

Alma Mater Studiorum Università di Bologna
Archivio istituzionale della ricerca

Erratum to: Global L_p estimates for degenerate Ornstein-Uhlenbeck operators with variable coefficients : L_p estimates for degenerate Ornstein-Uhlenbeck type operators (Mathematische Nachrichten, (2013), 286, 11-12, (1087-1101), 10.1002/mana.201200189)

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:

Bramanti M., Cupini G., Lanconelli E., Priola E. (2021). Erratum to: Global L_p estimates for degenerate Ornstein-Uhlenbeck operators with variable coefficients : L_p estimates for degenerate Ornstein-Uhlenbeck type operators (Mathematische Nachrichten, (2013), 286, 11-12, (1087-1101), 10.1002/mana.201200189). MATHEMATISCHE NACHRICHTEN, 294(9), 1839-1842 [10.1002/mana.202100054].

Availability:

This version is available at: <https://hdl.handle.net/11585/841679> since: 2021-12-14

Published:

DOI: <http://doi.org/10.1002/mana.202100054>

Terms of use:

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (<https://cris.unibo.it/>).
When citing, please refer to the published version.

(Article begins on next page)

This is the final peer-reviewed accepted manuscript of:

Bramanti, M., Cupini, G., Lanconelli, E., & Priola, E. (2021). Erratum to: Global L_p estimates for degenerate Ornstein-Uhlenbeck operators with variable coefficients : L_p estimates for degenerate Ornstein-Uhlenbeck type operators (Mathematische Nachrichten, (2013), 286, 11-12, (1087-1101), 10.1002/mana.201200189). Mathematische Nachrichten, 294(9), 1839-1842

The final published version is available online at <https://dx.doi.org/10.1002/mana.202100054>

Rights / License:

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (<https://cris.unibo.it/>)

When citing, please refer to the published version.

Errata to “Global L^p estimates for degenerate
Ornstein-Uhlenbeck operators with variable
coefficients”, Math. Nachr. 286, No. 11–12, 1087
– 1101 (2013)

Marco Bramanti, Giovanni Cupini,
Ermanno Lanconelli, Enrico Priola

May 24, 2021

In this note we want to point out and correct a mistake in our paper [1], where we consider a class of Ornstein-Uhlenbeck operators on \mathbb{R}^N

$$\mathcal{A} = \sum_{i,j=1}^{p_0} a_{ij}(x) \partial_{x_i x_j}^2 + \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j},$$

together with the corresponding Kolmogorov-Fokker-Planck operators on \mathbb{R}^{N+1}

$$L = \sum_{i,j=1}^{p_0} a_{ij}(z) \partial_{x_i x_j}^2 + \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j} - \partial_t$$

(here $z = (x, t)$). We will not recall here the structural assumptions on the matrix $B = (b_{ij})_{i,j=1}^N$. The matrix $A_0 = (a_{ij}(x))_{i,j=1}^{p_0}$ is a $p_0 \times p_0$ ($p_0 \leq N$) symmetric, bounded and uniformly positive definite matrix:

$$\frac{1}{\Lambda} |\xi|^2 \leq \sum_{i,j=1}^{p_0} a_{ij}(x) \xi_i \xi_j \leq \Lambda |\xi|^2 \quad (1)$$

for all $\xi \in \mathbb{R}^{p_0}$, $x \in \mathbb{R}^N$ (or $z \in \mathbb{R}^{N+1}$) and for some constant $\Lambda \geq 1$. The entries a_{ij} are assumed to satisfy a continuity condition which will be clarified in a moment. The main result in [1] is the following:

Theorem 0.1 (see [1, Thm. 1.1]) *For every $p \in (1, \infty)$ there exists a constant $c > 0$, depending on p, N, p_0 , the matrix B , the number Λ in (1) and the continuity modulus ω ,*

$$\omega(r) = \max_{i,j=1,\dots,p_0} \sup_{\substack{x,y \in \mathbb{R}^N \\ |x-y| \leq r}} |a_{ij}(x) - a_{ij}(y)|, \quad (2)$$

such that for every $u \in C_0^\infty(\mathbb{R}^N)$ one has:

$$\begin{aligned} \sum_{i,j=1}^{p_0} \left\| \partial_{x_i x_j}^2 u \right\|_{L^p(\mathbb{R}^N)} &\leq c \left\{ \|\mathcal{A}u\|_{L^p(\mathbb{R}^N)} + \|u\|_{L^p(\mathbb{R}^N)} \right\}, \\ \left\| \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j} u \right\|_{L^p(\mathbb{R}^N)} &\leq c \left\{ \|\mathcal{A}u\|_{L^p(\mathbb{R}^N)} + \|u\|_{L^p(\mathbb{R}^N)} \right\}. \end{aligned}$$

This result is derived from an analogous L^p estimate holding for L on a strip $S_T = \mathbb{R}^N \times [-T, T]$:

Theorem 0.2 (see [1, Thm. 3.1]) *Let L be as above, with uniformly continuous coefficients a_{ij} . For every $p \in (1, \infty)$ there exist constants $c, T > 0$ depending on p, N, p_0 , the matrix B , the number Λ in (1), c also depending on the modulus of continuity ω*

$$\omega(r) = \max_{i,j=1,\dots,p_0} \sup_{\substack{z_1, z_2 \in \mathbb{R}^{N+1} \\ |z_1 - z_2| \leq r}} |a_{ij}(z_1) - a_{ij}(z_2)|, \quad (3)$$

such that

$$\sum_{i,j=1}^{p_0} \left\| \partial_{x_i x_j}^2 u \right\|_{L^p(S_T)} \leq c \left\{ \|Lu\|_{L^p(S_T)} + \|u\|_{L^p(S_T)} \right\}$$

for every $u \in C_0^\infty(S_T)$.

Now, in the statement of the above theorem we assumed the coefficients a_{ij} to be uniformly continuous in S_T in *Euclidean sense*. However, the assumption that we actually use in the proof of Proposition 3.2 (which implies the above theorem), is the global uniform continuity of the coefficients *with respect to the local quasidistance d* in \mathbb{R}^{N+1} which is introduced in the paper. Although the topology induced by d coincides with the Euclidean topology, so that continuity with respect to the two structures is the same thing, global uniform continuity is a different issue. In particular, global uniform continuity in Euclidean sense does not imply global uniform continuity with respect to d . Accordingly, also the assumption in the statement of [1, Thm. 1.1] must be corrected. This means that the coefficients $a_{ij}(x)$, which now are defined in \mathbb{R}^N , must be uniformly continuous with respect to d if they are regarded as defined in a strip S_T . Let us make precise the above corrections.

The definition (3) of the continuity modulus $\omega(r)$ (when the coefficients are defined in the strip S_T) must be changed to:

$$\omega_{S_T}(r) = \max_{i,j=1,\dots,p_0} \left\{ \sup |a_{ij}(z) - a_{ij}(\zeta)| : d(z, \zeta) < r, z, \zeta \in S_T \right\}.$$

The definition (2) of the continuity modulus $\omega(r)$ (when the coefficients are defined in \mathbb{R}^N) must be changed to:

$$\omega_{\mathbb{R}^N}(r) = \max_{i,j=1,\dots,p_0} \left\{ \sup |a_{ij}(x) - a_{ij}(y)| : \inf_{|s| < T, |t| < T} d((x, t), (y, s)) < r, x, y \in \mathbb{R}^N \right\} \quad (4)$$

(where $T > 0$ must be small enough). The global estimates proved in [1, Thm. 1.1, Thm. 3.1] hold, with the same proof, with the constant depending on these moduli.

We end with a remark and an example that should better enlighten the relation between uniform continuity in the two senses.

Remark 0.3 *In the case of time-independent coefficients, let us compare global uniform continuity in Euclidean sense with global uniform continuity w.r.t. $\omega_{\mathbb{R}^N}(r)$ in (4). Since*

$$\inf_{|s| < T, |t| < T} d((x, t), (y, s)) \leq d((x, 0), (y, 0)) = \sum_{i=1}^N |(x - y)_i|^{1/q_i}.$$

(where q_i are the positive integers defined in [1, p.1091]), for $r < 1$ we have

$$|x - y| < r \Rightarrow \inf_{|s| < T, |t| < T} d((x, t), (y, s)) < Nr^{1/q_N}$$

so that

$$\max_{i,j=1,\dots,p_0} \{ \sup |a_{ij}(x) - a_{ij}(y)| : |x - y| < r \} \leq \omega_{\mathbb{R}^N}(Nr^{1/q_N}).$$

This shows that global uniform continuity w.r.t. $\omega_{\mathbb{R}^N}(t)$ implies global uniform continuity in Euclidean sense.

Example 0.4 *Let us consider the simplest example of degenerate KFP operator,*

$$\mathcal{L}u = u_{x_1 x_1} + x_1 u_{x_2} - u_t.$$

We have (keeping the notation in [1, p.1089])

$$E(s) = \begin{bmatrix} 1 & 0 \\ -s & 1 \end{bmatrix}$$

and, letting $y = (y_1, y_2)$,

$$E(s)y = (y_1, y_2 - sy_1).$$

Choosing, for $\varepsilon > 0$,

$$x = \left(\frac{1}{\varepsilon}, 1\right); y = \left(\frac{1}{\varepsilon}, 2\right); s = 0; t = \varepsilon$$

we have $(y, s) - (x, t) = (0, 1, -\varepsilon)$, so that $|(y, s) - (x, t)| \rightarrow 1$ as $\varepsilon \rightarrow 0$. On the other hand,

$$\begin{aligned} (y, s)^{-1} \circ (x, t) &= (x - E(t - s)y, t - s) = (x - E(\varepsilon)y, \varepsilon) \\ &= \left(\left(\frac{1}{\varepsilon}, 1\right) - \left(\frac{1}{\varepsilon}, 2 - \varepsilon \frac{1}{\varepsilon}\right), \varepsilon \right) = (0, 0, \varepsilon) \\ \left\| (y, s)^{-1} \circ (x, t) \right\| &= \|(0, 0, \varepsilon)\| = \sqrt{\varepsilon} \rightarrow 0. \end{aligned}$$

This shows that, if x, y are free to move in the whole \mathbb{R}^N ,

$$\left\| (y, s)^{-1} \circ (x, t) \right\| \rightarrow 0 \text{ does not imply } |(y, s) - (x, t)| \rightarrow 0.$$

For instance, let

$$a(x_1, x_2, t) = \sin\left(\frac{\pi}{2}x_2\right),$$

then a is uniformly continuous in \mathbb{R}^{N+1} , in Euclidean sense. Nevertheless, for (x_1, x_2, t) and (y_1, y_2, s) as above we have

$$d((x_1, x_2, t), (y_1, y_2, s)) = \sqrt{\varepsilon} \rightarrow 0$$

but

$$|a(x_1, x_2, t) - a(y_1, y_2, s)| = \left| \sin\left(\frac{\pi}{2}x_2\right) - \sin\left(\frac{\pi}{2}y_2\right) \right| = \left| \sin\left(\frac{\pi}{2}\right) - \sin\pi \right| = 1,$$

so that the function a is not uniformly continuous in any strip S_T , w.r.t. d .

Acknowledgment. We wish to thank Dr. Lukas Niebel for having brought to our attention the issue discussed in this note.

References

- [1] M. Bramanti, G. Cupini, E. Lanconelli, E. Priola: Global Lp estimates for degenerate Ornstein-Uhlenbeck operators with variable coefficients. *Mathematische Nachrichten*, Vol. 286, Issue 11-12 (2013), 1087–1101.

Marco Bramanti
Dipartimento di Matematica, Politecnico di Milano
Via Bonardi 9, 20133 Milano, Italy
marco.bramanti@polimi.it

Giovanni Cupini, Ermanno Lanconelli
Dipartimento di Matematica, Università di Bologna
Piazza di Porta S. Donato 5, 40126 Bologna, Italy
giovanni.cupini@unibo.it, ermanno.lanconelli@unibo.it

Enrico Priola
Dipartimento di Matematica “Felice Casorati”, Università di Pavia
Via Adolfo Ferrata, 5, 27100 Pavia, Italy
enrico.priola@unipv.it