of high-speed trains and low-speed trains on each section to be, respectively, 2 and 3 min. For each train, we set its maximum loading capacity equal to 60 passengers, its minimum dwelling time to 1 min, and its maximum dwelling time to 3 min. The minimum departure and arrival headway times are set to 2 min.

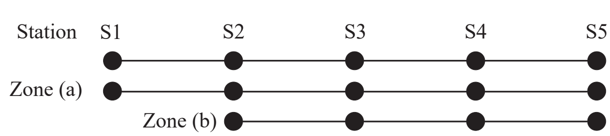

Fig. 2. The structure of the railway line

The time horizon is 20 min and the starting time of passenger demand is 0. The total number of passengers to be transported is 310. In the experiments, we consider different values (1, 2, 4, 5, 10 and 20 min) for the length δ of each time interval, and accordingly divide the time horizon into 20, 10, 5, 4, 2 and 1 time intervals. Figure 3 shows the passenger demand for each interval, when the length δ is 1 min. In this figure, the height of each rectangle is the distance travelled by the passengers along the line,

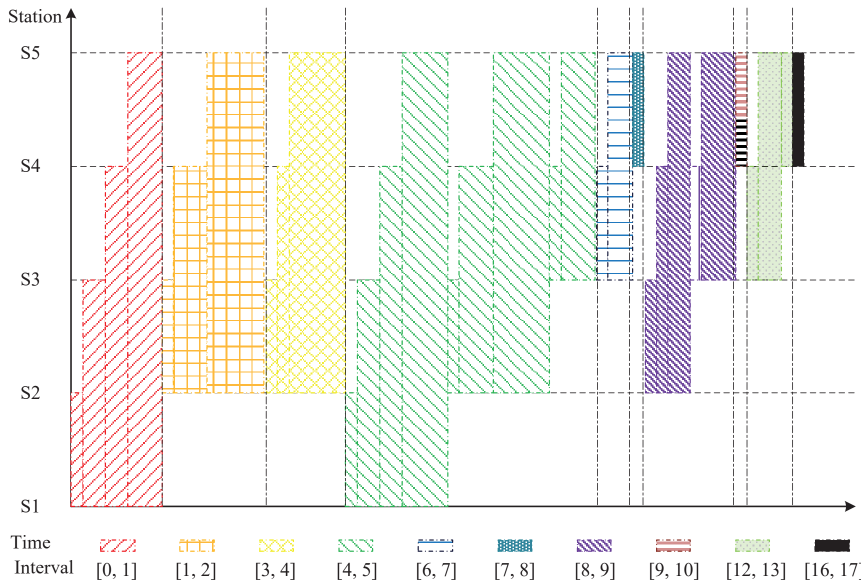

Fig. 3. The passenger demand with time interval length of 1 min

which also shows the origin and destination stations of the passengers, while the width of each rectangle represents the number of passengers traveling between the two stations of the corresponding rectangle. Clearly, according to the various trip purposes, the passengers desired departure times are different, and consequently, in Figure 3, the passenger demand varies in terms of volume and OD stations with the time intervals. For example, there are 9 different OD pairs during time interval $[4, 5]$ (from station S1 to S2, from station S1 to S3, from station S1 to S4, from station S1 to S5, from station S2 to S3, from station S2 to S4, from station S2 to S5, from station S3 to S4 and from station S3 to S5), while there is only one OD pair from station S4 to S5 during time interval $[7, 8]$. The detailed passenger demand with different time interval lengths δ are reported in the Appendix A.1 (see Table 9).

We tested the proposed ILP model with different time interval lengths by setting as terminating condition an optimality gap of 5%. Table 4 displays the obtained results. In Table 4, the first three columns report, respectively, the time interval length, the total travel time (i.e., the value of the objective function), and total number of stops of all trains. The fourth column shows the deviation time between the passenger desired departure times and the scheduled train departure time, when we consider that the passenger demand is known at every minute of the time horizon. Although the demand constraints ensure that the time-dependent demand is satisfied during the desired time interval, when longer time intervals are considered the deviation time can increase. Indeed, with the increase of the time interval length δ, passengers with different desired departure times are allowed to be transported together, regardless of their desired departure times. For computing the deviation time, we considered the train departure time closest to the desired passenger departure time. For example, if the passenger desired departure time is in interval $[10, 11]$ and the train departs at time instant 13, 11 is taken as the actual passenger departure time, and the corresponding deviation time is 2 min. Note that the deviation time when the time interval length is 1 min is 0, since constraints (20)-(22) guarantee that all the passengers can be transported by trains departing in the desired departure intervals. Finally, the last two columns show, respectively, the number of passengers that have to change their desired departure time and the average deviation time for these passengers.

Table 4. The computational results for the small-scale example with different time interval lengths.

Length	Travel time	# of stops	Dev. time	# of dev. passengers	Avg. dev. time
1	45	18	0	0	0
2	45	18	75	75	1
4	44	17	381	215	1.77
5	42	15	470	220	2.14
10	41	14	540	240	2.25
20	40	13	1040	260	4

From Table 4, we can see that with the increase of the time interval length, the total travel time and total number of stops decrease. On the contrary, the deviation time and number of passengers that need to change their desired departure time significantly increase, since their departure times are neglected. For example, with time interval length of 1 min, the total travel time and total number of stops are, respectively, 45 min and 18 stops. When we do not consider the desired departure times (i.e., δ is 20 min), the total travel time and total number of stops decrease to 40 min and 13 stops. However, the deviation time increases from 0 to 1040 min, which further shows the need of including time-dependent passenger demand in the optimization process for deriving the stop plan and timetable.

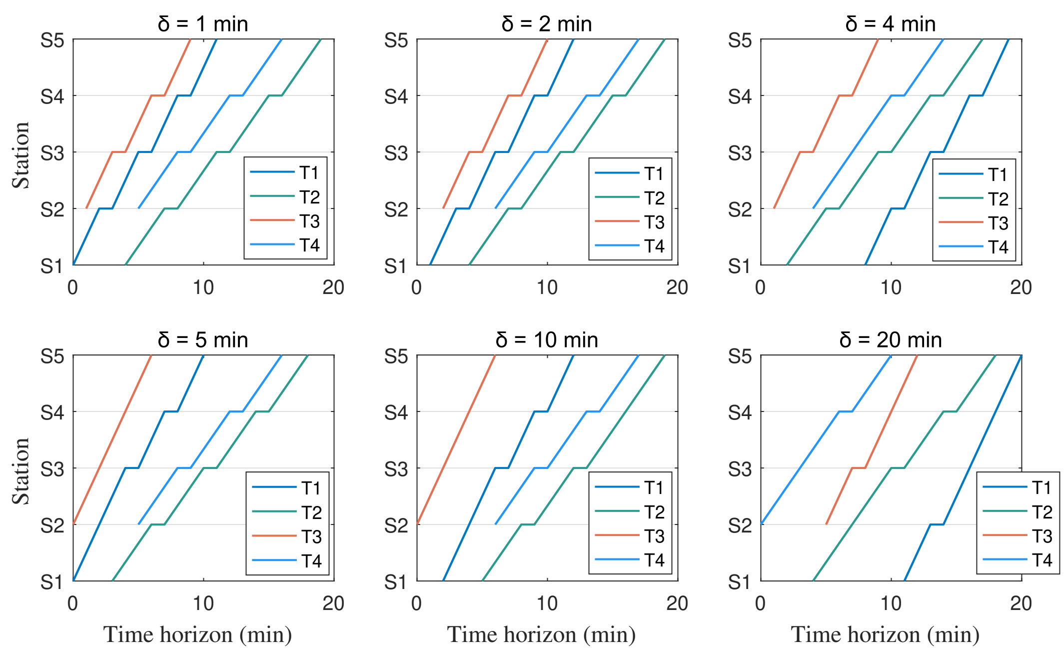

Fig. 4. The timetables with different lengths of discrete time interval (δ)

In order to provide a better understanding of the obtained solutions, Figures 4, 5 and 6 display the timetables, stop plans and passenger (ticket) distributions with different time interval lengths. In Fig. 4, we can see that with δ up to 4 min, the space-time train paths are more equally distributed over the time horizon and present a larger number of stops. When δ increases, the number of stops decreases because the goal is to minimize the total travel time and passengers are transported regardless of their desired departure time intervals.

In Figures 5 and 6, the stopping operations for loading and unloading passengers are represented by the solid dot "•", while the hollow dot "○" means that the train does not stop at that station, and the ticket distributions are shown in the same way as for Fig. 1. Clearly, with different values for the time interval length, the obtained train stop plans, and tickets distributions are quite different. Indeed, since passengers can be served only by the trains that depart during the desired departure time interval, a smaller δ implies that passengers with the same OD stations need to be distinguished. Thus, they typically need to be served by different trains, leading, as a consequence, to different ticket distributions. As an example, consider the passengers travelling from station S2 to S4 (see Table 9 with $\delta = 1$): there are 15 passengers with departure time interval $[1, 2]$, 5 with time interval $[3, 4]$, 15 with time interval $[4, 5]$ and 5 with time interval $[8, 9]$. When δ is 1 min, (i.e., when the maximum time deviation cannot exceed 1 min), the tickets corresponding to a trip between stations S2 and S4 need to be allocated on different trains (see Fig. 5 with $\delta = 1$): in particular, 5 tickets are assigned to train T1, 5 to T2, 15 to T3 and 15 to T4. With a larger time interval length, more passengers with the same OD stations may be served by the same train, thus reducing the number of stops of some trains. For example, when δ is 10 min (see Fig. 6 with $\delta = 10$), all of the passengers traveling from station S2 to S4 can choose any train departing from station S2 in time interval $[0, 10]$. Hence, all tickets corresponding to a trip between stations S2 and S4 are assigned to train T4. In general, we can see that, when the time interval length increases, more passengers with the same OD pair and different desired departure time intervals are grouped together on the same train: in this way, other trains can avoid some stops, and decrease their travel time.

In Figures 5 and 6, we can also see some empty slots (seats on some trains that are not assigned to specific OD pairs): indeed, tickets are assigned to trains based on the expected time-dependent passenger demand, and, once this demand is satisfied, there is no incentive in the ILP model for assigning more tickets. Therefore, these empty seats can be used to have a more robust plan, i.e., they can be assigned to passengers in real-time based on the actual demand.

Finally, we observe that by satisfying in a more accurate way (i.e., with shorter length δ) the passengers desired departure times, longer travel times may be experienced by the passengers, since, usually, more stops are needed to respect their preferred departure time interval. Therefore, the time interval length δ has to be chosen so as to achieve a good trade-off between time-dependent demand satisfaction and total train travel time. By solving the ILP model with (a few) different time lengths, we can provide the railway operators with some alternative solutions having different levels of quality of service. In addition, as will be shown in the next section, longer computing time is required when δ is shorter. Therefore, the choice of δ should also take into consideration the available computing time.

4.2 Wuhan-Guangzhou high-speed railway corridor

In this section, we consider a real-world instance of the Wuhan-Guangzhou high-speed railway corridor in China, and analyze the solutions obtained by solving the proposed ILP model for the TTSP-TD with different time interval lengths. Then, in Section 4.2.1, we report the results obtained with a time limit of three hours, while, in Section 4.2.2, we show the results obtained with different train capacity values. Finally, we present, in Section 4.2.3, a comparison with the ILP model, proposed in Qi et al. (2018a), that neglects the time-dependency of passenger demands, to highlight the importance of taking it into account.

The Wuhan-Guangzhou high-speed line consists of 18 stations, but, since Wulongquan East station is a reserved station without passengers boarding and alighting, we do not take it into consideration in our experiments, and consider a railway line with 17 stations. We consider a time horizon from 6:30 to 12:30, i.e., t_0=6:30 and t_T=12:30, and different time interval lengths with $\delta = 60, 90, 120, 180$ and 360 min corresponding to 5 time intervals for the smallest δ and 1 time interval for the largest one. We consider all possible OD pairs along the line (i.e., 136 OD pairs that are then split in time intervals according to

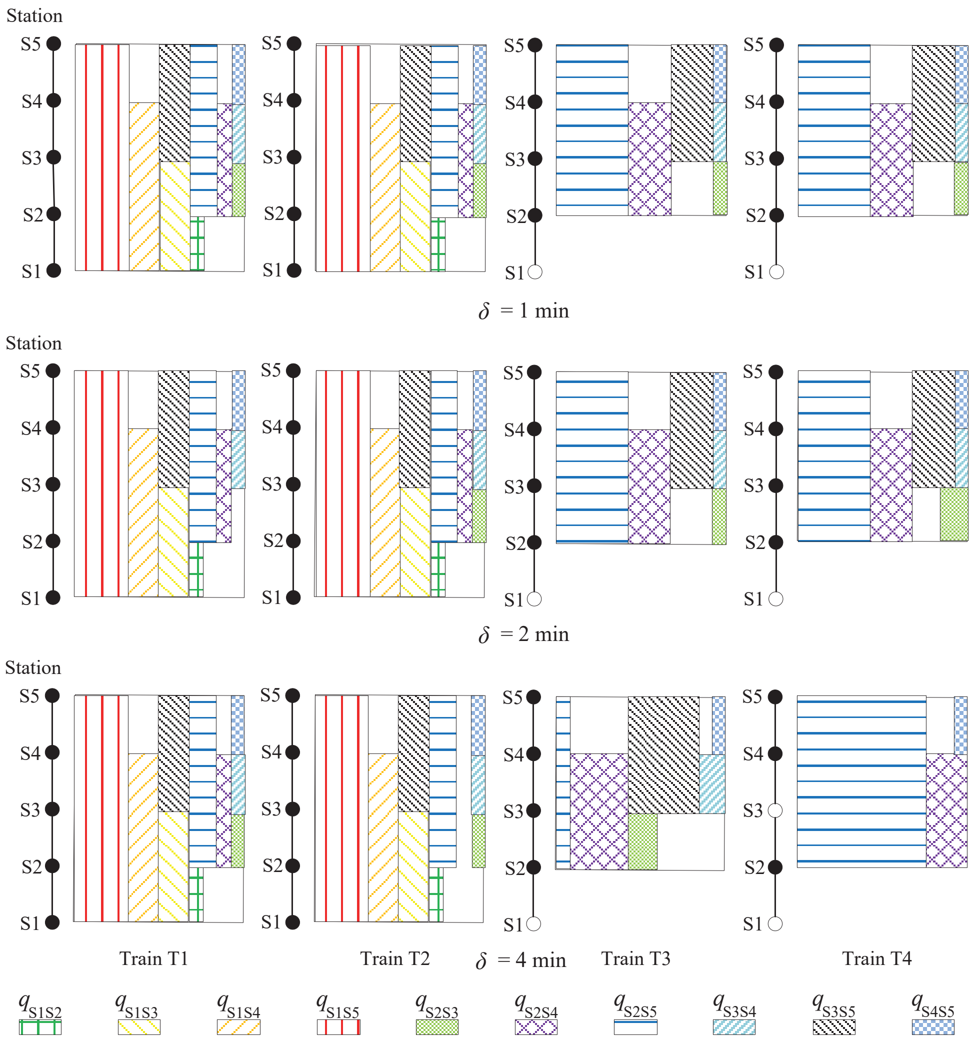

Fig. 5. The stop plans and tickets distributions with lengths of discrete time interval respectively as $\delta = 1, 2, 4$ min.

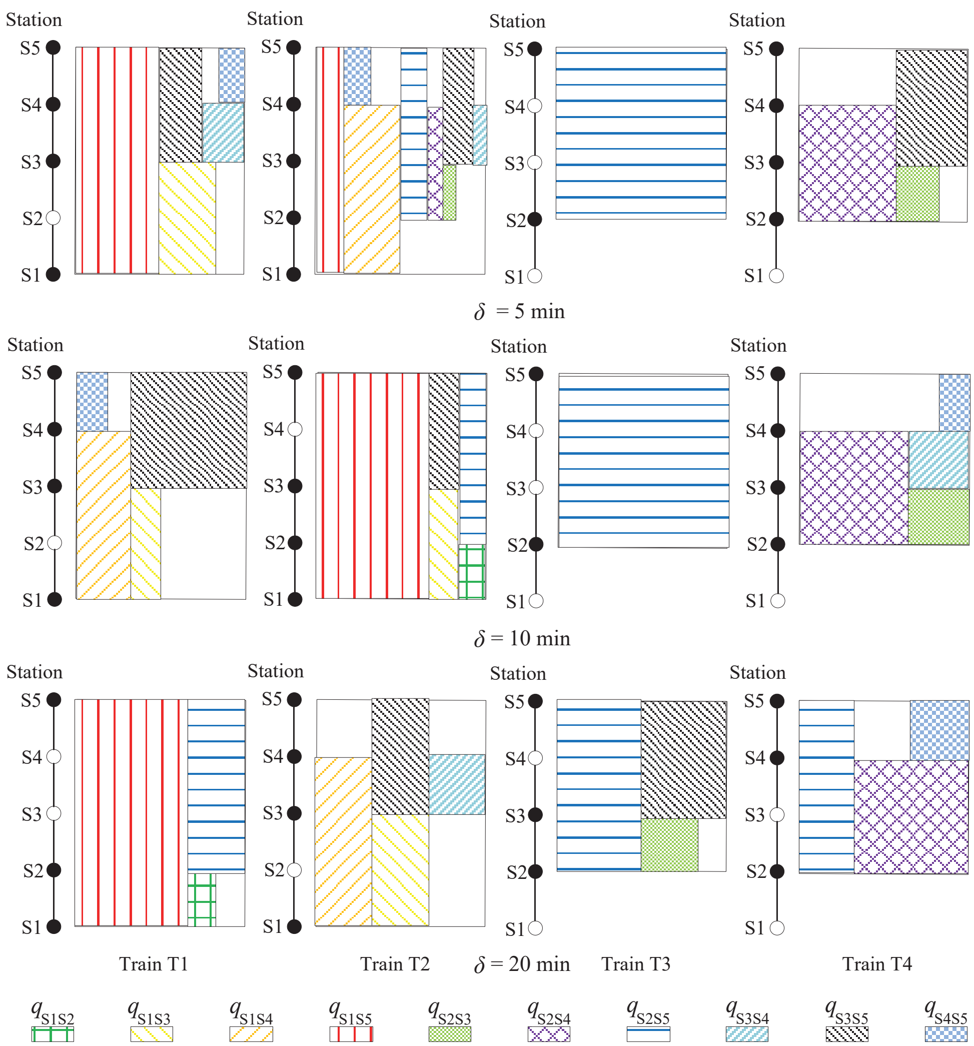

Fig. 6. The stop plans and tickets distributions with lengths of discrete time interval respectively as $\delta = 5, 10, 20$ min.

the desired passenger departure times), and 21854 passengers to be transported during the time horizon. The earliest departure time from the original station and latest arrival time at the terminal station of each train are respectively set as 6:30 and 23:30, i.e., T^E=6:30 and T^L=23:30.

In this line, 39 trains of three types (F-Trains, T-Trains and L-Trains), based on their operation zone, are operated during the considered time horizon. There are 3 trains (F-Trains) operated from Wuhan station to Changsha South station, 12 trains (T-Trains) from Changsha South station to Guangzhou South station, and the other 24 trains (L-Trains) from Wuhan station to Guangzhou South station. The distance and travel time for each train on each line segment are reported in Table 5. Note that the considered 39 trains must depart within the considered time horizon (t_0, t_T) of 6 hours.

Since some trains have their origin and terminal stations both on this line, other trains have either the origin or the terminal station (but not both) on this line, and other trains have both origin and terminal stations outside this line, but travel along the line, different train capacities are considered. In particular, the following values are used for the train capacity: 800 for trains of the first category, 300 for trains of the second one, and 200 for trains of the last one. In this way, we take into account that some train capacity is used along the other parts of the railway network. In particular, 4 T-Trains and 4 L-Trains have capacity 800, the maximum loading capacity of 3 F-Trains, 8 T-Trains and 17 L-Trains are set to 300, and the maximum loading capacity of 3 L-Trains is set to 200.

The minimum dwelling time at each station is set based on the passenger volume for that station. In particular, we consider the following values for the minimum dwelling time at each of the 15 stations of the line (excluding the first and last stations): 4, 4, 3, 3, 5, 3, 3, 3, 3, 3, 2, 3, 2, 3, 3 min. We set the maximum dwelling time at each station to 20 min: although this is a rather large value, we decided not to restrict it significantly in order to ensure feasibility of the time-dependent passenger demand, since shorter travel times will be preferred thanks to the objective function. Finally, we impose 2 min as minimum headway time between two consecutive train departures or arrivals at the same station.

Since we could not have access to historical data, the passenger demand between each OD pair and time interval has been determined by considering, according to the actual train timetable used in practice, the number of trains stopping at each station, the number of trains travelling along each section of the line, and the train departure times as a measure of the importance of the corresponding OD pair in specific time intervals.

Table 5. Distance and travel time for each train on each line segment.

Line segment	Distance (km)	Travel time (min)	Line segment	Distance (km)	Travel time (min)
Wuhan-Xianning North	85	18	Hengyang East-Leiyang West	55	13
Xianning North-Chibi North	43	13	Leiyang West-Chenzhou West	98	22
Chibi North-Yueyang East	87	19	Chenzhou West-Lechang East	106	23
Yueyang East-Miluo East	70	15	Lechang East-Shaoguan	44	10
Miluo East-Changsha South	77	18	Shaoguan-Yingde West	87	21
Changsha South-Zhuzhou West	52	13	Yingde West-Qingyuan	57	15
Zhuzhou West-Hengshan West	84	19	Qingyuan-Guanzhou North	36	11
Hengshan West-Hengyang East	41	11	Guanzhou North-Guanzhou South	47	16

Based on preliminary experiments, we used as terminating condition for the solution of the ILP model with CPLEX the optimality gap to be at most 10%. Indeed, we verified that the improvement of the optimality gap is rather slow, due to the complexity of the problem, and the computing time significantly increases when we consider small values of the time interval length. We report, in Section 4.2.1, the results obtained when the terminating condition is a given time limit.

The computational results are displayed in Table 6. The first column shows the considered time interval length, the second one reports the objective function value when the terminating condition is reached. In addition, we report the total number of train stops, since this is an element very closely related to the time-dependent demand satisfaction. Then, we report, in the fourth and fifth columns, respectively, the computing time (expressed in seconds) and the percentage optimality gap.

Table 6. The computational results for the Wuhan-Guangzhou high-speed railway corridor with different time interval lengths.

Length	Travel time	# of stops	Computing time	Optimality Gap%
60	9561	300	27564	9.93%
90	9544	276	1552	9.87%
120	8959	217	921	4.04%
180	9106	242	769	5.53%
360	8842	151	100	2.77%

From Table 6, we can clearly see that the solution process becomes harder when the time interval length is smaller, hence requiring larger computing times. In particular, more than 7 hours of computing time are required to solve the instance when δ is equal to 60 min, achieving a percentage optimality gap of 9.9%, while when δ is 360 min, a solution with gap 2.77% is obtained in less than 2 min. In addition, we observe that the total number of train stops increases when δ is smaller, in order to satisfy the time-dependent passenger demand: 300 stops are needed with δ equal to 60 min, while only 151 (about the half) with δ equal to 360 min. The travel time has a similar tendency, i.e., longer total travel time is experienced when δ is smaller, although this tendency is not sharp (most likely because we did not derive the optimal solutions). We can also notice that the travel time increase when δ changes from 360 to 60 is about 7.5%, which is a good compromise since, in the latter case, it is possible to satisfy all the desired departure intervals for every hour. As observed for the small scale example, also in the real-world case study, the appropriate interval length should be selected to achieve a good trade-off between the satisfaction of the passengers departure time intervals and the increase of train travel time and computing times: for the considered real-world instance, the computing time when $\delta = 60$ can still be considered as acceptable, since the TTSP-TD problem is solved in the planning phase. However, since shorter computing times would be preferred, we will show, in Section 4.2.1, the results obtained within three hours time limit.

To better analyze the obtained solutions, Figures 7 and 8 show, respectively, the timetables computed with δ equal to 60, and with larger time interval lengths ($\delta = 90$, 120, 180 and 360 min). In all figure, train types are distinguished by using different colors and line types.

By comparing the timetables displayed in Figures 7 and 8, we can see that the timetable obtained with time interval length of 60 min (Fig. 7) is the most balanced one, that provides service in a more homogeneous way along the time horizon. When we increase the length, trains with the same operation zone tend to be operated in a similar way: more tickets having the same OD pair are allocated to the same train, thus reducing the total travel time, but also disregarding the importance of time intervals.

4.2.1 Solutions obtained with three hours time limit

In this section, we analyze the results obtained when setting the computing time to three hours. For instances with δ between 90 and 360 min, the aim is to evaluate how the objective value and the optimality

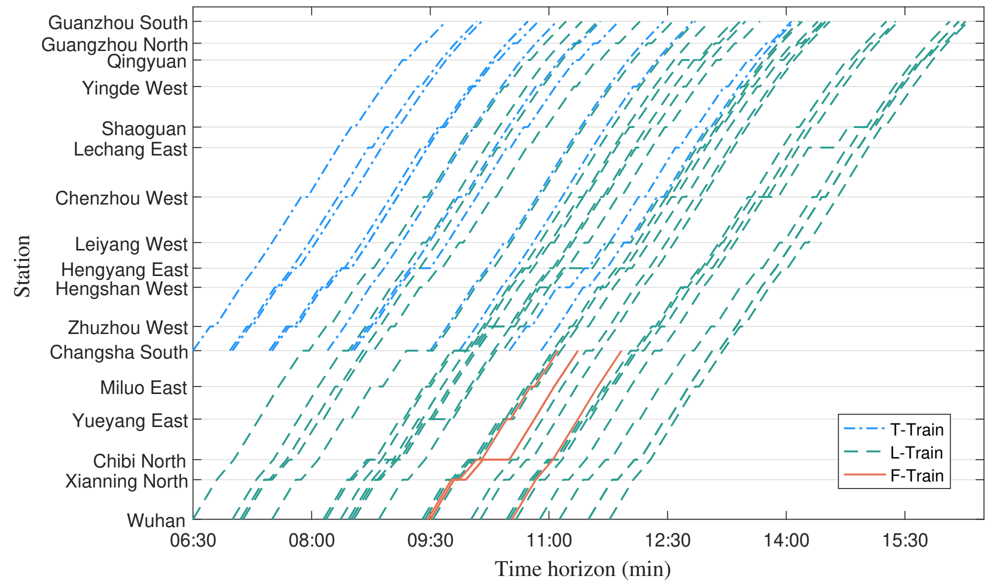

Fig. 7. The timetable with time interval length $\delta = 60$ min.

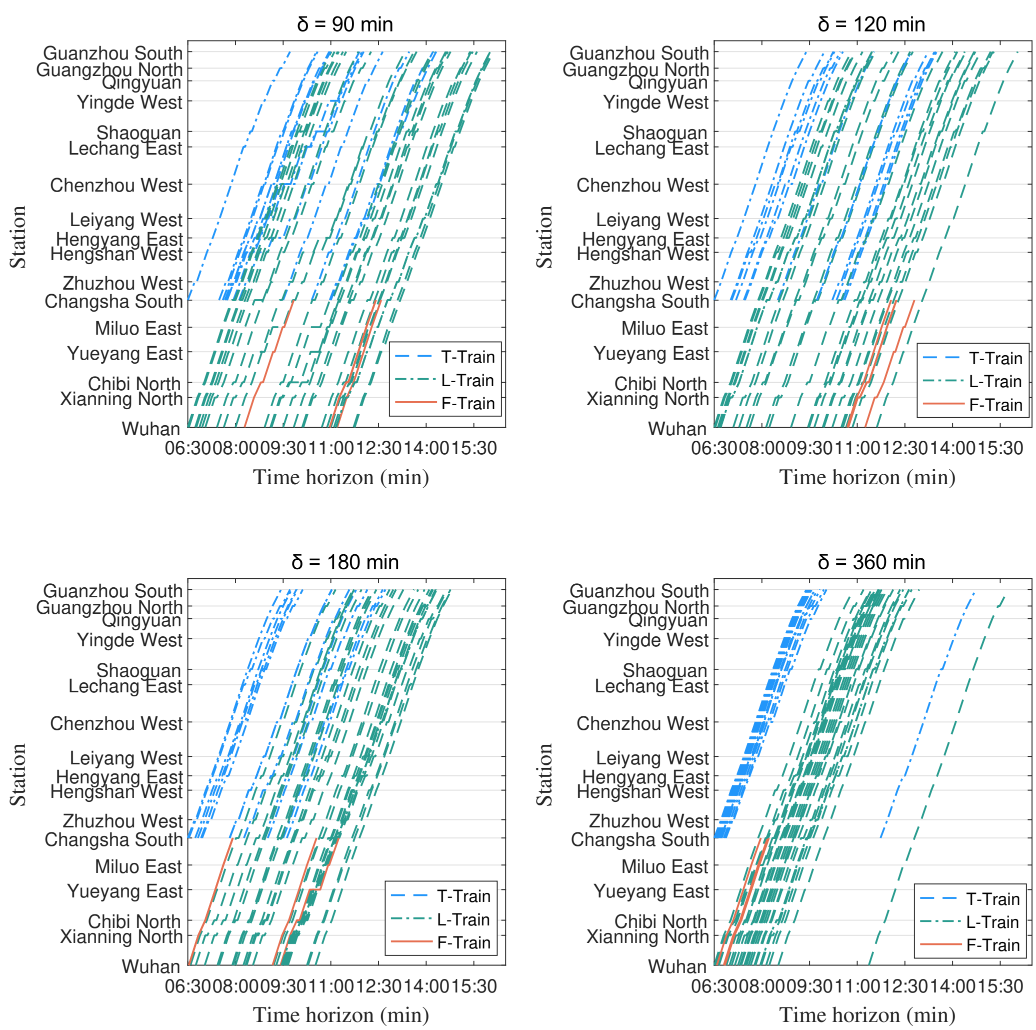

Fig. 8. The timetable with larger time interval lengths (δ = 90, 120, 180 and 360 min).

gap improve by allowing longer computing times, while for δ equal to 60 min we want to determine the quality of a solution found in this shorter computing time.

The results are reported in Table 7, where the columns have the same meaning as in Table 6. We can see that, for time interval length set to 60 min, the optimality gap increases to 12.66%, and the total travel time increases of about 3% with respect to the solution reported in Table 6: it is possible to fully satisfy the time-dependent demand in every hour by solving the ILP model within shorter computing time, at the expenses of an increase of the total train travel time. When we consider larger δ values, we can observe that, by allowing three hours of computing time, a significant improvement of 5.65% on the travel time is obtained for $\delta = 90$, for which the optimality gap is more than halved. Improvements can also be seen for the longer time interval lengths.

We can conclude that, for $\delta \geq 90$ minutes, a three hours time limit allows to obtain good quality solutions with a percentage optimality gap of at most 4.42%. When δ is set to 60 minutes, longer computing times are required: in this case, the total travel time (9561) increases of about 6% with respect to the case of $\delta = 90$ (9004) in order to satisfy the passenger desired departure time intervals for every hour, instead of every 90 minutes, and this increase represents the trade-off between demand satisfaction and travel time.

Table 7. The computational results for the Wuhan-Guangzhou high-speed railway corridor with three hours time limit.

Length	Travel time	# of stops	Optimality Gap
60	9858	326	12.66%
90	9004	226	4.42%
120	8885	193	3.18%
180	8831	179	2.56%
360	8726	138	1.40%

4.2.2 Results with different train capacity values

We consider $\delta = 60$ minutes, and change the train capacity of the different train categories. In the original case, the capacity value is 800 for trains of the first category, 300 for trains of the second one, and 200 for trains of the last one. We now evaluate the results obtained when all trains have the same capacity ranging from 800 down to 300. In addition, we consider as unchanged the capacities of the second and third categories, and decrease the capacity of the first category from 800 to 700. Finally, we again reduce the capacity of the first category to 700 or 600, but increase the capacity of the third category to 300. The time limit was set to 3 hours. The results are displayed in Table 8, where the first row corresponds to the original case.

We observe that, when all trains have the same capacity with value at least 400 (i.e., the capacity is increased for the trains of the second and third categories), the problem becomes easier to solve: the travel time is shorter and the number of stops smaller than in the original case. Moreover, the optimality gap is significantly reduced. However, when the capacity reaches value 300, no feasible solution can be found within the time limit. The same happens when we decrease the capacity of the first category to 700 while keeping the capacity unchanged for the other two categories. On the contrary, if we increase to 300 the capacity of trains in the last category while keeping the first category to 700, a good quality solution is obtained. Finally, if we further decrease the capacity of the first category to 600, we can observe that

Table 8. Results with different train capacity values (δ=60 min).

Train capacity			Travel time	# of stops	Optimality Gap %
First	Second	Third			
800	300	200	9858	326	12.66%
800	800	800	8988	228	4.34%
700	700	700	9084	243	5.33%
600	600	600	9204	240	6.43%
500	500	500	9127	262	5.50%
400	400	400	9229	272	6.20%
300	300	300	-	-	-
700	300	200	-	-	-
700	300	300	9138	255	5.66%
600	300	300	11698	416	26.17%

the solution value significantly increases and the optimality gap becomes very large. We also tested the latter case with a time limit of 7 hours: in this case, the total travel time becomes 9781, the number of stops 321 and the optimality gap 11.66%. We can conclude that, as expected, different train capacities lead to different results, and we can observe that improvements are achieved when the train capacity constraints are less strict.

4.2.3 Comparison with Qi et al. (2018a)

In this section, we report a comparison with the solution obtained by solving the model proposed in Qi et al. (2018a), where the time-dependency of the demand was not considered. In that case, to obtain timetables that are spread over the time horizon, a specific departure time window was imposed for each train. In this way, situations as in Fig. 8 with $\delta = 360$ were avoided. However, these departure time windows were predefined and not computed according to the desired time intervals. Hence the derived stop plan and timetable may not treat the time-dependency of passenger demand in an adequate way.

To show the advantage of the proposed model with respect to the one presented in Qi et al. (2018a), we compare the solutions obtained by solving these two models, and evaluate the number of passengers whose desired time interval is not satisfied by solving the model in Qi et al. (2018a). As mentioned in Section 3.5.1, to have a fair comparison, in the model by Qi et al. (2018a), we assign to each train as the departure time from its origin station the one obtained by solving the proposed model (24). In particular we consider the solution computed with time interval length 60 min. Then, with these given departure times, we solve the model by Qi et al. (2018a).

Figure 9 shows the obtained timetable: as we can see, trains are spread along the time horizon. The total travel time and total number of stops of the obtained solution are respectively 8925 min and 177, which are both smaller than those of the solution obtained by the proposed model. However, the model by Qi et al. (2018a) neglects the time-dependency of passenger demand. To evaluate the impact of this simplification, we fixed the timetable and stop plan obtained by solving the model of Qi et al. (2018a), and solved the evaluation model EM (see Section 3.5.1) to derive the number of passengers who were not served in their desired departure time interval. The outcome is that 7870 passengers out of 21854 cannot depart in their desired time interval, which is 36% (7870/21854=36.01%) of the total number of passengers. Therefore, a very large number of passengers could not find the expected train service (even with time interval length of one hour), and it is evident that time-dependency is an important factor in train stop planning and timetabling.

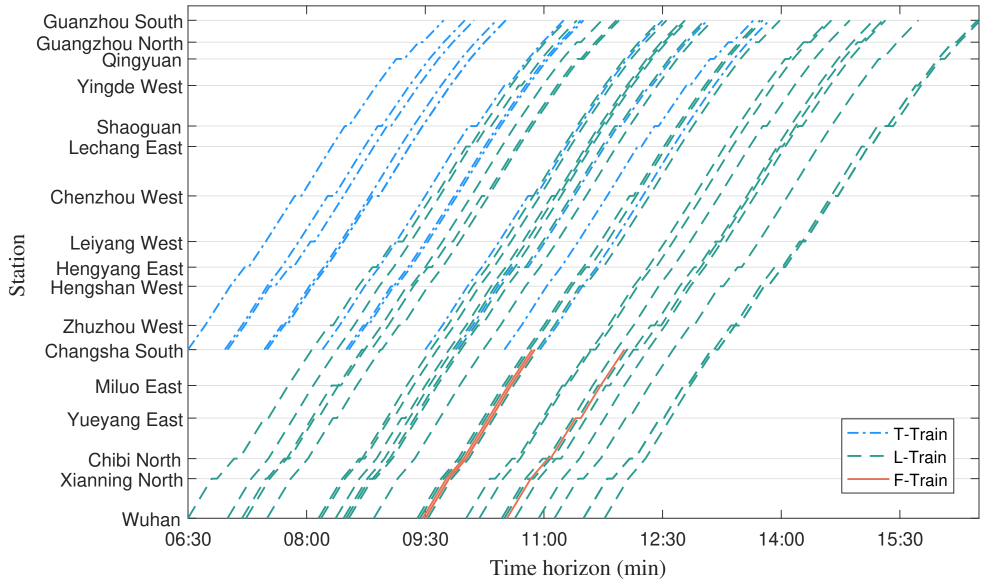

Fig. 9. The timetable generated by the model in Qi et al. (2018a).

To further compare the outcome of proposed model with respect to the one presented in Qi et al. (2018a), we compute a posteriori the deviation times between the passengers desired departure times and the train departure times for the two models. Although the deviation times are not minimized in the objective function of the proposed ILP model, the maximum deviation time is limited by δ, since all passengers depart in their desired time interval. On the contrary, the deviation times are completely neglected in the model by Qi et al. (2018a). The deviation times are computed after the stop plans, timetables and seat allocations have been obtained by solving each of the two models. To compute the best total deviation time for a given solution, we solve a Linear Programming (LP) model (called deviation time model, DTM) that determines for each passenger the selection of the train that minimizes the total deviation time. DTM contains constraints to impose that passengers of all OD pairs travel on a train that stops at both origin and destination stations, and has available seats, according to the stop plans, timetables and seat allocations derived by solving each of the two ILP models. DTM is reported in the Appendix A.2.

DTM is solved by CPLEX in less than a second, since stop plans, timetables and seat allocations are fixed. For the proposed ILP model, the total deviation time is $472,709$ and the average deviation time 21.63 minutes. For the model by Qi et al. (2018a), the total deviation time is $1,339,744$ and the average deviation time 61.30 minutes. Therefore, the total and average deviation times of the proposed model, although not minimized, are significantly smaller than those of the other model, showing that the former is able to appropriately satisfy the time-dependent passenger demand.

In summary, we can conclude that stop plans and timetables that satisfy the passenger desired departure time intervals can be derived by solving the proposed ILP model in reasonable computing times. Different time interval lengths δ lead to different levels of satisfaction of the time-dependent demand and total travel times: the value for δ has to be selected based on the available computing time, and, for the considered instance, $\delta = 60$ or $\delta = 90$ min turn out to be the most appropriate options.

5 Conclusions and Further Research

Aiming at generating a practical operation plan, this paper studies the integrated train stop planning and timetabling problem with time-dependent passenger demand, i.e., the desired departure time intervals of the passengers are taken into account. We formulated an Integer Linear Programming (ILP) model for this integrated problem, which contains several constraints that link the individual problems (stop planning, timetabling and passenger flow distribution) in order to consider them in a joint way. The CPLEX solver was used to solve the model: we executed two sets of experiments, on a small-scale example and on an instance of the Wuhan-Guangzhou high-speed railway corridor. The time horizon was split into time intervals of different lengths to show the influence of the length of the passengers desired departure time intervals on the obtained solutions. On one hand, a smaller length guarantees shorter deviation times from the desired departure times, but, on the other hand, it causes an increase of train travel times and requires longer computing times for solving the problem. We also compared the obtained stop plan and timetable with those determined by solving a model from the literature that neglects the time-dependency of the passenger demand, and showed that, in the latter case, 36% of the passengers could not depart in the desired time interval.

Further research will focus on the following two aspects. First, given the complexity of the integrated problem, efficient heuristic algorithms will be designed in order to deal with larger scale instances and short time interval length (e.g., 30 minutes), where the latter element becomes relevant if the passenger demand is known with high precision. In this case, an interesting extension is to include the minimization of the time deviation between the train departure times and the passenger desired departure times in the objective function. Second, as the problem is also related to the ticket distribution issue, optimization of ticket pricing, considering different travel classes, and choice behavior of passengers can be incorporated into the integrated problem to evaluate how the economic aspects can influence the stop plans and timetables.

Acknowledgement

This research was supported by the State Key Laboratory of Rail Traffic Control and Safety (Contract No. RCS2020ZT002), Beijing Jiaotong University, and the National Natural Science Foundation of China (Nos. 72001019, 71825004, 71621001).

References

Borndörfer, R., Klug, T., Lamorgese, L., Mannino, C., Reuther, M., Schlechte, T., (Eds.). 2018. Handbook of Optimization in the Railway Industry (Vol. 268). Springer.

Barrena, E., Canca, D., Coelho, L. C., Laporte, G., 2014a. Exact formulations and algorithm for the train timetabling problem with dynamic demand. Computers & Operations Research 44, 66-74.

Barrena, E., Canca, D., Coelho, L. C., Laporte, G., 2014b. Single-line rail rapid transit timetabling under dynamic passenger demand. Transportation Research Part B: Methodological 70, 134-150.

Cacchiani, V., Qi, J., Yang, L. 2020. Robust optimization models for integrated train stop planning and timetabling with passenger demand uncertainty. Transportation Research Part B: Methodological 136, 1-29.

Cai, X., Goh, C., 1994. A fast heuristic for the train scheduling problem. Computers & Operations Research 21(5), 499-510.

Canca, D., Barrena, E., Algaba, E., Zarzo, A., 2014. Design and analysis of demand-adapted railway timetables. Journal of Advanced Transportation 48, 119-137.

Cao, Z., Yuan, Z., Li, D., 2014. Estimation method for a skip-stop operation strategy for urban rail transit in China. Journal of Modern Transportation 22(3), 174-182.

Deng, L., Shi, F., Zhou, W., 2009. Stop schedule plan optimization for passenger train. China Railway Science 30 (4), 103-107.

Dong, X., Li, D., Yin, Y., Ding, S., Cao, Z., 2020. Integrated optimization of train stop planning and timetabling for commuter railways with an extended adaptive large neighborhood search metaheuristic approach. Transportation Research Part C: Emerging Technologies 117, 102681.

Fu, H., Nie, L., 2011. Economy and efficiency oriented passenger flow organization: train stop scheduling for high-speed railways. Journal of Belting Institute of Technology 20, 79-83.

Fu, H., Nie, L., Sperry, BR., He, Z., 2012. Train stop scheduling in a high-speed rail network by utilizing a two-stage approach. Mathematical Problem in Engineering 5, 123-132.

Fu, H., Sperry, BR., Nie, L., 2013. Operational impacts of using restricted passenger flow assignment in high-speed train stop scheduling problem. Mathematical Problems in Engineering 78(70), 143-175.

Gao, Y., Kroon, L., Yang, L., Gao, Z., 2018. Three-stage optimization method for the problem of scheduling additional trains on a high-speed rail corridor. Omega 80, 175-191.

Hao, W., Meng, L., Corman, F., Long, S., Jiang, X., 2019, September. A train timetabling and stop planning optimization model with passenger demand. In RailNorrköping 2019. 8th International Conference on Railway Operations Modelling and Analysis (ICROMA), Norrköping, Sweden, June 17th-20th, 2019 (Vol. 69, pp. 390-406). Linköping University Electronic Press.

Higgins, A., Kozan, E., Ferreira, L., 1997. Heuristic techniques for single line train scheduling. Journal of Heuristics 3(1), 43-62.

Jiang, F., Cacchiani, V., Toth, P., 2017. Train timetabling by skip-stop planning in highly congested lines. Transportation Research Part B: Methodological 104, 149-174.

Lusby, R. M., Larsen, J., Ehrgott, M., Ryan, D., 2011. Railway track allocation: models and methods. OR spectrum 33(4), 843-883.

Meng, L., Zhou, X., 2019. An integrated train service plan optimization model with variable demand: A team-based scheduling approach with dual cost information in a layered network. Transportation Research Part B: Methodological 125, 1-28.

Mo, P., Yang, L., Wang, Y., Qi, J., 2019. A flexible metro train scheduling approach to minimize energy cost and passenger waiting time. Computers & Industrial Engineering 132, 412-432.

Niu, H., Zhou, X., 2013. Optimizing urban rail timetable under time-dependent demand and oversaturated conditions. Transportation Research Part C: Emerging Technologies 36, 212-230.

Niu, H., Zhou, X., Gao, R., 2015. Train scheduling for minimizing passenger waiting time with time-dependent demand and skip-stop patterns: Nonlinear integer programming models with linear constraints. Transportation Research Part B: Methodological 76, 117-135.

Parbo, J., Nielsen, O., Prato, C., 2016. Passenger perspectives in railway timetabling: A literature review. Transport Reviews 36(4), 500-526.

Parbo, J., Nielsen, O. A., Prato, C. G., 2018. Reducing passengers' travel time by optimising stopping patterns in a large-scale network: A case-study in the Copenhagen Region. Transportation Research Part A: Policy and Practice 113, 197-212.

Qi, j., Li, S., Gao, Y., Yang, K., Liu, P., 2018a. Joint optimization model for train scheduling and train stop planning with passengers distribution on railway corridors. Journal of the Operational Research Society 69(4), 556-570.

Qi, J., Cacchiani, V., Yang, L., 2018b. Robust train timetabling and stop planning with uncertain passenger demand. Electronic Notes in Discrete Mathematics 69, 213-220.

Qi, J., Yang, L., Di, Z., Li, S., Yang, K., Gao, Y., 2018c. Integrated optimization for train operation zone and stop plan with passenger distributions. Transportation Research Part E: Logistics and Transportation Review 109, 151-173.

Robenek, T., Maknoon, Y., Azadeh, S., Chen, J., Bierlaire, M., 2016. Passenger centric train timetabling problem. Transportation Research Part B: Methodological 89, 107-126.

Robenek, T., Azadeh, S., Maknoon, Y., Lapparent, M., Bierlaire, M., 2018. Train timetable design under elastic passenger demand. Transportation Research Part B: Methodological 118, 19-38.

Schöbel, A., 2012. Line planning in public transportation: models and methods. OR spectrum 34(3), 491-510.

Shang, P., Li, R., Liu, Z., Yang, L., Wang, Y., 2018. Equity-oriented skip-stopping schedule optimization in an oversaturated urban rail transit network. Transportation Research Part C: Emerging Technologies 89, 321-343.

Szpigel, B., 1973. Optimal train scheduling on a single track railway. Operations Research 72, 343-352.

Sun, L., Jin, J. G., Lee, D. H., Axhausen, K. W., Erath, A., 2014. Demand-driven timetable design for metro services. Transportation Research Part C: Emerging Technologies 46, 284-299.

Yang, L., Qi, j., Li, S., Yuan, G., 2016. Collaborative optimization for train scheduling and train stop planning on high-speed railways. Omega 64, 57-76.

Yang, S., Liao, F., Wu, J., Timmermans, H. J., Sun, H., Gao, Z., 2020. A bi-objective timetable optimization model incorporating energy allocation and passenger assignment in an energy-regenerative metro system. Transportation Research Part B: Methodological 133, 85-113.

Yue, Y., Wang, S., Zhou, L., Tong, L., Saat, M., 2016. Optimizing train stopping patterns and schedules for high-speed passenger rail corridors. Transportation Research Part C: Emerging Technologies 63, 126-146.

Yin, J., Yang, L., Tang, T., Gao, Z., Ran, B., 2017. Dynamic passenger demand oriented metro train scheduling with energy-efficiency and waiting time minimization: mixed-integer linear programming approaches. Transportation Research Part B: Methodological 97, 182-213.

Zhou, X., Zhong, M., 2005. Bicriteria train scheduling for high-speed passenger railroad planning applications. European Journal of Operational Research 167, 752-771.

Zhu, Y., Mao, B., Bai, Y., Chen, S., 2017. A bi-level model for single-line rail timetable design with consideration of demand and capacity. Transportation Research Part C: Emerging Technologies 85, 211-233.

A Appendix

A.1 Passenger demand of the small-scale example

Table 9 displays the detailed passenger demand with different time interval lengths. The first column shows the length of time interval, and the second column has one row for each specific interval. For each time interval, the third column reports the passenger demand during that time interval. From Table 9, we can see that with the increase of the time interval length, the difference between the desired departure times of passengers traveling between the same OD stations is less evident, and passengers with different desired departure times are grouped together. In particular, when we consider only one time interval, all passengers traveling between the same OD stations can be transported by any train.

Table 9. The passenger demand for the small-scale example with different time interval lengths.

Length	Interval	Passenger demand
1 min	$[0, 1]$	$q_{S1S2} = 5, q_{S1S3} = 10, q_{S1S4} = 10, q_{S1S5} = 20;$
	$[1, 2]$	$q_{S2S3} = 5, q_{S2S4} = 15, q_{S2S5} = 25;$
	$[3, 4]$	$q_{S2S3} = 5, q_{S2S4} = 5, q_{S2S5} = 10;$
	$[4, 5]$	$q_{S1S2} = 5, q_{S1S3} = 10, q_{S1S4} = 10, q_{S1S5} = 20,$ $q_{S2S3} = 5, q_{S2S4} = 15, q_{S2S5} = 25, q_{S3S4} = 5, q_{S3S5} = 15;$
	$[6, 7]$	$q_{S3S4} = 5, q_{S3S5} = 10;$
	$[7, 8]$	$q_{S4S5} = 5;$
	$[8, 9]$	$q_{S2S3} = 5, q_{S2S4} = 5, q_{S2S5} = 10, q_{S3S4} = 5, q_{S3S5} = 15;$
	$[9, 10]$	$q_{S4S5} = 5;$
	$[12, 13]$	$q_{S3S4} = 5, q_{S3S5} = 10, q_{S4S5} = 5;$
	$[16, 17]$	$q_{S4S5} = 5;$
2 min	$[0, 2]$	$q_{S1S2} = 5, q_{S1S3} = 10, q_{S1S4} = 10, q_{S1S5} = 20, q_{S2S3} = 5, q_{S2S4} = 15, q_{S2S5} = 25;$
	$[2, 4]$	$q_{S2S3} = 5, q_{S2S4} = 5, q_{S2S5} = 10;$
	$[4, 6]$	$q_{S1S2} = 5, q_{S1S3} = 10, q_{S1S4} = 10, q_{S1S5} = 20,$ $q_{S2S3} = 5, q_{S2S4} = 15, q_{S2S5} = 25, q_{S3S4} = 5, q_{S3S5} = 15;$
	$[6, 8]$	$q_{S3S4} = 5, q_{S3S5} = 10, q_{S4S5} = 5;$
	$[8, 10]$	$q_{S2S3} = 5, q_{S2S4} = 5, q_{S2S5} = 10, q_{S3S4} = 5, q_{S3S5} = 15, q_{S4S5} = 5;$
	$[12, 14]$	$q_{S3S4} = 5, q_{S3S5} = 10, q_{S4S5} = 5;$
	$[16, 18]$	$q_{S4S5} = 5;$
4 min	$[0, 4]$	$q_{S1S2} = 5, q_{S1S3} = 10, q_{S1S4} = 10, q_{S1S5} = 20, q_{S2S3} = 10, q_{S2S4} = 20, q_{S2S5} = 35;$
	$[4, 8]$	$q_{S1S2} = 5, q_{S1S3} = 10, q_{S1S4} = 10, q_{S1S5} = 20,$ $q_{S2S3} = 5, q_{S2S4} = 15, q_{S2S5} = 25, q_{S3S4} = 10, q_{S3S5} = 25, q_{S4S5} = 5;$
	$[8, 12]$	$q_{S2S3} = 5, q_{S2S4} = 5, q_{S2S5} = 10, q_{S3S4} = 5, q_{S3S5} = 15, q_{S4S5} = 5;$
	$[12, 16]$	$q_{S3S4} = 5, q_{S3S5} = 10, q_{S4S5} = 5;$
	$[16, 20]$	$q_{S4S5} = 5;$
5 min	$[0, 5]$	$q_{S1S2} = 10, q_{S1S3} = 20, q_{S1S4} = 20, q_{S1S5} = 40, q_{S2S3} = 15,$ $q_{S2S4} = 35, q_{S2S5} = 60, q_{S3S4} = 5, q_{S3S5} = 15;$
	$[5, 10]$	$q_{S2S3} = 5, q_{S2S4} = 5, q_{S2S5} = 10, q_{S3S4} = 10, q_{S3S5} = 25, q_{S4S5} = 10;$
	$[10, 15]$	$q_{S3S4} = 5, q_{S3S5} = 10, q_{S4S5} = 5;$
	$[15, 20]$	$q_{S4S5} = 5;$
10 min	$[0, 10]$	$q_{S1S2} = 10, q_{S1S3} = 20, q_{S1S4} = 20, q_{S1S5} = 40, q_{S2S3} = 20,$ $q_{S2S4} = 40, q_{S2S5} = 70, q_{S3S4} = 15, q_{S3S5} = 40, q_{S4S5} = 10;$
	$[10, 20]$	$q_{S3S4} = 5, q_{S3S5} = 10, q_{S4S5} = 10;$
20 min	$[0, 20]$	$q_{S1S2} = 10, q_{S1S3} = 20, q_{S1S4} = 20, q_{S1S5} = 40, q_{S2S3} = 20,$ $q_{S2S4} = 40, q_{S2S5} = 70, q_{S3S4} = 20, q_{S3S5} = 50, q_{S4S5} = 20;$

A.2 Deviation time model

Let TP be the set of all passengers desired departure times, and p_{ij}^{τ} the number of passengers of OD pair (i, j) who want to depart at time $\tau \in TP$ ($t_0 \leq \tau \leq t_T$, $i, j \in S$ with $i < j$). The stop plans, timetables and seat allocations are obtained by solving either the proposed ILP model for the TTSP-TD problem or the model presented in Qi et al. (2018a), and are input to the deviation time model. Based on the computed stop plans, we let $\bar{S}_k$ be the set of stations at which train $k \in K$ stops (including its origin and destination). Based on the obtained timetables, we let $\bar{t}_{ki}^{d}$ be the departure time of train $k \in K$ from station $i \in \bar{S}_k$, and $\bar{q}_{ij}^{kt_{ki}}$ the number of seats allocated on train $k \in K$, with departure time interval t_{ki}, to passengers of OD pair (i, j) ($i, j \in \bar{S}_k$ with $i < j$): note that t_{ki} is the unique departure time interval of train k from station i according to its departure time $\bar{t}_{ki}^{d}$ in the computed timetable. Let $z_{ij}^{k\tau}$ be a non-negative variable representing the number of passengers with desired departure time τ who travel from station i to station j on train k ($k \in K$, $i, j \in \bar{S}_k$ with $i < j$). The deviation time model reads as follows:

$$\min \sum_{k\in K} \sum_{i\in \bar{S}_k} \sum_{j\in \bar{S}_k : j>i} \sum_{\tau\in TP} |\tau - \bar{t}_{ki}^{d}|\ z_{ij}^{k\tau} \tag{29}$$

$$\sum_{k\in K: i,j\in \bar{S}_k} z_{ij}^{k\tau} = p_{ij}^{\tau} \quad \forall\ i, j \in S \text{ with } i < j,\ \tau \in TP \tag{30}$$

$$\sum_{\tau\in TP} z_{ij}^{k\tau} \leq \bar{q}_{ij}^{kt_{ki}} \quad \forall\ k \in K,\ i, j \in \bar{S}_k \text{ with } i < j \tag{31}$$

$$0 \leq z_{ij}^{k\tau} \leq \min\{p_{ij}^{\tau}, \bar{q}_{ij}^{kt_{ki}}\} \quad \forall\ k \in K,\ i, j \in \bar{S}_k \text{ with } i < j,\ \tau \in TP \tag{32}$$

The objective function (29) minimizes the sum of all deviation times, computed by considering the difference between the desired departure time τ and the train departure time $\bar{t}_{ki}^{d}$ for all passengers. Constraints (30) require that all passengers of each OD pair (i, j) and each desired departure time τ are transported by trains that stop at both stations i and j. Constraints (31) ensure that, for each train k, the allocated seats for transporting passengers of each OD pair (i, j) are respected. Finally, constraints (32) define the non-negativity of the variables and impose an upper bound on the variable value: indeed, $z_{ij}^{k\tau}$ cannot be larger than the minimum between the number of passengers of OD pair (i, j) with desired departure time τ and the number of seats allocated on train k from i to j. Note that, when we compute the deviation times for the proposed TTSP-TD model, the upper bound of $z_{ij}^{k\tau}$ is zero for all trains that do not depart in an interval containing τ (indeed, $\bar{q}_{ij}^{kt_{ki}} = 0$ for these trains), i.e., passengers will depart in their desired time interval. In addition, it is not necessary to impose the integrality constraints on the variables, since the constraint matrix is Totally Unimodular and the right-hand-sides are integer.