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Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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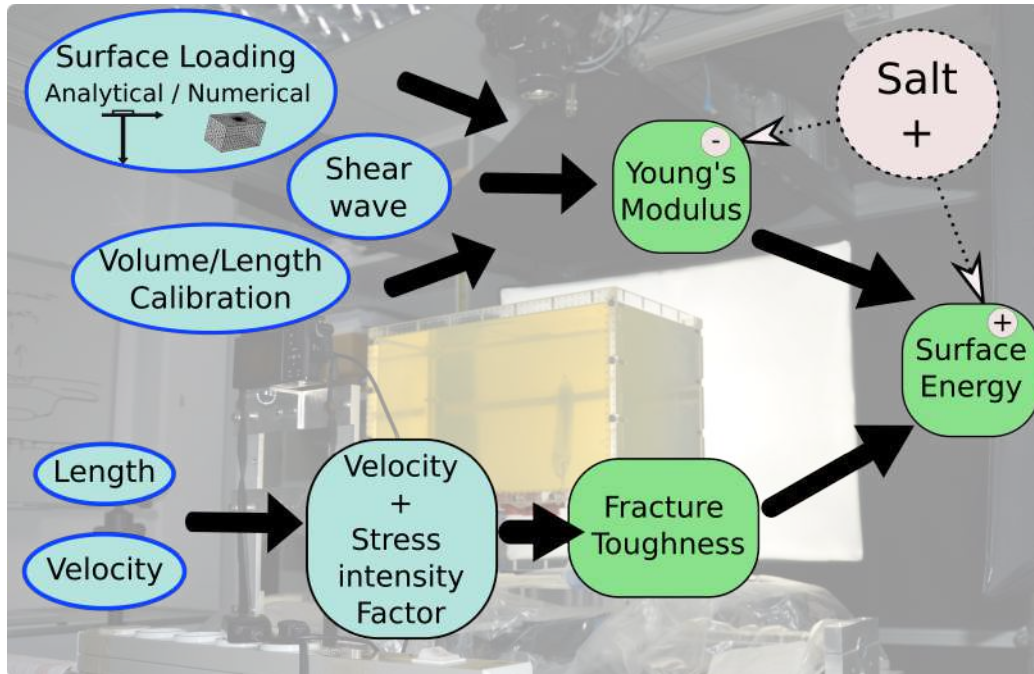
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Graphical Abstract

Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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Highlights

Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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- Numerical models improve the gelatin Young's modulus estimation by surface loading.
- Gelatin Young's modulus may be overestimated by 5 % when using an analytical solution.
- The gelatin Young's modulus can be derived from the length for an air-filled crack.
- Addition of salt to gelatin decreases its rigidity and increases its surface energy.

Characterizing the physical properties of gelatin, a classic analog for the brittle elastic crust, insight from numerical modeling

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Abstract

Precise characterization of the mechanical properties of gelatin, a classic analog of the elastic crust, is necessary for scaling the mechanical models of the Earth's crust behavior in laboratory experiments. Here we reassess how to accurately calculate the Young modulus (E) of gelatin contained in experimental tanks. By means of dedicated analog experiments and finite element simulations, we estimate the bias introduced by using equations appropriate for a half-space to interpret the subsidence due to a cylindrical surface load applied on the gelatin. In the case of a standard experimental setup with gelatin adhering to the tank wall, we find E is overestimated by at least 5 % for a box with lateral size smaller than 20 times the cylinder diameter. In addition, we deduce a correction factor to be applied when using an analytical formula. We confirm that measuring the shear velocity leads to accurate estimates for the rigidity of gelatin. We also propose a new method for in situ Young's modulus estimation, relying on the length of air-filled propagating crack. Indeed, for a given injected volume, this length depends only on the density contrast between air and gelatin and on the Young's modulus

of the gelatin. The fracture toughness of the gelatin is estimated independently. Direct comparison between fracture toughness and Young's modulus shows that for a given Young's modulus, salted gelatin has a higher fracture toughness than unsalted gelatin.

Keywords: Analog modeling, Gelatin, Young's modulus, Fracture toughness, Crack propagation

1. Introduction

Gelatin, a transparent animal-derived biopolymer in its sensu stricto appellation [1], has been used as an analog of the crust and lithosphere, in a wide range of laboratory experiments [2]. It has proven useful to study the seismic cycle in subduction zones [e.g. 3], the deformation of the upper crust around magma storage zones [e.g. 4, 5] or magma transport through either open conduits [e.g. 6] or magma-filled cracks [e.g. 7, 8, 4, 9, 10, 11, 12, 13]. A physical understanding of the Earth can be gained from analog experiments provided that they are geometrically, kinetically and dynamically scaled [14]. As a consequence the physical and rheological characterization of the gelatin has been addressed by numerous studies in Earth Sciences [e.g. 1, 15, 16] as a complement to work done in the food industry [e.g. 17].

Gelatin has been shown to behave in its gel-like state as a visco-elastic medium, which makes it particularly appropriate for tectonic studies [1]. The balance between the viscous and elastic behavior has been investigated by measuring the storage (G') and the loss (G'') moduli over a broad range of deformation rates. This balance depends mainly on gelatin concentration, temperature and aging [1, 16, 15]. If prepared with a concentration between 2 and 5 wt %, gelatin behaves elastically at low temperatures (6-14°C) for time scales up to a few hours [1, 16, 15]. The main focus of the present study is the characterization of gelatin elastic behavior during fluid-filled crack propagation experiments, with application to the study of magma transport through the crust.

Elastic behavior of gelatin can be characterized by its Poisson's ratio ν and its Young's modulus E . Poisson's ratio of gelatin is generally assumed to be 0.5, which means that gelatin is incompressible [18, 19]. Slightly smaller Poisson's ratio with values around 0.45 for gelatin concentration greater than 3 wt % were measured by van Otterloo and Cruden [15] but the values they obtained for smaller concentration were unrealistic [15]. Pansino and Taisne

[20] inferred a value larger than 0.47 with a 2.7 wt % concentration. Because these values are very close to 0.5, we will further consider gelatin as an incompressible medium. In contrast, crustal rocks generally have a Poisson’s ratio around 0.25.

The Young’s modulus can be derived from the limit of the storage modulus G' when the frequency tends to infinity and thus measured on small samples of gelatin with a rheometer [1]. However this method is destructive and limited to a reduced size sample. The gelatin Young’s modulus increases with the gelatin concentration and decreases with the temperature. It also depends on the gelatin composition, on the preparation protocol and on the cooling history [16]. It might sometimes be useful to add salt to the gelatin in order to slightly increase its density and improve the scaling in specific conditions [21, 22, 23]. In particular, when attempting to model viscous effects on magma propagation, the use of salted gelatin is required in order to guarantee a sufficient buoyancy of the oils injected inside the gelatin. Brizzi et al. [24] have shown that the addition of salt dramatically affects not only material behavior, but also gel structure stability. Adding salt induces a decrease of both the Young’s modulus and the viscosity, an increase of the time required for cooling down to a stable state and it tends to promote the elastic behavior compared to the viscous one. They also noted that the transparency might be reduced by salt addition and that the mechanical properties become highly sensitive to the preparation protocol such that it is more difficult to control the reproducibility of the experiments. Due to the complex behavior of the gelatin and the variability of experimental conditions, it is recommended to quantify the Young’s modulus associated with each gelatin tank, which requires the use of non destructive and *in situ* measurements.

The Young’s modulus of crustal rocks can be measured in the laboratory by means of uniaxial strain-stress experiments [25]. However, interpolating laboratory measurements on small rock samples to infer the Young’s modulus value at depth and the crustal behavior is not trivial. The main issues in laboratory experiments are related to the scaling and use of non-fractured samples at low pressure and temperature that may not reflect the “in situ” conditions at depth in the crust. There are basically two ways of quantifying the in-situ crustal Young’s modulus: either using the surface displacement induced by surface loading or unloading events [26, 27, 28], often referred to as a “static” estimation, or using seismic wave velocities derived from local tomography surveys [29, 30], often referred to as “dynamic” estimation. A systematic discrepancy has been evidenced between the dynamic and static

68 Young’s moduli. For rocks, the dynamic Young’s modulus is always larger
69 by a factor which depends on the porosity [31, 32].

70 For gelatin in a tank, the most commonly used in-situ and non-destructive
71 method is based on surface loading. It consists of measuring the vertical dis-
72 placement induced by a circular load applied at the surface and inverting for
73 the Young’s modulus using an analytic formula for a circular rigid load ap-
74 plied on an infinite half-space [e.g. 33, 11, 16, 34, 35]. Due to the assumption
75 of an infinite medium, the diameter of the load should be smaller by a factor
76 of ten than the smaller dimension of the tank (either vertically or laterally)
77 [16].

78 However, gelatin tanks used for experiments are usually of limited size
79 mainly to ensure a volume which is manageable to produce. It follows that
80 instrumentalists always have to be cautious of boundary effects. One way to
81 account for the rigid boundaries at the box walls consists in using numerical
82 models to infer the actual state of stress within the gelatin [e.g. 11]. For
83 instance, Corbi et al. [36] calculated the state of stress inside their gelatin
84 tank using a 2D-axisymmetric Finite Element Model (FEM) in order to in-
85 terpret the crack path observed in experiments. Pinel et al. [37] improved
86 the estimate of the stress field acting within the gelatin tank previously used
87 by Watanabe et al. [38] to quantify the influence of the external stress field
88 on the fluid-filled crack path. This was done by computing the stress field in-
89 duced by a surface load using a 3D numerical simulation, taking into account
90 the geometry of the tank and the geometry of the load, and by comparing
91 it to the analytical solution previously used. In the same way, Maccaferri
92 et al. [13] computed both the local stress field and the surface displacement
93 induced by a load of given size, accounting for the rigid boundaries of the
94 box and used this information to obtain an accurate value of the Young’s
95 modulus for gelatin tanks.

96 More recently, Pansino and Taisne [39, 20] proposed to derive the Young’s
97 modulus from the propagation velocity of shear waves. They took advantage
98 of gelatin being a birefringent photo-elastic material. Its refractive index
99 varies with the stress applied such that, using a pair of polarizing filters
100 enables to visualize the deviatoric stress and track the propagation of shear
101 waves within the gelatin. This method can be thought of as the equivalent
102 of crustal Young’s modulus estimations from seismic waves. The results
103 were compared with estimations made using the static loading method and
104 showed to be in good agreement [20]. In addition, Pansino and Taisne [20]
105 proposed that the shear wave method could potentially be used to quantify

106 any variations of the Young's modulus inside the tank.

107 Experiments of fluid-filled crack propagation also require the characteri-
108 zation of the brittle behavior of the gelatin. A key parameter is the fracture
109 toughness. Crack propagation will only occur once the stress intensity factor
110 at the tip, which depends on the applied stress and the shape of the crack,
111 exceeds the fracture toughness of the surrounding medium. The fracture
112 toughness K_c is linked to the Young's modulus by the following equation
113 expressed by Griffith [40]:

$$K_c = \sqrt{2\gamma_s E}, \quad (1)$$

114 with γ_s the surface energy of the solid, which for gelatin is estimated around
115 $1.0 \pm 0.2 \text{ J.m}^{-2}$ [33, 16]. This relationship is often used to derive the fracture
116 toughness but it should be kept in mind that the value of γ_s is expected to
117 depend on the gelatin composition and might be different when salt is added
118 [22]. Alternatively, the fracture toughness can be quantified by measuring
119 the pressure required to propagate a pre-existing fluid-filled crack [33, 16].
120 Another method consists in retrieving the stress intensity factor of propa-
121 gating cracks for various velocities of propagation: the fracture toughness is
122 then given by the limit of the stress intensity factor when the propagation
123 velocity tends to zero [41].

124 In the current study, we further detail how numerical simulations may
125 improve the determination of gelatin Young's modulus by surface loading.
126 In particular, we provide an accurate estimate of the error resulting from
127 using the analytical formula when deriving the Young's modulus by surface
128 loading. We also compare Young's modulus obtained by surface loading to
129 values derived by the shear wave velocity method. Then, Young's modulus
130 estimates are used to derive a calibration of the relation between the injected
131 volume and the crack length in case of air-filled cracks. We thus provide a
132 new method for Young's modulus estimation. Finally, we characterize the
133 fracture toughness of the gelatin to quantify the effects of adding salt to
134 gelatin on its brittle behavior.

135 2. Methods

136 2.1. Laboratory technique

Table 1: Gelatin properties for the 37 tanks. E_{an}^{load} is calculated only when a circular load was applied to the surface with Eq. 5 whereas E_{num}^{load} is calculated with the Finite Element model with the rigid load condition taking into account the exact geometry of the load applied. Injections is for the number of injections performed and used in this study.

Tank number	Tank geometry	Height (cm)	Volume (L)	Gel. (%)	Salt (%)	ρ_{gel} kg.m ⁻³	T_{gel} (°C)	Duration (h)	Injections (.)	$E_{an}^{load} \pm std$ (Pa)	$E_{num}^{load} \pm std$ (Pa)	Shear velocity $\pm std$ (cm/s)	$E^{shw} \pm std$ (Pa)	K_c^{2D} Pa.m ^{1/2}	K_c^{3D} Pa.m ^{1/2}
1701	big	20.00	16.00	2	0	1020		1.6	11	1637 \pm 27				49	54
1702	big	20.00	16.00	2	0	1020		1.7	9	2277 \pm 136				51	56
1703	big	20.00	16.00	2	0	1020		1.1	7	2315 \pm 118				58	64
1704	big	20.00	16.00	2	0	1020		1.8	6	2312 \pm 105				48	53
1705	big	20.00	16.00	2	0	1020		2.1	8	2408 \pm 354				54	60
1706	big	20.00	16.00	2	0	1020		1.6	7	2286 \pm 142				48	53
1707	big	20.00	16.00	2	0	1020		1.6	7	2342 \pm 276				52	58
1708	big	20.00	16.00	2	0	1020		1.3	6	2117 \pm 130				44	48
1801	big	9.62	7.70	2.5	11	1080		1.8	5	1308					
1802	big	22.00	17.60	3	10	1060		3.3	4	2001 \pm 174					
1803	big	22.65	18.12	3.5	15	1120		2.9	6	630					
1804	big	22.71	18.17	3.5	15	1120		4.4	7	2201 \pm 352				66	73
1805	big	21.81	17.45	3.5	15	1120		3.8	6	834 \pm 128					
1806	big	23.00	18.40	3.5	15	1120		4.0	7	966 \pm 121				42	47
1807	big	22.30	17.84	3.5	15	1120		5.2	7	845 \pm 257				41	45
1808	big	21.92	17.54	3.5	15	1120		3.5	4	909 \pm 98					
1809	big	22.32	17.86	3.5	15	1120		4.0	8	800 \pm 101					
1810	big	21.96	17.57	3.5	15	1120		2.5	4	938 \pm 100					
1901	big	12.06	9.65	2	0	1020	10	0.4	7	2201				56	62
1902	big	18.76	15.01	2	0	1020	?-12	0.2	7	1937				57	62
1903	big	18.81	15.05	2	0	1020	?-12	1.1	9	2085				59	65
1904	big	19.19	15.35	2	0	1020		0.8	7	1885		54.32 \pm 3.54	903 \pm 11.52		
1905	big	15.88	12.70	3.5	15	1120		1.6	5	1995		61.46 \pm 10.89	1269 \pm 40.16		
1906	big	13.05	10.44	3.5	15	1120		2.1	5	1769 \pm 94	1443 \pm 77	65.97 \pm 7.14	1462 \pm 28.27	74	82
1907	small	22.56	4.36	2.5	15	1120		0.5	1	1783 \pm 55	1290 \pm 40	64.73 \pm 5.00	1408 \pm 19.43		
1908	cylinder	26.18	4.20	3.5	15	1120	10	1.7	1	3649 \pm 297	2892 \pm 236	98.07 \pm 13.11	3232 \pm 77.15		
1909	big	21.92	17.54	2	15	1120	14	2.9	8	306 \pm 61		17.25 \pm 2.53	100 \pm 2.618	17	19
1910	small	23.09	4.46	1.5	15	1120	8	0.4	1	1532 \pm 477	846 \pm 263	34.71 \pm 2.30	405 \pm 4.78		
1911	cylinder	25.66	4.12	1.5	15	1120	8-11	1.7	3	700 \pm 50	441 \pm 32	34.86 \pm 3.34	408 \pm 6.99		
1913	big	21.37	17.10	2	15	1120	14	2.1	5	597 \pm 47		22.64 \pm 2.39	172 \pm 3.25		
1914	small	21.53	4.16	2	15	1120	8-12	0.8	1	1382 \pm 137	762 \pm 76	46.72 \pm 2.00	733 \pm 5.62		
1915	cylinder	28.81	4.63	2	15	1120	8-14	0.7	3	1500 \pm 265	950 \pm 168	59.20 \pm 2.70	1177 \pm 9.59		
1916	big	21.53	17.22	2	15	1120	14	3.5	12	595 \pm 19		25.14 \pm 2.71	212 \pm 4.09	30	33
2002	big	22.50	18.00	1.8	0	1020	5.2-7.2	1.5	7	1847 \pm 20				51	57
2003	big	22.50	18.00	1.6	0	1020	10.2-13.3	1	10	1005 \pm 20				36	40
2004	big	22.50	18.00	1.5	0	1020	8.2-13.3	1	9	1113 \pm 16				38	42
2005	big	22.50	18.00	2	15	1120		1	9	666 \pm 16				33	37

2.1.1. Gelatin preparation

We used type A pig-skin gelatin (Bloom number 280g) purchased from Italgelatine S.p.A., in the form of solid granules. Gelatin with different concentrations were prepared and salt was added to some preparations in order to increase the gelatin density. In total we made 37 tanks of gelatin, 15 at 1.5 to 2 wt % without adding salt and 22 varying the gelatin concentration from 1.5 wt % to 3.5 wt % and the salt concentration from 10 to 15 wt % (Tab. 1).

To prepare the unsalted gelatin, we completely dissolved the granules of gelatin with hot water ($\sim 60^\circ\text{C}$). In order to get a transparent solid, a perfect dissolution of the gelatin granules is required. We proceeded in several steps, dissolving only a fraction of the total mass needed to prepare one tank in a 2 L beaker and holding the solution on warm plate at 60°C . When all the granules were dissolved we diluted with warm water to reach the desired volume and concentration. We let it cool down to room temperature for a few hours. When the preparation had reached $\sim 32^\circ\text{C}$, we mixed it to homogenize the gelatin and we transferred the liquid gelatin into the tank, before placing it into the refrigerator at $\sim 8^\circ\text{C}$ for 20 hours. According to Kavanagh et al. [16] and van Otterloo and Cruden [15] liquid gelatin solidifies, behaving as a visco-plastic solid below 25°C and as an elastic solid below 15°C . Note that during the preparation, the temperature of the gelatin should never exceed 70°C or drop below 4°C to avoid the denaturation of the peptide chains by heating or freezing, respectively [1]. Following Brizzi et al. [24]’s protocol, in order to add salt to the recipe, we first dissolved the salt in hot water, then we used the salty water to dissolve the gelatin granules, as previously described. It is important to first dissolve the salt and then the gelatin granules to avoid the precipitation of the peptide chains and to preserve the transparency of the solid.

To prevent the formation of a tough “skin” on the gelatin surface caused by water evaporation, a thin layer of vegetable oil was poured on top of the gelatin (salted and unsalted) before placing it into the fridge. The oil was removed before starting the experiments.

The gelatin density was computed as the ratio between its mass and the volume. Using a graduated cylinder, we measured a volume of 50 ± 1 mL of liquid gelatin and weight it with a gram-accurate scale. Density of unsalted and salted (15 wt %) gelatin were estimated to 1020 ± 40 kg.m $^{-3}$ and 1120 ± 40 kg.m $^{-3}$, respectively. Although, we have a quite large formal abso-

lute error due to the poor scale resolution, we obtain very good reproducibility in our measurements from one tank to another. We checked that density changes due to cooling and solidification were negligible by repeating density measurements at different temperatures, and checking that the gelatin level in the tank did not change after solidification.

2.1.2. *Experimental setup and recordings*

We used three different plexiglas tanks: a cylindrical one with a diameter of 14.3 cm and a height of 30 cm and two cuboids with dimensions $L \times l \times H$ of 13.9 cm \times 13.9 cm \times 24 cm for the smaller one and 40 cm \times 20 cm \times 25 cm for the larger one. The larger tank corresponds to the one used for experiments described by Maccaferri et al. [13] (see Fig. 1). The height of gelatin was usually close to 20 cm (see Tab. 1). For experiments 2002, 2003 and 2004, both the room and the gelatin temperature were continuously recorded during the whole duration of the experiment by thermocouples, whereas for some other experiments the gelatin temperature was measured by an infrared thermometer. However, we do not have temperature records for all the experiments: when available the gelatin temperature, or temperature values for the beginning and the end of the experiment are given in Tab. 1 together with the duration of the whole experiment.

In order to measure the gelatin Young's modulus by static deformation, the gelatin surface was loaded with rectangular (6 \times 14 cm) or circular shapes (with diameters ranging from 2 to 4 cm) and masses ranging from 3 g to 331 g. With the corresponding pressure range, we induced surface displacement large enough to be accurately measured, without damaging the gelatin. We used a digital caliper, whose accuracy is 10^{-2} mm, to measure the subsidence. We fixed the caliper to a rigid support on the top of the tank (Fig. 1c). We measured the distance to the gelatin surface without load d_1 and the distance to the top of the load d_2 . Knowing the load thickness e , we can derive the vertical surface displacement u induced by the load (Eq. 2):

$$u = d_2 + e - d_1 \quad (2)$$

The main limitation to the measurement accuracy comes from the ability of experimentalists to use the caliper without deforming the gelatin surface while measuring the subsidence. To reduce the uncertainties, we repeated the measurements for each tank at least three times using the same load until getting 3 values less than 0.2 mm apart, and when possible, we used several loads with increasing mass.

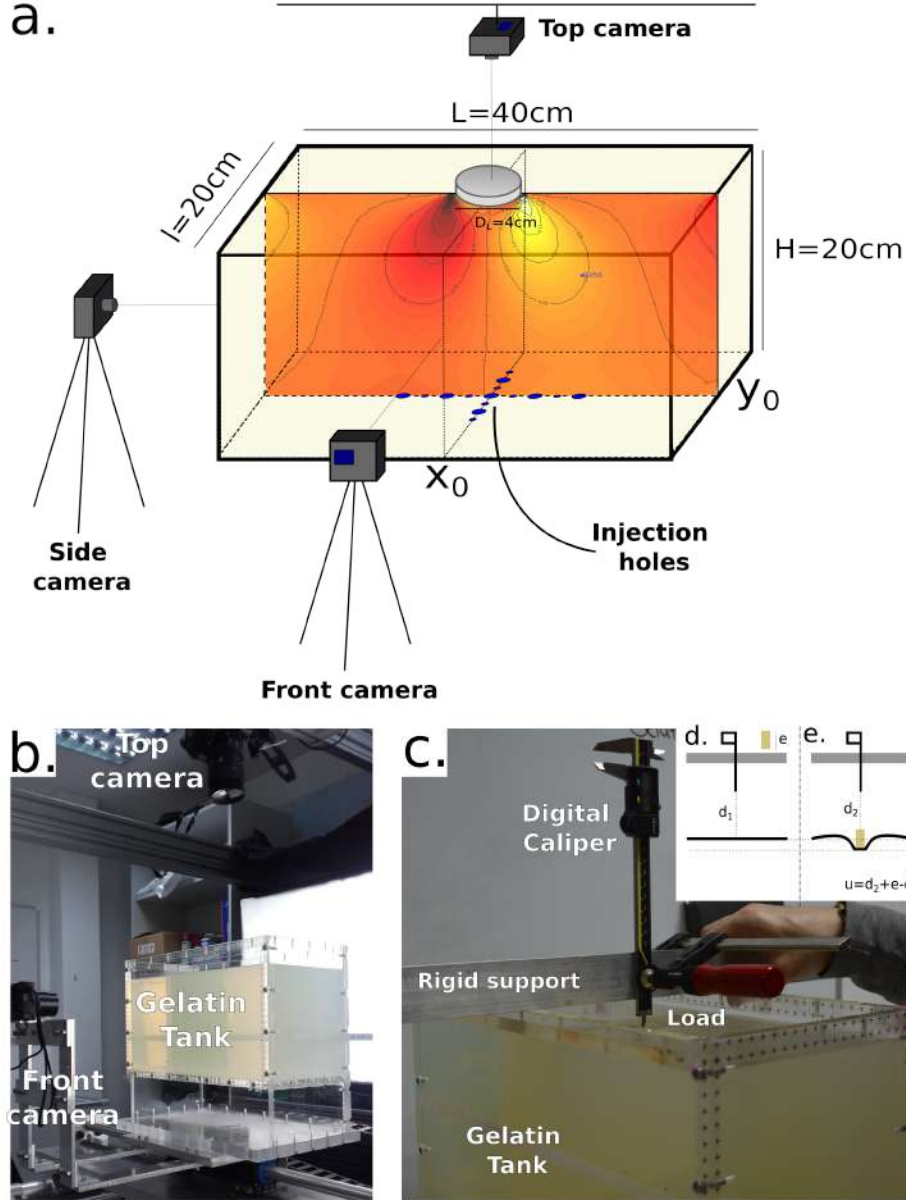


Figure 1: Experimental set-up. (a) Sketch of the experimental set-up and location of the three cameras. Section along plane $y = 0$ shows shear stress induced by the loading (grey cylinder). The 15 injection holes (2 sizes) are marked by blue circles on the underside of the tank. (b) Photograph of the experimental set-up. Two lamps illuminate the tank from the back and right sides through the white screens. (c) Photograph of the measurement of the surface vertical downward displacement induced by a load applied at the surface. Inset diagrams schematize the two steps of a measurement (d) measuring the reference distance d_1 before putting the load and (e) measuring the distance d_2 to the top of the load of known thickness e .

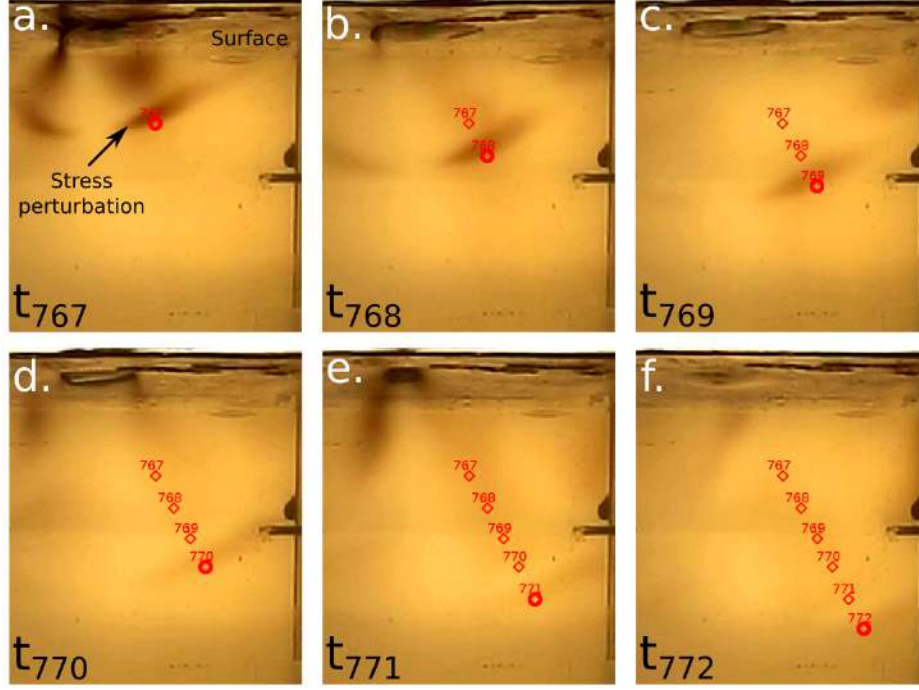


Figure 2: Shear wave velocity measurements with TRACKER. Panels (a to f) display screenshot of six successive images (t_{767} to t_{772}) from the video records of tank 1910 showing shear wave propagation front after exciting the surface. Red dots indicate the manual picking of the propagation front position to compute the velocity. Note that the shear wave attenuates quickly and it becomes increasingly difficult to track it manually.

209 In order to estimate the gelatin Young's modulus by measuring the shear-
 210 waves propagation velocity, we added polarizing filters on both the front and
 211 back sides of the tank. Following Pansino and Taisne [39, 20], we excited the
 212 gelatin surface with a spoon, and we recorded the shear wave propagation
 213 with the front camera. Using the open source software TRACKER [12], we
 214 measured the propagation velocity (Fig. 2). For each tank, we performed
 215 velocity measurements on several wave trains by manually picking the prop-
 216 agation front on at least 4 to 5 successive images. Then we computed the
 217 mean velocity and the standard deviation associated to each tank.

218 In order to study the fluid-filled crack propagation, we used a syringe to
 219 inject a finite volume of air from some holes at the bottom of the tank. The
 220 fracture orientation was controlled by carefully orienting the needle used for
 221 the injection. No slit was needed, we let the air create its own fracture. On

the bottom of the larger rectangular tank, 15 holes with 2 cm in between allow to perform several injections into the same gelatin. Three perpendicular cameras recorded the fluid-filled crack shape and path. Two spotlights illuminated the tank from the back and right sides (Fig. 1). Videos subsampling was done with the video editing and open source software Shotcut [42]. The software TRACKER was used to measure the length L' of the air-filled cracks and to extract the path and the velocity of the crack. When all injections were completed in a tank, we took several pictures of a ruler at the location of the cracks in order to measure the calibration factor F needed to scale the videos. The crack length L was computed as $L = F \times L'$. Thus, the statistical error on the crack length is given by:

$$\frac{\sigma_L}{L} = \sqrt{\left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_{L'}}{L'}\right)^2} \quad (3)$$

Where σ is the standard deviation. Given that $\frac{\sigma_F}{F} = 0.024$ and $\frac{\sigma_{L'}}{L'} = 0.026$, we obtain $\frac{\sigma_L}{L} = 0.035$.

2.2. Numerical simulations

In order to take into account the finite size of the tank and the exact shape and size of the circular load, we use a 3D FEM to compute the surface displacement induced by the applied load. Numerical simulations are performed with the commercial software COMSOL [43] applying a zero displacement condition to the lateral and bottom boundaries of the gelatin to reproduce the adherence of the gelatin to the tank walls. We use a mesh made of about 330,000 tetrahedral units, refined in a vertical plane centered below the load as well as on the upper surface around the load (minimum size of the mesh was set to 2 mm). The upper surface is considered as a free surface except where the load is applied. To simulate the loading, the easiest boundary condition to be considered would be a constant pressure

$$P_{Load} = \frac{4m_L g}{\pi D_L} \quad (4)$$

with g the standard acceleration due to gravity and m_L the load mass applied on the circular surface of diameter D_L (Fig. 3a). We will refer to this condition, as the "uniform pressure condition". However, this boundary condition is not fully satisfactory as rigid loads are applied to the surface inducing a uniform vertical displacement below the load. To better reproduce this

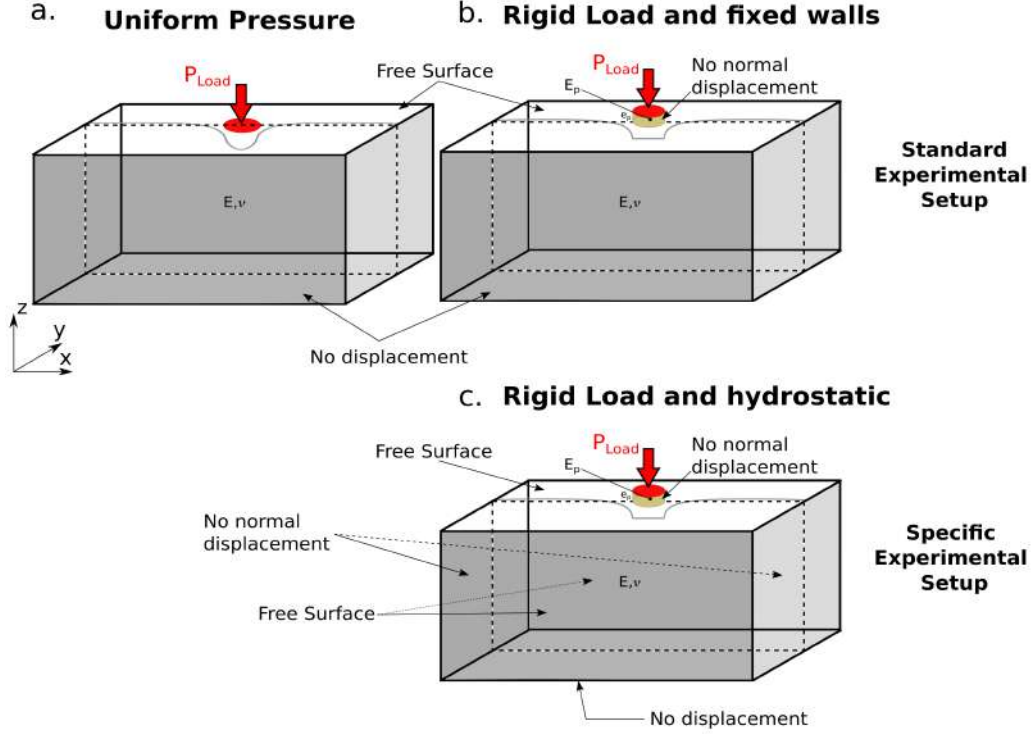


Figure 3: Boundary conditions applied in the numerical model and modeled surface displacements. The gray line shows the shape of the vertical component of the surface displacement induced by the boundary condition applied to the surface: (a) uniform pressure with gelatin adhering to the tank walls, (b) rigid load with gelatin adhering to the tank walls and (c) rigid load with gelatin in contact with water on two of the lateral sides of the gelatin block.

condition we simulate a thin (thickness e_p) rigid plate characterized by a large Young's modulus E_p . On the upper surface of this plate we apply the pressure P_{Load} . We also apply a condition of zero horizontal displacement to the lateral edge of this rigid plate (Fig. 3b). We will refer to this condition as the "rigid load condition". We set the values of E_p and e_p to 10^9 Pa and 4 mm respectively, to ensure a uniform vertical displacement below the applied load. As a theoretical value of 0.5 for the Poisson's ratio cannot be handled numerically, we set it to 0.49.

We compare the simulations using constant pressure and rigid load in similar conditions. We also run a set of simulations for a given load, increasing the box size.

263 Additionally, several studies [34, 44], have presented an experimental
 264 setup where gelatin is in contact with water on two lateral sides and de-
 265 tached from the other two sides of the tank. In order to discuss the influence
 266 of such a specific setup, we also applied another set of lateral conditions with
 267 a free surface on the two proximal lateral sides and a roller condition (zero
 268 displacement in the direction perpendicular to the tank wall) on the distal
 269 lateral sides (Fig. 3c).

270 3. Young’s modulus determination by surface loading

271 3.1. Validity domain of the analytical solution

272 The relationship between the vertical surface displacement (U_z) beneath
 273 a rigid load of mass m_L applied to the surface of a half-space is given by the
 274 analytical formula (see Timoshenko et al. [45]):

$$U_z = \frac{m_L g (1 - \nu^2)}{D_L E}, \quad (5)$$

275 where D_L is the load diameter. This method has been followed by most ana-
 276 log modelers to estimate the Young’s Modulus of gelatin [e.g. 33, 11, 16, 34,
 277 35]. However, assuming an infinite half space, such formula may introduce
 278 an error. This error was reported to remain small providing that the tank
 279 minimum size is ten times larger than the load diameter [16]. This estimation
 280 was derived from a correlation study between the Young’s modulus estimate
 281 and the ratio of the load versus tank dimensions [16]. As the tank size is
 282 usually limited for practical reasons, this condition might be difficult to ful-
 283 fill, in particular, because the load applied should be large enough to ensure
 284 significant displacement and it should be applied over an area large enough
 285 to avoid damaging the gelatin. In our case, even with our largest cuboid
 286 tank, the tank minimum dimension is indeed only five times larger than the
 287 load we applied. In order to quantify the error performed when using Eq. 5
 288 as a function of the tank size, we compare the surface displacement calcu-
 289 lated from the numerical simulation with the one predicted by the analytical
 290 formula.

291 Numerically, we apply a circular load (mass $m = 8$ g and diameter $D_L =$
 292 40.04 mm) to the center of our largest rectangular tank and we progressively
 293 multiply the tank size by a factor f up to 80. Calculations are done both for
 294 the uniform pressure condition (Fig. 3a) and the rigid load condition (Fig. 3b)

295 with gelatin adhering to the tank walls. The gelatin Young's modulus is set
 296 to 1000 Pa. Displacement profiles are shown in Fig. 4. As expected, the
 297 uniform pressure solution (dashed lines in Fig. 4) results in a maximum
 298 displacement below the load center while the rigid load condition produces
 299 a smaller and uniform displacement below the load which better reproduces
 300 our experimental conditions. We also represent the analytical solution for a
 301 uniform pressure load, which can be derived from Sneddon [46] as proposed
 302 by Sigmundsson and Einarsson [47] and Pinel et al. [28]:

$$U_z = \frac{4}{\pi} \frac{m_L g (1 - \nu^2)}{D_L E} \quad (6)$$

303 We find that using a uniform pressure (Eq. 6) instead of a rigid load (Eq. 5)
 304 as boundary condition for the loading, would produce an overestimate of the
 305 Young's modulus E by a factor $4/\pi \sim 1.27$ for an infinite half-space. More-
 306 over, both analytical formulas (Eq. 5 and 6) predict more vertical displace-
 307 ment than the corresponding numerical solution. Therefore, if such formula
 308 is applied to our largest cuboid tank (Tab. 2), E gets overestimated by 15%
 309 and 21% for the uniform pressure and the rigid load conditions, respectively.

310 In Tab. 2, displacements at the center of the load are also estimated
 311 using analytical formulas and a Poisson's ratio value of 0.49 (instead of 0.5)
 312 similarly to numerical simulations. Using a Poisson's ratio of 0.49 for the rigid
 313 load condition (Eq. 5) produces an overestimate of the vertical displacement
 314 by 1% (0.02mm). Such an overestimate is negligible in comparison to the
 315 error made by ignoring the boundary effect.

316 Both Fig. 4 and Tab. 2 show that when the tank size increases, numerical
 317 solutions tend to the value given by the analytical ones, which confirms the
 318 validity of the numerical model.

319 We compare the estimation of the Young's modulus when using the ana-
 320 lytical formula (Eq. 5) to the numerical solution for the rigid load conditions
 321 (Tab. 2) as a function of the relative size of the load and the tank (Fig. 5).
 322 The caliper we used to measure the subsidence has an accuracy of 0.01 mm
 323 which corresponds to an error of 5% on the Young modulus estimate. Ac-
 324 cording to Fig. 5, the ratio between the shortest dimension of the tank and
 325 the load diameter must be at least 21 for the error being less than 5% in
 326 the case of an experimental setup with gelatin adhering to the tank walls.
 327 This ratio is twice larger than the ratio previously recommended based on
 328 correlation studies [16]. For ratios below twenty, side effects due to the rigid
 329 walls of the tank cannot be neglected and the analytical formula significantly

330 overestimates the displacement induced by a given load thus producing an
 331 overestimation of the Young's modulus of the gelatin (see also Tab. 1). Fig. 5
 332 also shows that, when a specific experimental setup that ensures hydrostatic
 333 conditions on the proximal lateral sides of the gelatin block is used, the use
 334 of the analytical solution is less problematic. Then the analytical solution
 335 can be used provided that the shortest dimension of the tank remains 7 times
 336 larger than the load diameter. However even in the few studies that consider
 337 these specific lateral conditions, the Young's modulus was measured before
 338 the gelatin sides were melted and replaced by water meaning that the gelatin
 339 adhered to wall during the Young's modulus measurement. While most ex-
 340 perimenters indicate the size of the tank they are using, the size of the load
 341 applied during the Young's modulus measurement is never provided except
 342 by Kavanagh et al. [16]. In the literature, the minimum size of the tank
 343 ranges from 8.6 cm [16] to 80 cm [6], which means that the maximum size
 344 of the applied load should be less than 0.4 cm for the smallest tanks and
 345 less than 3.8 cm for the largest. We provide a solution to this problem as
 346 from our results, it is possible to derive a correction factor to be applied to
 347 the value derived using the analytical formula in order to take into account
 348 the actual size of the tank (Fig. 5) and thus obtain a reliable value for the
 349 Young's modulus.

350 3.2. Young's modulus estimations by surface loading

351 Based on the results presented in the previous paragraph, the best method
 352 to estimate the Young's modulus by surface loading is to use the numerical
 353 simulation with a rigid load condition. We thus follow this strategy. Using
 354 the FEM simulation, with a geometry corresponding to each tank and each
 355 load considered, we compute the surface displacement U_z^{FEM} induced by
 356 the load with a Young's modulus value of $E^{FEM} = 1000$ Pa, Poisson's ratio
 357 $\nu = 0.49$ and gravity $g = 9.81$ m.s⁻². For each measurement of surface
 358 subsidence in the analog setup U_z^{mes} , we can determine a Young's modulus
 359 value E^{load} using the following equation:

$$E^{load} = \frac{E^{FEM} U_z^{FEM}}{U_z^{mes}} \quad (7)$$

360 The Young's modulus is estimated for the 37 gelatin blocks (15 tanks filled
 361 with unsalted gelatin at 1.5 to 2 wt % and 22 tanks filled with salted gelatin
 362 at 1.5 to 3.5 wt % of gelatin and 10 to 15 wt % of salt). We estimate a value

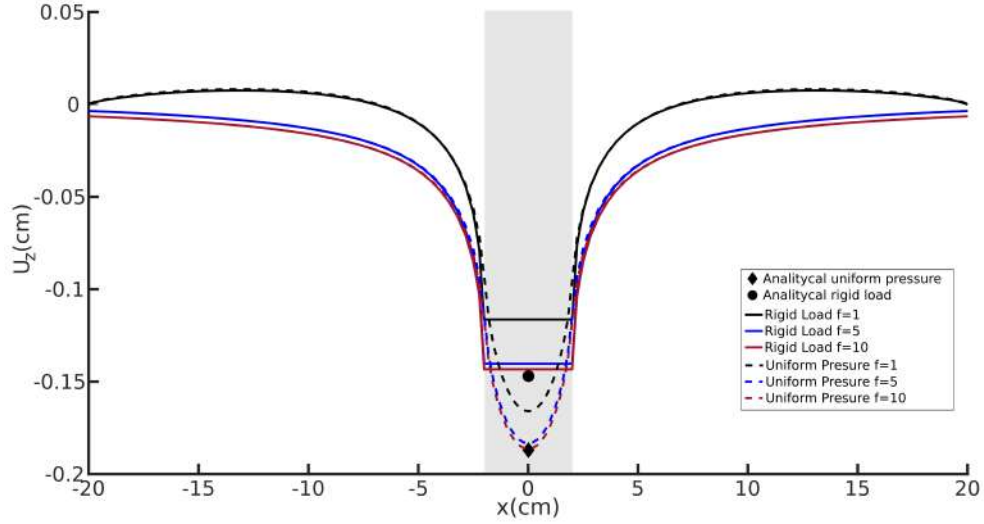


Figure 4: Profile of vertical surface displacement along x-axis ($y=0$, profile at the load center) for a circular load (diameter $D_L = 40.04$ mm, mass $m_L = 8$ g) applied to the surface of a rectangular tank (dimensions $L \times l \times H$ of $40 \text{ cm} \times 20 \text{ cm} \times 20 \text{ cm}$ multiplied by a factor f , the value $f = 1$ corresponding to the largest rectangular tank we used in our experiments). The Young's modulus is set to 1000 Pa. Black, blue and red solid lines are profiles calculated with the FEM for, respectively $f = 1$, $f = 5$ and $f = 10$. Dashed lines are for solutions derived with the uniform pressure condition, whereas plain lines are for solutions derived with the rigid load condition (in the case of the experimental setup with gelatin adhering the tank walls). Black diamond and circle represent the vertical displacement from analytical formulas (in $x=0$ at the load center) considering, the uniform pressure and the rigid load condition, respectively.

Table 2: Comparison of modeled vertical displacement induced by a circular load (diameter $D_L = 40.04$ mm, mass $m_L = 8$ g) applied at the surface of a rectangular tank (dimensions $L \times l \times H$ of $40 \text{ cm} \times 20 \text{ cm} \times 20 \text{ cm}$ multiplied by a factor f , the value $f = 1$ corresponding to the largest rectangular tank we used in our experiments). The Young's modulus is set to 1000 Pa. Two analytical solutions are tested also with three numerical solutions ("uniform pressure condition" with gelatin adhering to the walls of the tank, "rigid load condition" with gelatin adhering to the walls of the tank and "rigid load condition" with gelatin in contact with water on two of the lateral sides of the gelatin block) for several values of the tank size. In numerical simulations, the Poisson's ratio value is set to 0.49. FEM: Finite Element Model ; P: uniform pressure ; rgdL: rigid load; Swall: gelatin adhering to the tank walls; Fwall: gelatin in contact with water on two of the lateral sides of the gelatin block.

Model		Tank size l (dm)	Ratio l/D _L	U _z (mm)
Analytic rgdL	$\nu = 0.5$	∞	∞	1.4695
Analytic rgdL	$\nu = 0.49$	∞	∞	1.4889
Analytic P	$\nu = 0.5$	∞	∞	1.8711
Analytic P	$\nu = 0.49$	∞	∞	1.8958
FEM P-Swall	f=1	2	5	1.5919
FEM P-Swall	f=10	20	50	1.8648
FEM rgdL-Swall	f=1	2	5	1.1640
FEM rgdL-Swall	f=1.5	3	7.5	1.2633
FEM rgdL-Swall	f=2	4	10	1.3146
FEM rgdL-Swall	f=3	6	15	1.3660
FEM rgdL-Swall	f=4	8	20	1.3915
FEM rgdL-Swall	f=5	10	25	1.4064
FEM rgdL-Swall	f=6	12	30	1.4162
FEM rgdL-Swall	f=7	14	35	1.4237
FEM rgdL-Swall	f=8	16	40	1.4286
FEM rgdL-Swall	f=9	18	45	1.4323
FEM rgdL-Swall	f=10	20	50	1.4381
FEM rgdL-Swall	f=20	40	100	1.4521
FEM rgdL-Swall	f=50	100	250	1.4594
FEM rgdL-Swall	f=60	120	300	1.4623
FEM rgdL-Swall	f=80	160	400	1.4634
FEM rgdL-Fwall	f=0.5	1	2.5	1.2944
FEM rgdL-Fwall	f=1	2	5	1.3785
FEM rgdL-Fwall	f=1.5	3	7.5	1.4056
FEM rgdL-Fwall	f=2	4	10	1.4217
FEM rgdL-Fwall	f=3	6	15	1.4375
FEM rgdL-Fwall	f=4	8	20	1.4453
FEM rgdL-Fwall	f=6	12	30	1.4521
FEM rgdL-Fwall	f=8	16	40	1.4555
FEM rgdL-Fwall	f=10	20	50	1.4597
FEM rgdL-Fwall	f=20	40	100	1.4628

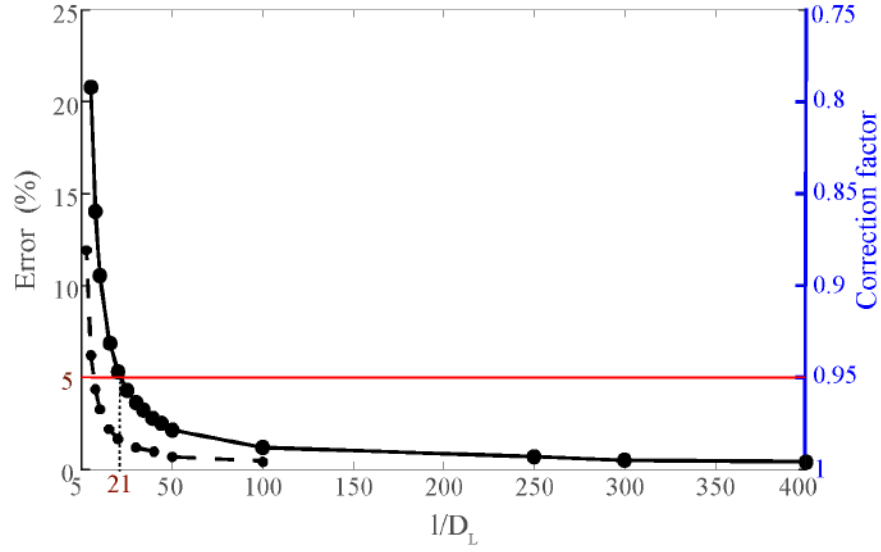


Figure 5: Estimation of the error resulting from the use of the analytical solution for a rigid circular load applied on a half-space (Eq. 5) when estimating the surface displacement induced by a circular load as a function of relative size of the load applied and the tank. The right axis gives the corresponding correction factor to be applied to the value derived analytically in order to obtain the actual value of the Young's modulus inside the tank. The plain and dashed lines are, respectively, for the experimental setup with gelatin adhering to the tank walls and for the case where gelatin is in contact with water on the two proximal lateral sides. Circles indicate the numerical simulations we performed. The red line is for a 5 % error.

of the Young's modulus for each measurement of the vertical displacement. Then for each tank we compute the mean and standard deviation values from all available measurements (Fig. 6 and Tab. 1). For unsalted gelatin at 2 wt %, we estimate $E = 2150 \pm 230$ Pa, with a good reproducibility. Only the first tank (1701) is found to have a lower value $E = 1640$ Pa. However, because neither the length, nor the shape nor the velocity of the cracks in this gelatin block is significantly different from the others, we believe that this low value is due to an error on the measurement of the vertical displacement induced by the load.

Varying gelatin and salt concentration, Young's modulus of salted gelatin range between 300 and 2900 Pa (Tab. 1). As found by Brizzi et al. [24], 2 wt % gelatin with 15 wt % salt has lower Young's modulus than unsalted gelatin at 2 wt %. By increasing the gelatin concentration to 3.5 wt %, we prepared salted gelatin with Young's modulus similar to unsalted gelatin at 2 wt %. We made 3 tanks at 3.5 wt % and 15% salt and we obtain $E = 2065 \pm 120$ Pa, a value close to the Young's modulus estimated for unsalted gelatin at 2 wt %.

4. Comparison of Young's modulus estimates either by shear-wave velocity or surface loading

An alternative method for non-destructive and in-situ measurements of Young's modulus has been proposed recently by Pansino and Taisne [39, 20]. It takes advantage of the birefringent photo-elastic property of the gelatin. This property allows the measurement of the shear wave velocity ν_s and the determination of Young's modulus E^{shw} with the following equation:

$$E^{shw} = 2(1 + \nu)\rho_g\nu_s^2 \quad (8)$$

where ρ_g is the gelatin density and ν is its Poisson's ratio, here, assumed to be 0.5.

Shear wave velocities measurements are performed for 12 gelatin blocks (1 unsalted gelatin and 11 salted gelatin) enabling for a direct comparison with the previous estimates based on surface loading. We compute the Young's modulus E^{shw} for each measured value of shear wave velocity, then we use the mean value and the standard deviation obtained for each gelatin block (Fig. 6 and Tab. 3). Young's modulus estimates using both the surface loading method and the shear wave method have the same order of magnitude.

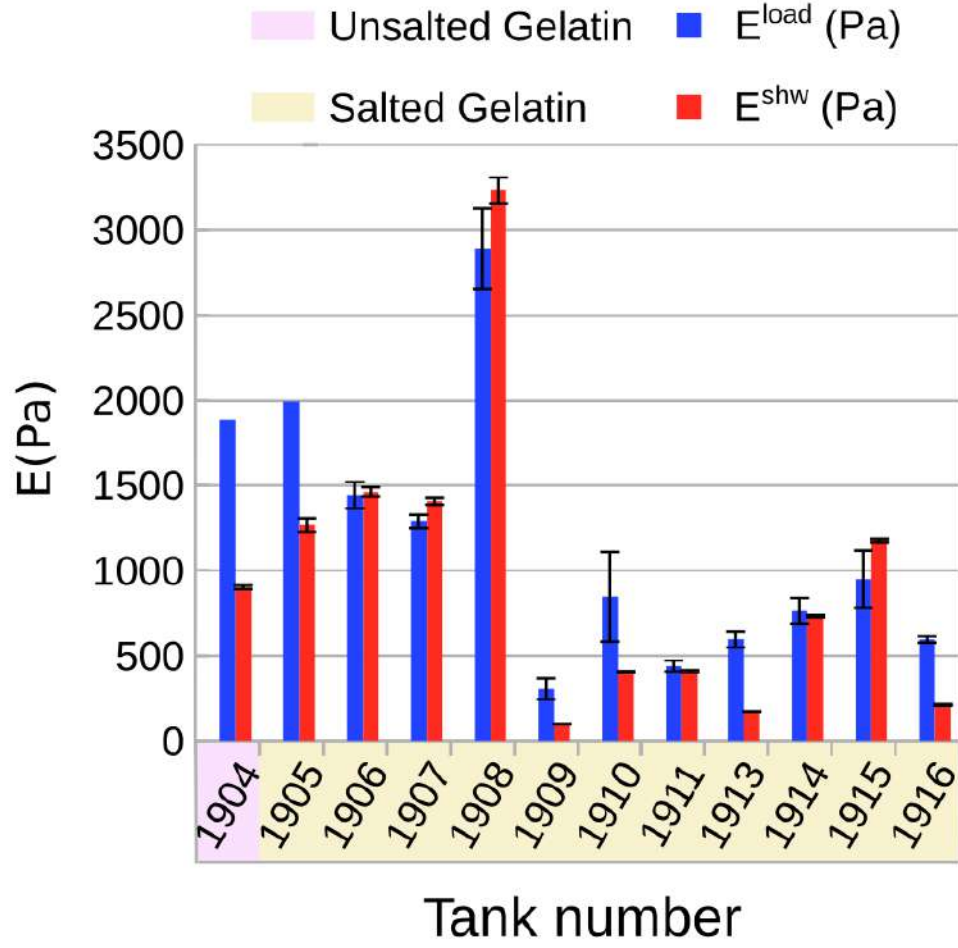


Figure 6: Comparison of Young's Modulus estimated for 12 gelatin tanks and two methods. The average value obtained from several measurements is represented, either derived by the surface loading method (E^{load} in blue), or from shear waves velocity measurements (E^{shw} in red). Tank numbers of unsalted gelatin and salted gelatin are highlighted in purple and orange respectively. Error bars represent the standard deviation.

Table 3: Comparison of Young’s modulus estimation by surface loading E^{load} and shear-wave velocity measurement E^{shw} for 12 tanks. ΔE is the absolute difference in Pa, $\frac{\Delta E}{E^{load}}$ is the percentage of difference between both measurements.

Tank	E^{load} (Pa)	E^{shw} (Pa)	ΔE (Pa)	$\frac{\Delta E}{E^{load}}$ (%)	$\frac{E^{load}}{E^{shw}}$
1904	1885	905	980	52 %	2.08
1905	1995	1270	725	36 %	1.57
1906	1445	1460	-15	-1 %	1.00
1907	1290	1410	-120	-9 %	0.91
1908	2890	3230	-340	-12 %	0.89
1909	305	100	205	67 %	3.05
1910	845	405	440	52 %	2.09
1911	440	410	30	6 %	1.07
1913	600	170	430	72 %	3.53
1914	760	735	25	3 %	1.03
1915	950	1170	-220	-23 %	0.81
1916	595	210	385	64 %	2.83

Discrepancies over 500 Pa are found only for 2 tanks (1904, 1905) and discrepancies over 60% are found for 3 other tanks (1909, 1913 and 1916) having the smallest values of Young’s modulus. In Tab. 3 we report the ratio between E^{load} and E^{shw} , which is smaller in our case (between 0.81 and 3.53) than in Pansino and Taisne [20] (between 0.79 and 5.29). In both studies, a better agreement between estimation by surface loading and by shear waves velocity is observed for more rigid gelatins. Pansino and Taisne [20] attributed the largest values given by the surface loading method to the non homogeneous cooling of the gelatin. Whereas the shear wave method allows to quantify the strength of the interior region of the tank, the surface loading method quantifies the strength of the upper layer, whose hardening by cooling is much quicker than the interior of the tank. This conclusion was supported by larger discrepancy between E^{load} and E^{shw} being observed after a shorter duration of cooling. However, because precise dimension of the load are not provided for each tank of Pansino and Taisne [20]’s study, an overestimation of the Young’s modulus by surface loading due to the use of the analytical formula cannot be completely excluded.

413 5. Young's modulus estimation by crack length measurement

414 5.1. Principles and theoretical background

415 The length of propagating air-filled cracks in a gelatin block depends on
 416 the injected air volume and on the physical properties of the gelatin [10]
 417 such that cracks shape might be used to estimate the Young's modulus [8].
 418 Whereas the critical fluid volume required to ensure a buoyant crack propaga-
 419 tion is a function of the fracture toughness [48], the relationship between the
 420 injected volume and the crack length only depends on the Young's modulus
 421 and buoyancy.

422 In the framework of the Weertman's theory, a static crack of length $L =$
 423 $2a$, filled with an incompressible fluid, is characterized by the following half-
 424 opening profile w along z direction [49, 50]:

$$w(z) = \frac{1 - \nu^2}{E} \Delta \rho g \sqrt{a^2 - z^2} (a + z); -a \leq z \leq a, \quad (9)$$

425 where a is the half-length of the crack and $\Delta \rho = \rho_{solid} - \rho_{fluid}$ is the density
 426 contrast between the fluid and the host rock. Integrating the opening profile
 427 (Eq. 9) over the crack's length gives the area A of the crack's cross section:

$$A = \frac{\pi(1 - \nu^2)}{8} \frac{\Delta \rho g}{E} L^3 \quad (10)$$

428 The influence of fluid compressibility on Eq. 10 can be shown by using a 2D
 429 numerical, boundary-element model (see Fig. 7). In 3D, one could expect for
 430 the volume V of the crack :

$$V = \alpha(1 - \nu^2) \frac{\Delta \rho g}{E} L^4, \quad (11)$$

431 with α a constant.

432 Considering that in side view a rising crack has a shape close to an ellip-
 433 soid in its upper part and close to a rectangular in its lower part (Fig. 8a),
 434 and that the crack half breadth or lateral dimension r is comprised between
 435 $3/4a$ and a [51], it follows that α is expected to range between 0.22 and 0.30.

436 5.2. Calibration of the Volume-Length relation in 3D

437 Volumes ranging between 0.4-20 mL were injected with syringes of differ-
 438 ent sizes (2 ± 0.2 , 10 ± 1 or 20 ± 2 mL). Measurements of the crack length from

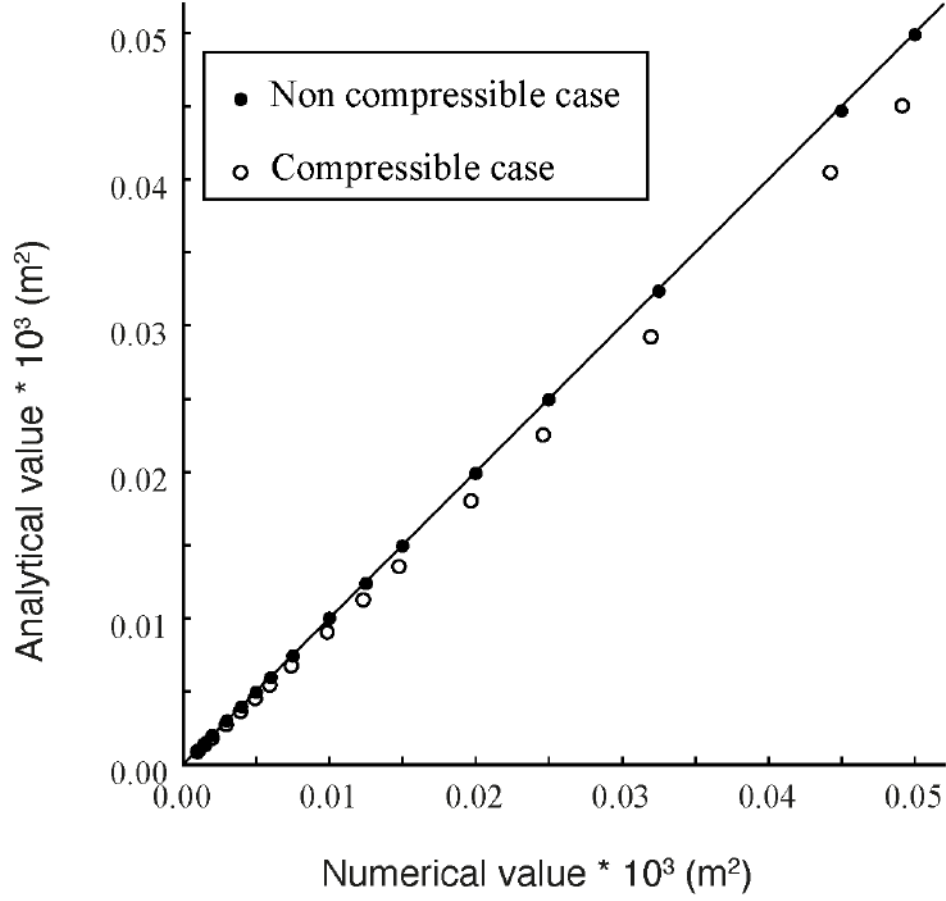


Figure 7: Comparison of the analytical value and the numerical value for the 2D cross-sectional area of a finite length static crack in an infinite elastic medium. Analytical value is given by Eq. 10, the numerical value is calculated with the Boundary Element model described in Maccaferri et al. [52], considering increasing crack lengths. For the compressible case, the bulk modulus of the fluid is set to 100 Pa whereas for the non compressible case it is set to 10^6 Pa.

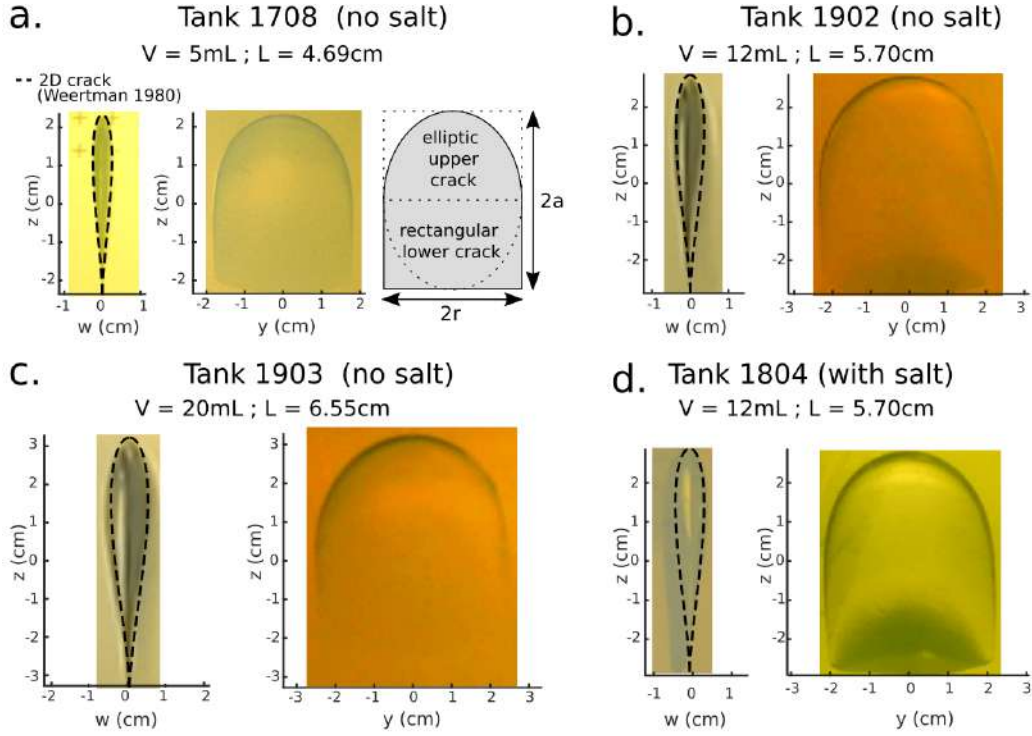


Figure 8: Panels (a), (b), (c) and (d) show crack shapes in cross section (left) and front (right) view of four experiments. Profile opening along cross sections is computed with Eq. 9 (dashed line). Panel (a) also shows a scheme of the theoretical shape of front view as a combination of an elliptic upper part which radius are r (half-width) and a (half-length) and a rectangular lower part which sides are $2r$ and a .

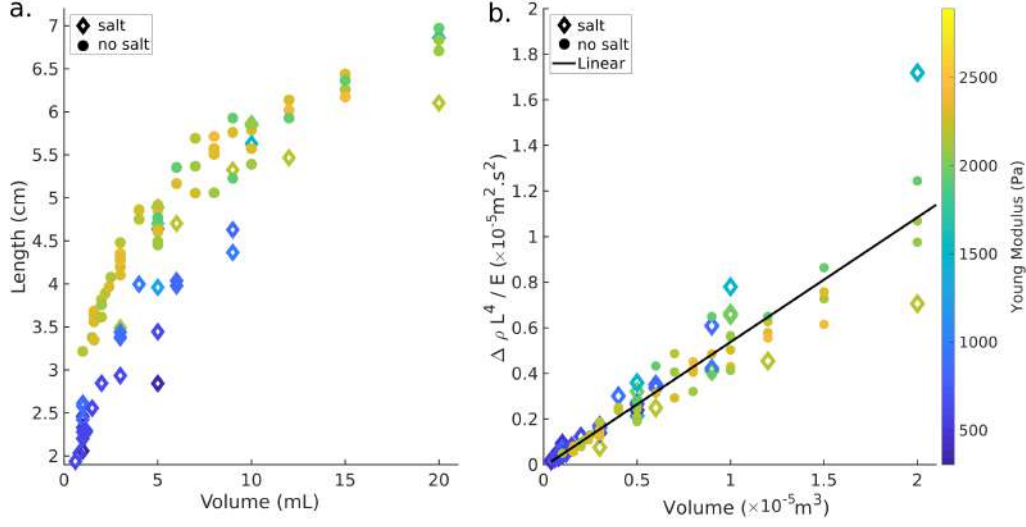


Figure 9: Relationship between crack length (L) and injected volume (V) for air-filled cracks (for all injections performed in tanks 1701 to 1916 listed in Tab. 1). Circles and diamonds represent injections inside unsalted gelatin and salted gelatin, respectively. Colors represent the Young's modulus ($E = E^{load}$) of the tank estimated by surface loading. (a) Crack length (L) as a function of injected volume of air (V). (b) $\frac{\Delta \rho L^4}{E}$ as a function of the injected volume of air (V), the linear tendency gives $\alpha = 0.25$ for Eq. 11.

both cameras are consistent and, for most of the injections, the crack length remains constant during crack propagation.

Fig. 8 shows a selection of cross-section and front views of cracks compared to the opening profile predicted by the Weertman's theory (Eq. 9). As shown by experiments performed injecting 12 mL in tank 1902 (unsalty) and in tank 1804 (salty) which have similar Young's modulus, the crack length is not affected by the addition of salt into the gelatin.

Fig. 9a shows the crack length (L) as a function of the injected volume (V) for each air injection. For a given injected volume, shorter cracks form in gelatin with lower Young's modulus. In order to test the validity of Eq. 11, Fig. 9b represents $\frac{\Delta \rho L^4}{E}$ as a function of the injected volume V . We use our numerically estimated Young's modulus for the first 33 tanks listed in Tab. 1 to calibrate Eq. 11 by determining the proportionality coefficient α . The linear tendency allows to estimate α_{air} to 0.25.

The crack length versus injected volume relationship (Eq. 11 with α_{air} set to 0.25) can thus be used to infer the Young's modulus inside any gelatin tank by measuring the crack length when injecting a known volume of air or

456 of any non-viscous buoyant fluid.

457 In order to further validate this method, we derive the Young's modulus,
 458 from Eq. 11 (E_α) with $\alpha=0.25$ considering several injected volumes in four
 459 tanks (tanks 2002, 2003, 2004 and 2005 as listed in Tab. 1), which were not
 460 used to derive α . We then compute the relative differences between E_α and
 461 E^{load} (Eq. 12) for several injections (Fig. 10):

$$\frac{\Delta E}{E}(\%) = \frac{|E^{load} - E_\alpha|}{E^{load} + E_\alpha} \times 200 \quad (12)$$

462 We get a mean error and a standard deviation of 17 ± 9 % which reflects
 463 the dispersion of our data due to measurements uncertainties on length,
 464 volume and $\Delta\rho$. Whereas the absolute accuracy of this method does not
 465 seem better than 15%, it enables us to evidence potential changes in the
 466 Young's modulus along the crack path as discussed below.

467 5.3. Evidence of a Young's modulus vertical gradient in some experimental 468 tanks

469 If the Young's modulus of the gelatin tank is homogeneous, the length of
 470 the crack does not vary, except in the close vicinity of the upper free surface
 471 where it is expected to decrease [53]. Otherwise, a progressive change in the
 472 crack length of an ascending crack may reflect a gradient in the rigidity of
 473 the gelatin. In two tanks (1806 and 1807), we injected the air a few hours
 474 after taking the gelatin out of the fridge. In those cases, cracks are getting
 475 shorter and thicker during their ascent consistently with a decrease of the
 476 Young's modulus value at shallower depths. In tank 1806, we measure the
 477 crack length at several depths for injections 1834 and 1835. Using Eq. 11,
 478 we evidence a vertical gradient of rigidity of 35 Pa.cm^{-1} and 43 Pa.cm^{-1} ,
 479 respectively (Fig. 11).

480 Those tanks are filled with salted gelatin but the observed gradient can-
 481 not originate from a gradient in salt concentration. If it had been the case
 482 larger salt concentration would have been expected in the lower part of the
 483 gelatin tank resulting in smaller values of Young's modulus at the bottom
 484 of the tank. An effect of a gradient in gelatin concentration cannot be ex-
 485 cluded, however it is unlikely as such a gradient was never observed in other
 486 tanks obtained following the same protocol. Such a decrease of the Young's
 487 modulus with decreasing distance to the gelatin surface cannot be due to
 488 an ongoing cooling either, as it would result in the reverse gradient. One
 489 explanation would be a progressive re-heating of the gelatin tank due to the

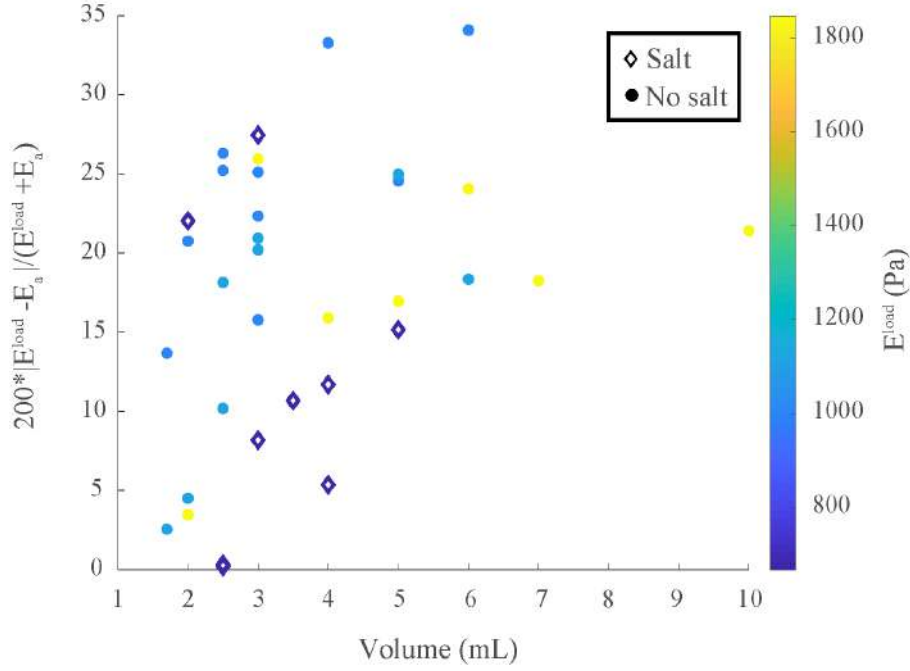


Figure 10: Relative difference between the Young's modulus estimated by air-filled crack length (E_a) and by surface loading (E^{load}) for four tanks (2002, 2003, 2004 and 2005 as listed in Tab 1). Circles and diamonds represent injections inside unsalted gelatin and salted gelatin, respectively. Colors represent the Young's modulus ($E = E^{load}$) of the tank estimated by surface loading. E_a is computed with $\alpha = 0.25$ in Eq. 11.

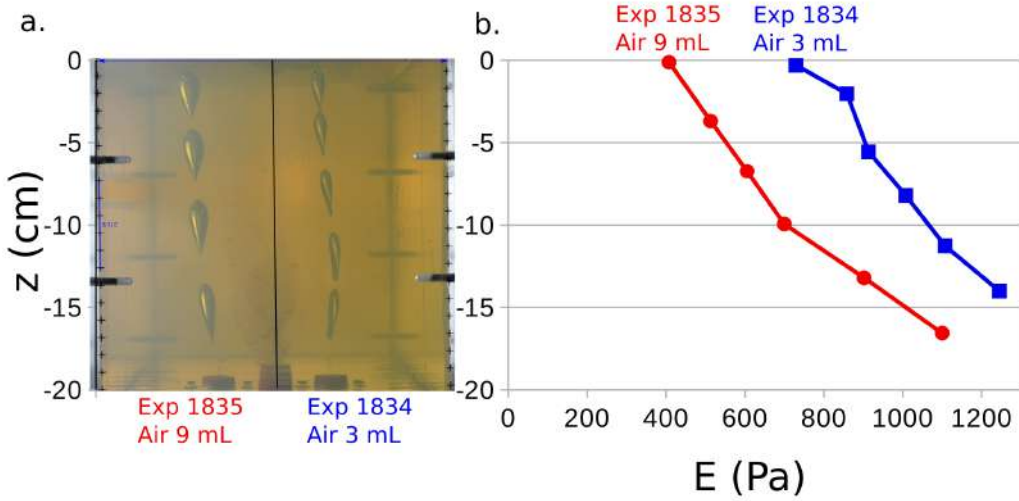


Figure 11: A rigidity gradient affects the tank 1806. (a) Screenshots showing the evolution of the shape of the cracks 1834 (3 mL) and 1835 (9 mL) during propagation. (b) Evolution of E with depth, blue and red curves represent experiments 1834 and 1835 respectively.

lighting. This is consistent with the fact that both experiments were run after a long stay (more than 4 hours) at room temperature. The slightly larger value obtained for the Young's modulus derived from injection 1834, performed before injection 1835, is consistent with this explanation. But the difference between both estimations might also reveal lateral variations in the tank. In particular, the crack in 1835 is injected in the backward side of the tank which is closer to the light source.

6. Fracture toughness characterization

Magma transport through the upper crust occurs mainly by dike propagation. Magma flows inside a planar fracture such that the velocity is partly controlled by fracturing, at least in the tip area, and depends on the crustal fracture toughness [41]. Fracture toughness is thus a key physical parameter of the gelatin for fluid-filled crack propagation experiments. This property is linked to the Young's modulus through the surface energy γ_s (see Eq. 1) [40], which is usually poorly known. Kavanagh et al. [16] estimated the value of surface energy for unsalted gelatin with concentration ranging from 5 to 8 % to be around $1.0 \pm 0.2 \text{ Jm}^{-2}$. It was done measuring independently the Young's modulus by surface loading and the fracture toughness

by quantifying the pressure required to propagate a pre-existing crack, using a two-dimensional approximation. Here we followed the strategy proposed by Heimpel and Olson [41] to estimate the fracture toughness. We estimated the velocity for several finite size air-filled cracks of various volumes injected inside the same tank of gelatin. We computed the stress intensity factor based on the crack length. Following Secor and Pollard [49], in 2D, the stress intensity factor in mode I K_I^{2D} , for a buoyant crack, can be expressed as:

$$K_I^{2D} = \Delta\rho g a \sqrt{\pi a} \quad (13)$$

Dahm [51] better characterized the 3D shape of buoyancy-driven propagating fractures with an approximately circular and straight line boundary at the upper and lower ends, respectively. He showed that fractures are self similar with the lateral extent (half breadth r) linked to the vertical extent (half length a) by the relationship: $r = (3/4)a$. Using this approximation and the expression for the stress intensity factor in 3D proposed by Heimpel and Olson [41], we can derive a scaling factor between 3D (K_I^{3D}) and 2D (K_I^{2D}) stress intensity factors:

$$K_I^{3D} = \frac{2\sqrt{3}}{\pi} K_I^{2D} = \frac{2\sqrt{3}}{\sqrt{\pi}} \Delta\rho g a \sqrt{a} \approx 1.103 K_I^{2D} \quad (14)$$

Fig. 12 shows the evolution of K_I^{3D} as a function of the crack velocity in three different tanks. The critical value for the stress intensity factor is the minimum which allows for crack propagation and equals the fracture toughness of the host medium. Therefore, a linear regression is used to quantify the fracture toughness of the gelatin in each tank, which corresponds to the vertical intercept.

Derived fracture toughness values are listed in Tab. 1 for each tank where this property was estimated. We use our estimates of K_I^{3D} , in combination with estimates of the Young's modulus E^{load} , to compute the surface energy γ_s for our gelatins (Eq. 1). Our results (Fig. 13) are mostly consistent with the previous estimate of γ_s [33, 16]. For unsalted gelatin, we derive a value for γ_s equal to 0.77 Jm^{-2} , which is slightly below the range derived by Kavanagh et al. [16] with more rigid gelatin. Importantly, our results clearly show that, for the same Young's modulus, a salted gelatin is characterized by a higher fracture toughness than an unsalted one. Using all available salted gelatin, for γ_s we obtain a value of 1.32 Jm^{-2} . When omitting tank 1906, which could be considered as an outlier in Fig. 13, we obtain for γ_s a value of 1.10 Jm^{-2}

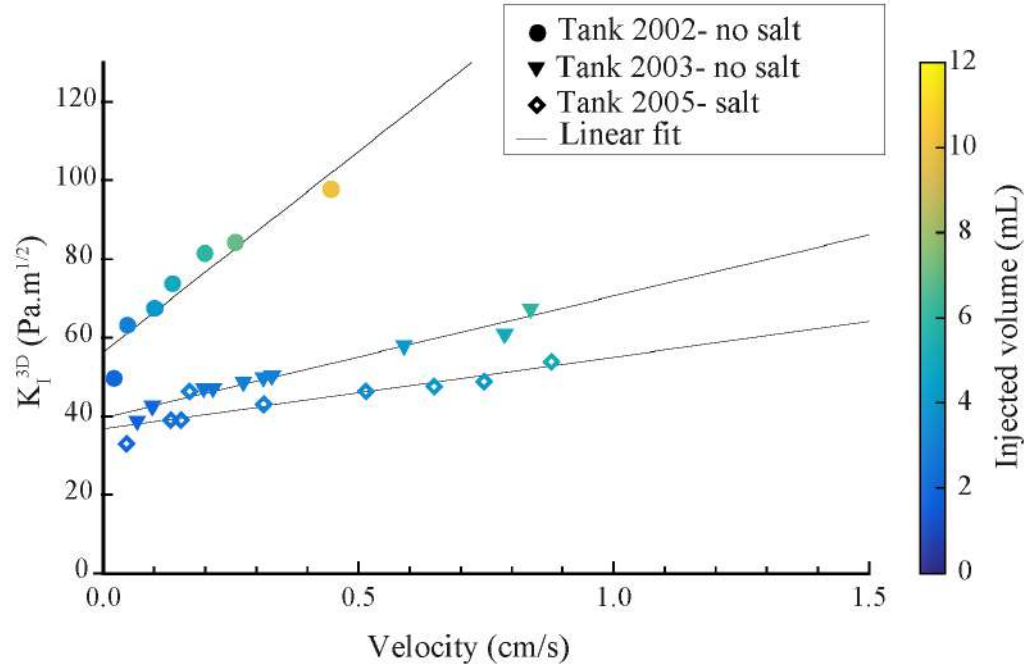


Figure 12: Stress intensity factor K_I^{3D} as a function of vertical velocity. 26 air injections for 3 different tanks (2002 and 2003 unsalted and 2005 salted) are represented. The stress intensity factor is estimated from the 3D theory (Eq. 14). For each tank, the limit of K_I when the vertical velocities tends to zero corresponds to the fracture toughness of the gelatin.

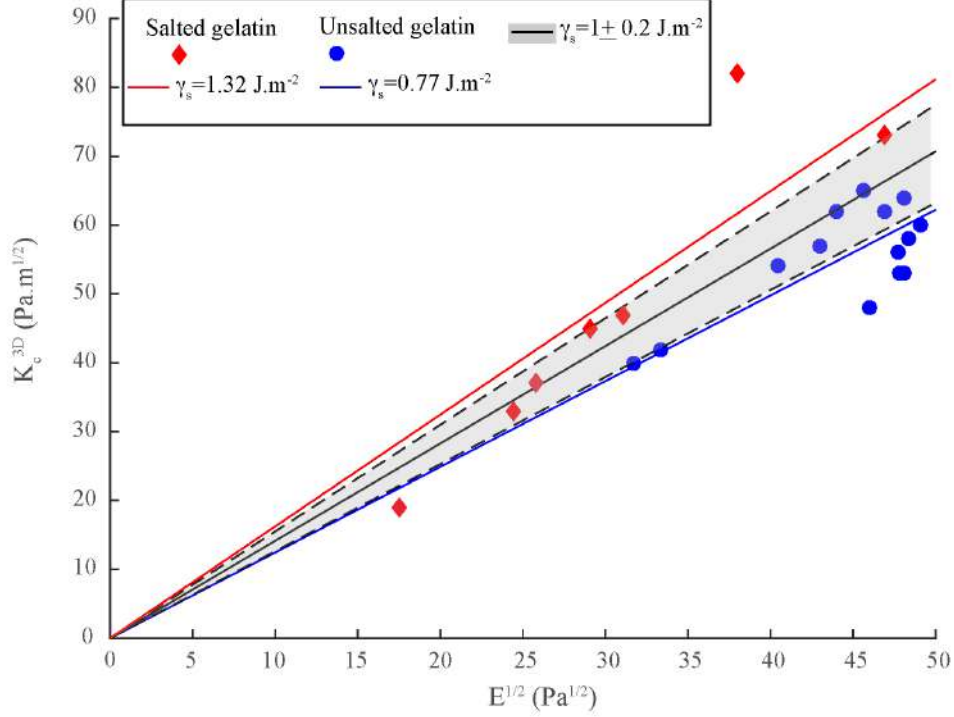


Figure 13: Relationship between the fracture toughness and the Young's modulus. Fracture toughness is estimated using the 3D approximation (Eq. 14). Linear fit, with uncertainties, corresponding to γ_s equal to $1 \pm 0.2 \text{ J.m}^{-2}$ [33, 16] is represented by the black line, whereas fits obtained from the salted and unsalted tanks are represented, respectively, by red and blue lines.

for the salted gelatin, which is significantly larger than the value obtained for the unsalted one. Further experiments varying the salt concentration would be useful to better characterize the influence of salt on the surface energy of gelatin.

7. Discussion

7.1. Comparison of the various methods for Young's modulus estimation

Our numerical simulations pointed out that the most common method used by experimentalists to infer Young's modulus is reliable (error $< 5 \%$) only if the diameter of the load applied at the surface of the gelatin is 20 times smaller than the minimum dimension of the tank. Otherwise, a numerical

551 model is required to link the vertical displacement to the Young's modulus of
 552 gelatin taking into account the boundary effect of the rigid tank walls. Here,
 553 we also provide a correction factor that can be applied to the value derived
 554 from the analytical formula. Even considering the improvement brought by
 555 the numerical model, this method still suffers some bias. The major bias is
 556 that the measurement is done at the surface, which makes it difficult to reveal
 557 potential heterogeneities or layering of the gelatin. The same limitation is
 558 encountered when deriving a static value of the crustal Young's modulus from
 559 surface loading or unloading events. The lateral extent of the load determines
 560 the crustal depth over which the Young's modulus is effectively averaged. In
 561 particular, one expects that loads applied on a broader area will probe a
 562 thicker layer of the underlying medium. In order to characterize this effect,
 563 we perform several numerical simulations in 2D axisymmetry using a FEM.
 564 Numerical calculations are performed either to match the experiments (with
 565 the Poisson's ratio set to 0.49 and a rigid load applied at the surface) or the
 566 crustal Earth case (with the Poisson's ratio set to 0.25 and a uniform pressure
 567 applied at the surface). The numerical box size is set to 2000 times the size
 568 of the load applied at the surface to match the ideal case of a half-space. We
 569 set the Young's modulus to be a linear function of depth at shallow level and
 570 constant deeper:

$$\begin{aligned}
 E(z) &= E_{surf} + \nabla E \times z \text{ for } z < Z_d \\
 E(z) &= E_{surf} + \nabla E \times Z_d \text{ for } z \geq Z_d
 \end{aligned} \tag{15}$$

571 where E_{surf} is the value of the Young's modulus at the surface (in $z=0$), ∇E
 572 is its vertical gradient and Z_d is the depth at which the Young's modulus
 573 becomes constant. The numerical simulation is used to calculate the value of
 574 the surface displacement induced by the load. From this value, we can esti-
 575 mate the corresponding Young's modulus (E_{eq}) for a homogeneous medium
 576 (using Eq. 5 for the rigid load condition and Eq. 6 for the uniform pressure
 577 condition). We then calculate an effective depth Z_{eq} corresponding to the
 578 depth over which the actual Young's modulus inside the medium should be
 579 averaged in order to obtain E_{eq} . It can be expressed by:

$$Z_{eq} = 2 \frac{E_{eq} - E_{surf}}{\nabla E}, \tag{16}$$

580 providing that Z_{eq} remains smaller than Z_d . Z_{eq} gives an estimate of the
 581 penetration depth reached by the surface load for Young's modulus measure-
 582 ments.

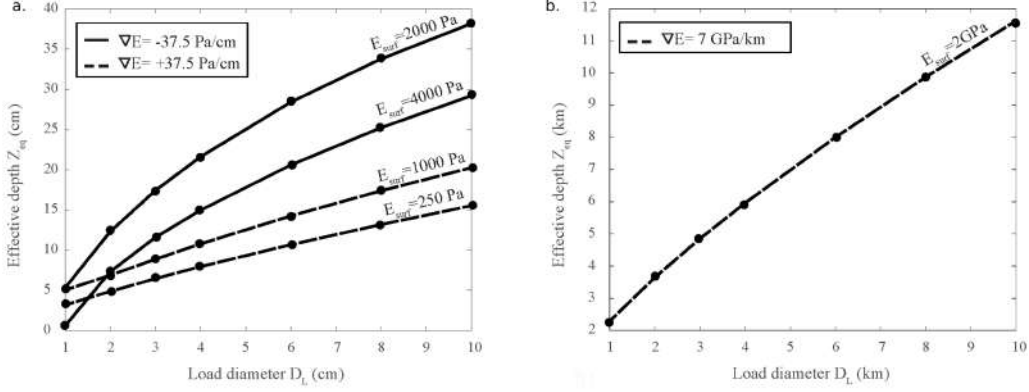


Figure 14: Effective depth over which the Young's modulus is probed as a function of the size of the load applied when there is a vertical gradient. Plain and dashed lines are for a decrease and an increase of the Young's modulus with depth, respectively. Different curves are obtained for various values of the Young's modulus at the surface. Circles are for the numerical simulations performed. a) Cases relevant for the gelatin tank. The depth Z_d at which the Young's modulus becomes constant is here set to 40 cm, the Poisson's ratio to 0.49 and a rigid load is applied at the surface. b) Case relevant for the Earth's crust. The depth Z_d at which the Young's modulus becomes constant is here set to 20 km, the Poisson's ratio to 0.25 and a uniform pressure load is applied at the surface.

Fig. 14 shows Z_{eq} as a function of the size of the load for different Young's modulus profiles. Results are obtained with the depth Z_d set to 40 cm for the experimental case (Fig 14a) and to 20 km for the crustal case (Fig 14b). In all cases, as expected, Z_{eq} increases with the diameter of the load applied. Consequently loads of various diameter might be used to evidence a vertical gradient of the Young's modulus in a gelatin block. In case a load of significant diameter is required, a numerical solution to interpret the subsidence should then be used. Note that applying the load at various location of the tank surface might also be useful to reveal potential lateral gradients. Once again, the use of the numerical model might be necessary.

To infer the gelatin Young's modulus, the alternative method based on the measurement of shear wave velocities as recently proposed by Pansino and Taisne [39, 20] or the new method based on the measurement of the length of a finite volume crack proposed in this study, enable to detect spatial variations of the Young's modulus. They can additionally be used to quantify the potential gradients, which is not possible with surface load measurements. The accuracy of the shear wave velocities method strongly depends on the absolute value of the Young's modulus. Higher rigidity, will produce faster

601 seismic waves, thus reducing the ability to follow a wave train with sufficient
602 resolution before any reflection occurred on the rigid walls. In contrast, the
603 method based on the air-filled crack propagation cannot be considered as
604 strictly non-destructive because once the crack has propagated through it,
605 the gelatin remains cut along the path followed by the crack.

606 Going back to the Earth’s crust, seismic tomography has been used to
607 infer spatial variations of the Young’s modulus at depth, usually showing
608 an increase of the rigidity with depth [54, 29, 30], whereas measurements
609 made by surface loading or unloading only provide a value averaged over
610 a given crustal thickness below the surface. Fig. 14b presents the depth
611 probed by surface loading as a function of the lateral extent of the load for
612 values corresponding to a typical crust. Note that usually when using surface
613 displacements induced by surface loading to infer the crustal rheology, the
614 load size is imposed by the natural phenomenon at play (e.g. lake level
615 change, ice thickness variations, etc.). Also, similarly to what we propose for
616 the gelatin, the length of magma intrusions could potentially be used to bring
617 insight into the crustal Young’s modulus. However it might be difficult to
618 have a precise knowledge of the volume of magma involved. Besides, here we
619 derived the relationship between length and volume for a non viscous fluid,
620 which might be not fully appropriate in case of dynamic magma propagation.

621 7.2. Critical length for crack propagation

622 The critical volume required to ensure liquid-filled fracture propagation is
623 key information both in the hydraulic fracturing domain and in volcanology.
624 Using a numerical model and analytical derivation, Davis et al. [48] and Sal-
625 imzadeh et al. [55] provided an expression for the critical volume for vertical
626 propagation of a buoyant crack in three dimensions. They underlined that
627 previous estimations for critical ”volumes” were only given in terms of crit-
628 ical fracture length and based on analyses performed in 2-D. In particular,
629 following Secor and Pollard [49] and using the fact that the stress intensity
630 factor is equal to the fracture toughness at the upper tip and zero at the
631 lower tip of the crack, the critical half-length is given by:

$$a_c = \left(\frac{K_c}{\Delta \rho g \sqrt{\pi}} \right)^{2/3}, \quad (17)$$

632 Using again the approximation $r = (3/4)a$ and the expression for the stress
633 intensity factor in 3D proposed by Heimpel and Olson [41], we end up with

634 an expression for the critical length in 3D:

$$a_c = \left(\frac{\sqrt{\pi}}{2\sqrt{3}} \frac{K_c}{\Delta\rho g} \right)^{2/3}, \quad (18)$$

635 Using Eq. 18 and Eq. 11, we obtain an expression for the critical volume and
 636 can express it the same way used by Davis et al. [48]. Using α equal to 0.25
 637 in Eq. 11, we have:

$$V_c = \alpha \frac{2^{13/3}}{9} \frac{1-\nu}{16\mu} \left(\frac{9\pi^4 K_c^8}{\Delta\rho^5 g^5} \right)^{1/3}, \quad (19)$$

$$V_c \approx 0.56 \frac{1-\nu}{16\mu} \left(\frac{9\pi^4 K_c^8}{\Delta\rho^5 g^5} \right)^{1/3}, \quad (20)$$

638 with μ the shear modulus. Eq. 20 is very close to the expression numer-
 639 ically derived by Davis et al. [48], who gave a coefficient of 0.75 instead of
 640 0.56. This expression of the critical volume can be useful to interpret the
 641 volume of magmatic dikes keeping in mind that it was derived neglecting
 642 potential viscous effects.

643 7.3. Young's modulus decrease and surface energy increase in presence of salt

644 We used the independent estimation of Young's modulus and fracture
 645 toughness to estimate the surface energy of the gelatin. We evidenced that
 646 the surface energy is increased by addition of salt. It follows that for the same
 647 value of the Young's modulus, a salted gelatin will have a higher fracture
 648 toughness. This is consistent with the roughly six times higher velocity
 649 measured in unsalted gelatin, for a similar injected volume and a similar
 650 Young's modulus (Fig. 8). In the same way the critical volume for crack
 651 propagation is larger for the salted gelatin than for the unsalted one. It is thus
 652 important to take into account this influence of the salt on the surface energy
 653 of the gelatin when using salted gelatin as a crustal analog. In particular, to
 654 enable the injection of viscous fluids like vegetable or silicon oils, the use of
 655 salted gelatin might be required in order to guarantee a sufficient buoyancy.
 656 In this case, the fracture toughness cannot be simply derived using the surface
 657 energy for unsalted gelatin.

658 8. Conclusion

659 We illustrated the added value of using numerical simulations to improve
660 the interpretation of analog experiments. In particular, we quantified the
661 errors associated with the use of the analytical formula corresponding to an
662 elastic half-space [45] for a finite medium. An overestimation of 5 % is ex-
663 pected when using this analytical formula to derive the Young’s modulus if
664 the tank size is not 20 times larger than the load diameter when a standard ex-
665 perimental setup is used with gelatin adhering to the tank walls. We showed
666 that using a 3D numerical model removes the constraint of only applying sur-
667 face loads of limited diameters to derive the Young’s modulus. This enables,
668 for instance, to check for potential heterogeneous elastic properties inside a
669 gelatin tank. Two others methods are suitable to quantify the Young’s mod-
670 ulus and can alternatively reveal its variations inside a tank. One consists
671 of measuring shear wave velocities, which is fully non-destructive. The other
672 is based on the calibration performed in this study and requires the quan-
673 tification of the length of cracks filled with a known volume of a non-viscous
674 buoyant fluid, which can be done with limited alteration of the gelatin. In
675 addition we highlighted the influence of salt on gelatin physical properties.
676 While salt was known to decrease the Young’s modulus value of the gelatin,
677 we showed that it also increases its surface energy. More generally, the infor-
678 mation provided by numerical models regarding the depth probed by surface
679 loading, might prove to be useful when interpreting Young’s modulus values
680 for the Earth’s crust derived by static loading/unloading events. Based on
681 our numerical model results in the case of a linear increase of the crustal
682 Young’s modulus with depth, we confirm that the lateral size of the surface
683 loading (or unloading) considered should be the same order of magnitude as
684 the crustal thickness to be probed.

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