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Electro-Osmotic Non-Isothermal Flow in Rectangular Channels with Smoothed Corners

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Abstract

Microchannel heat sinks are able to provide high cooling capabilities in terms of heat flux rates. This makes them particularly interesting for the thermal management of electronic components such as CPUs, which have high power density and small dimensions. Pressure drop of the coolant across the microchannels may, however, be significant and give rise to viscous heating, thereby preventing the practical use of these devices. When the coolant is a polar fluid and the channel walls possess a net electric charge, an alternative means of moving the fluid is through an applied external electric field. The flow which originates is called electro-osmotic (EOF). EOF does not require moving parts, is free of vibrations and does not need lubrication, but is subject to Joule heating of the fluid and has flow and heat transfer characteristics which differ from those of pressure-drive flows. In spite of several previous investigation on EOF, no attention has been paid to the changes in velocity and temperature distributions caused by modifying the base cross-section of the channels which may be circular, rectangular or polygonal, thanks to the current capabilities of microfabrication. This work investigates numerically the influence of smoothing the corners of the cross-section at fixed hydraulic diameter on the values of the Poiseuille and Nusselt numbers for the laminar, steady and fully developed, electro-osmotic flow in a rectangular channel subject to uniform heat flux and Joule heating. Several aspect ratios are considered, as are different values of the ratio of Joule heating to heat flux through the walls. The results highlight

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a very slight increase of the Poiseuille number with the radius of curvature, whereas the Nusselt number experiences a significant improvement. Correlations are obtained for both the Poiseuille and Nusselt number as a function of the radius of curvature, aspect ratio and Joule heating-to-heat flux ratio.

Keywords: Electro-osmotic flow, microchannel, rounded corners, heat transfer enhancement

1. Introduction

In the last two decades microchannels have been the subject of a large body of fundamental research and gradually evolved into constitutive elements of so-called micro-flow devices (MFDs), as can be easily realised when comparing older reviews on the subject, [1, 2], to more recent ones [3–5].

In spite of some research still being addressed to fundamental subjects [6–11], MFDs have now progressed beyond the level of prototypes and find applications in several fields [12], e.g. micro heat exchangers (MHXs) are employed in air conditioning systems [13] and heat pumping equipment [14]. Among other applications, microchannel heat sinks are able to provide high cooling capabilities in terms of heat flux rates. This makes them particularly interesting for the thermal management of electronic devices, owing to the ever-increasing compactness of the latter and, subsequently, power density. Even for very low flow rates, though, pressure drop across such devices may be significant, especially when liquids are employed as coolants, with the reduction in hydraulic diameter of the ducts quickly leading to viscous heating of the fluid circulated [15] and unviable pumping costs.

At small scales, however, flow devices can take advantage of micro-effects such as electro-osmosis, which allows the motion of a liquid relative to a charged surface, induced by an applied external potential gradient across a microchannel [16, 17]. When the coolant used is a polar fluid and the channel walls have a net electrical charge, an uneven charge distribution develops in the liquid, which may be used to move it by applying an electric field at the two ends of the channel, which acts on a layer of mobile ions close to

the walls. The flow which originates is called electro-osmotic (EOF). EOF represents a method to circulate the fluid alternative to pressure gradient, which is applicable just when channel dimensions drop. EOF do not require moving parts, do not produce noise nor vibrations and do not need lubrication. Also, liquid reservoirs tiny volumes, making them ideal for direct connection to the chips. EOF have been reported to yield experimental Nusselt numbers about 10% larger than pressure-driven flow (PDF) for the same geometry, although Joule heating may become significant when applied voltage between the electrodes increases beyond a certain threshold [18]. Further advantages of EOF lie in the straightforwardness of its operation through voltage signals sent to electrodes as compared to the complications of manufacturing and controlling a micro mechanical pump. Among drawbacks, the velocity profiles differs significantly to that of pressure-driven flows, the chemical composition at the interface exerts a strong influence on it, and the circulation of ions causes Joule heating in the fluid, which partly offsets its cooling capabilities [16]. Electro-osmotic flows in microchannels have received considerable theoretical attention, both for traditional geometries, such as circular ducts and parallel plates [19, 20], but also for more complex geometries such as polygonal, elliptical and triangular ducts [21–23] and the use of electro-osmotic pumps (EOPs) has been investigated both theoretically and experimentally [24–27] to circulate a polar fluid through the channels of heat sinks with the specific purpose of electronics cooling, [18, 28, 29]. The use of non-Newtonian fluids ([30, 31]) and nanofluids has also been investigated in more recent years, [32].

Concerning the single microchannel, which is the building block of every MHX, several aspects have been the subject of investigation. The circular geometry was one of the earliest to be investigated, as per the work of Rice and Whitehead, [19], who dealt with non-negligible thickness of the EDL compared to the classical treatment by Smoluchowski and predicted a maximum in the apparent viscosity of the flow (the so-called electro-viscous effect). Maynes and Webb studied the convective heat transfer for purely electro-osmotic [33] and both pressure and electro-osmotically driven flows, [34] considering the relative extension of the Debye length to the channel half-width,

52 indicated with Z . Their analysis was limited to circular and parallel-plate ducts and to
53 fully-developed velocity profiles and revealed how the temperature profile and Nusselt
54 number strongly depend on the values of Z , of the non-dimensional Joule heating and,
55 where applicable, on the relative magnitude of the external electric field to pressure
56 gradient imposed. For simple geometries, Moghadam ([35] obtained the exact solution
57 for the entrance region of a circular capillary considering the contribution of viscous
58 dissipation, which was compared to Joule heating. The results outlined a decrease of
59 the Nusselt number in the entrance region and a possible temperature overshoot near
60 the wall due to high velocity gradients.

61 Vocale et al. [23] investigated numerically the fully-developed, isothermal flow in trian-
62 gular microchannels for both purely electro-osmotic and combined pressure- and electro-
63 osmotically driven cases. Their results remarked the strong influence on the velocity
64 field of the aspect ratio and of the electrokinetic diameter of the duct: in particular,
65 the former affects the maximum achievable flowrate, whilst the latter determines the
66 maximum pressure gradient which may be overcome. The role of the relative ratio of
67 pressure forces and electrical forces acting on the fluid was also discussed and charac-
68 teristic curves for EOPs with triangular channels were obtained. Vocale et al., [22],
69 also carried out a numerical study on the non-isothermal electro-osmotic flow in elliptic
70 ducts under so-called H1 thermal boundary conditions (i.e. uniform heat flux and uni-
71 form temperature over the heated perimeter) for different aspect ratios, electrokinetic
72 diameters and relative magnitude of Joule heating to heat flux through the walls, M_z ,
73 also comparing the results with those for rectangular cross-sections of similar aspect
74 ratios. Their results highlight again a strong influence of the aspect ratio and of the
75 electrokinetic diameter, with the Nusselt number increasing with the latter quantity.
76 Joule heating affected the heat transfer adversely, but was negligible for $M_z \leq 0.1$.

77 From the above analysis it can be appreciated how several aspects of electro-osmotic
78 flow in microchannels have been investigated, in particular referring to the type of fluids
79 involved, the relative magnitude of the external electric field, the type of flow (purely
80 electro-osmotical or a combination of electro-osmosis and pressure gradient) and ge-

ometry of the channels, which can take several forms (circular, trapezoidal, rectangular, etc.) thanks to the current development of micro-fabrication technologies. Optimising the cross-section can lead to better performance in terms of heat transfer, pressure drop or entropy production, yet, to the Author's best knowledge, no analysis has been carried out so far as regards the smoothing of corners of the cross-section, which has been demonstrated to generate improvements in the transport phenomena, sometimes with increases in heat transfer that overcome those in frictional losses, [36, 37], and has already been investigated for pressure-driven flows in microchannels with and without viscous dissipation, [38–40]. To this purpose, a purely electro-osmotic, fully developed flow of a Newtonian fluid in a microchannel of base rectangular cross-section subject to H1 boundary conditions is numerically analysed. Several aspect ratios are considered, starting from square ducts to configurations resembling parallel plates, and several values of M_z are also studied. The corners of the reference geometry are progressively rounded and the effect of the change of cross-section on the Poiseuille and Nusselt numbers, which are related to the parameters of interest by simple correlations. The results can also be used to optimise the channels e.g. in terms of performance evaluation criteria ([41]).

2. Model of the EOF

Electro-osmosis refers to the bulk fluid motion observed through capillaries or microchannels when electric fields are applied across them. A detailed description of the phenomenon and of its treatment can be found in the excellent book by Kirby, [16], a more succinct treatment is given in [17, 42]. As mentioned earlier, bulk movement of the fluid in EOF is the result of the application of an external electric field which interacts with the locally non-uniform charge density close to the walls. The problem can receive an integral, one-dimensional treatment considering the bulk fluid motion only, replacing the phenomena occurring at the wall-fluid interface with an effective slip velocity condition, [16], or the phenomena in the tiny region where charge density is non-uniform, the so-called electric double layer (EDL) may be treated to various

degrees of complexity. In this work the so-called Gouy-Chapman EDL model will be adopted, which consists of some simplifying assumptions. The model assumes that the EDL is at equilibrium, so that electrostatic forces balance Brownian thermal motion. As a result, the net charge of the walls, deriving from adsorption and ionization, is balanced by a mobile diffuse charge density. The two regions form the electric double layer, and the potential on the ideal plane of separation is called Stern potential, ζ : since measuring the actual electric potential at the wall is ridden with experimental difficulties, the Stern potential is used instead as a boundary condition. To describe and predict the behaviour of EOF in the Gouy-Chapman approximation, the model must therefore determine the charge distribution within the fluid, the resulting electric potential distribution, the velocity and temperature fields.

2.1. Ion distribution

It is assumed that the velocity for the i -th particle at a given point is given by the sum of three components, the mean flow velocity of the electrolyte, \vec{v} , the velocity of the ions acted upon by the electric field, \vec{v}_{drift} , and the diffusion velocity, \vec{v}_{diff} , which is generated by Brownian motion:

$$\vec{v}_i = \vec{v} + \vec{v}_{diff} + \vec{v}_{drift} \quad (1)$$

Concentration c_i for the i -th ionic species is obtained writing the steady-state mass balance in the absence of species generation, Eq. (2)

$$\nabla \cdot (D_i \nabla c_i) - \nabla \cdot (c_i \vec{v}) - \nabla \cdot (c_i \vec{v}_{drift}) = 0 \quad (2)$$

where the diffusion velocity has been expressed in terms of diffusion coefficient, D_i and concentration gradient, by using Fick's law. If an electrostatic force \vec{F}_{es} generated by an electric field \vec{E} is exerted on the ions, it is assumed that they move with a drift velocity such that

$$\vec{v}_{drift} = \mu \vec{F}_{es} \quad (3)$$

131 where μ is the electron mobility. \vec{F}_{es} can be expressed for irrational electric fields
 132 in terms of the electric potential ψ for a ion of valence z_i as

$$F_{es} = z_i e \vec{E} = -z_i e \nabla \psi \quad (4)$$

133 where e is the magnitude of the unit electron charge. The electric field and gra-
 134 dient concentration generate two currents, called drift current and diffusion current
 135 respectively, which must balance out at equilibrium:

$$J_{drift} = -\mu \rho \nabla \psi \quad J_{diff} = -D \nabla \rho_e \quad (5)$$

136 In Eq. (5) ρ_e is the charge density. Assuming that ions can be treated as point
 137 charges, Maxwell-Boltzmann statistics holds and the diffusion coefficient can be related
 138 to the Boltzmann constant, κ_B and thermodynamic temperature, T by the so-called
 139 Einstein-Smoluchowski equation

$$D = \mu \kappa_B T \quad (6)$$

140 Substituting Eq. (6) into Eq. (3) and recalling Eq. (4), the drift velocity becomes

$$\vec{v}_{drift} = -\frac{z_i e c_i}{\kappa_B T} \nabla \psi \quad (7)$$

141 Since the fluid can be assumed as incompressible, $\nabla \cdot \vec{v} = 0$, and Eq.(2) yields

$$\nabla^2 c_i - \frac{\vec{v}}{D_i} \cdot \nabla c_i + \nabla \cdot \left(\frac{z_i e c_i}{\kappa_B T} \nabla \psi \right) = 0 \quad (8)$$

142 Equation (8) can be expressed in non-dimensional form through the hydraulic di-
 143 ameter D_h , the mean concentration of the i-th species, c_0 and the norm of the velocity
 144 vector $||\vec{v}||$ by introducing the quantities below

$$\nabla^{2*} = D_h^2 \nabla^2 \quad \nabla^* = D_h \nabla \quad Pe_m = \frac{||\vec{v}||}{D_i} D_h \quad c_i^* = \frac{c_i}{c_0} \quad ||\vec{v}|| \cdot \hat{i}_v = \vec{v} \quad (9)$$

so that

$$\nabla^{2*} c_i^* - Pe_m \hat{i}_v \cdot \nabla^* c_i^* + \nabla^* \cdot \left(\frac{z_i e c_i^*}{\kappa_B T} \nabla^* \psi \right) = 0 \quad (10)$$

145 By further assuming that the mass Peclet number, Pe_m is negligible (i.e. no diffusive
146 mass transport along the axial direction), one obtains

$$\nabla^* \left(\nabla^* c_i^* + c_i^* \nabla^* \left(\frac{z_i e}{\kappa_B T} \right) \right) = 0 \quad (11)$$

147 It can be proven that the solution of Eq.(11) is given by

$$c_i^* = \exp \left(\mp \frac{z_i e}{\kappa_B T} \psi \right) \quad (12)$$

148 where the sign depends on whether concentration refers to co-ions or counter-ions
149 respectively.

150 2.2. Electrical potential

151 Knowledge of the concentration of co-ions and counter-ions allows the calculation
152 of the net free charge density, ρ_e in the fluid:

$$\rho_e = z_i e (c_+ - c_-) = -2z_i e c_0 \sinh \left(\frac{z_i e}{\kappa_B T} \right) \quad (13)$$

153 which is employed in the Poisson equation $\nabla \cdot \varepsilon \vec{E} = -\rho_e$ to obtain the electrical
154 potential ψ

$$\nabla^2 \psi = \frac{2z_i e c_0}{\varepsilon} \sinh \left(\frac{z_i e}{\kappa_B T} \right) \quad (14)$$

155 where the electrical permittivity ε is assumed to be constant. Two more non-
156 dimensional quantities are introduced, namely

$$x^* = \frac{x}{D_h} \quad * = \frac{z_i e}{\kappa_B T} \quad (15)$$

157 and Eq.(14) becomes

$$\nabla^{2*} \psi^* = (\lambda_D D_h) \sinh(\psi^*) \quad (16)$$

where $\lambda_D = \sqrt{2e^2 c_0 z_i^2 / \varepsilon \kappa_B T}$ is the Debye-Hückel parameter, reciprocal to the Debye length. The latter quantity is a property of the electrolyte solution, i.e. the bulk fluid, and gives an estimate of the thickness of the EDL. To solve Eq.(16) the value of the potential on the Stern plane, ζ , is used as a boundary condition, thus neglecting the thickness of the compact layer. In non-dimensional form

$$\psi^*(x^* \approx 0) = \psi_0^* = \zeta^* = \frac{z_i e}{\kappa_B T} \zeta \quad (17)$$

Observing Eqs.(16) and (17) it is apparent that ψ^* depends on the geometry, on ζ^* and on $\lambda_D D_h$.

2.3. Velocity field

The non-dimensional velocity distribution in the channel is obtained from the solution of the Navier-Stokes equation. The assumptions made in this work are steady, fully-developed laminar flow of a Newtonian fluid with thermophysical properties independent of temperature inside a microchannel of uniform cross-section, A_c , with rigid, non-porous walls and of length L . Further, no pressure gradients act along the flow direction and the only body force present is an external electric field along the channel axis, z , $\vec{E}_{ext} = E_z \hat{i}_z$. As a consequence, only the velocity component along z is non-zero, and its magnitude at each point of the cross-section depends on the other two coordinates: $\vec{u} = u_z(x, y) \hat{i}_z$, which is written as u . The momentum transport equation

$$\mu \nabla^2 \vec{v} + \rho_e \vec{E}_{ext} = \vec{0} \quad (18)$$

only has one component along z and can be written in non-dimensional form introducing the quantities below

$$y^* = \frac{y}{D_h} \quad A_c^* = \frac{A_c}{D_h^2} \quad u^* = \frac{u}{U}, \quad E_z^* = \frac{E_z L}{\zeta} \quad (19)$$

So that

$$\nabla^{2*} u^* = \frac{2n_0 z e \zeta D_h^2}{\mu U L} = M E_z^* \sinh(\psi^*) \quad (20)$$

178 The normalizing velocity U is chosen such that $M = 1$, and Eq. (20) becomes

$$\nabla^{2*} u^* = E_z^* \sinh(\psi^*) \quad (21)$$

179 The boundary conditions for the problem are given by the no-slip velocity condition
180 at the walls:

$$u_{wall} = u_0 = 0 \Rightarrow u_0^* = 0 \quad (22)$$

181 Considering how Eq.(21) was obtained, it is immediate to recognise that the non-
182 dimensional velocity depends on the geometry, the zeta potential the product of λ_D
183 times D_h and the external electric field.

184 2.4. Temperature Field

185 The temperature distribution is obtained from the energy equation, once the velocity
186 field has been determined. Joule heating caused by the current density flux \vec{j} operates
187 as a source term q_g , which results from the scalar product of $\vec{j} = \kappa_0 \vec{E}_{ext}$, κ_0 being the
188 electrical conductivity of the fluid, and the electric field \vec{E}_{ext}

$$q_g = \vec{j} \cdot \vec{E}_{ext} = \kappa_0 E_z^2 \quad (23)$$

189 A thermally fully-developed, steady flow is assumed, so that the variation of local
190 temperature $T(x, y, z)$ along the flow direction can be related to the variation of bulk
191 temperature $T_b(z)$:

$$\frac{\partial T}{\partial z} = \frac{dT_b}{dz} \quad (24)$$

192 The variation of the bulk temperature with the axial coordinate is obtained by an
193 energy balance over a fluid element occupying a length dz of the channel

$$q' dz + \kappa_0 E_z^2 A_c dz = \rho u_b A_c c_p dT_b \quad (25)$$

194 so that

$$\frac{\partial T}{\partial z} = \frac{q' + \kappa_0 E_z^2 A_c}{\rho u_b c_p A_c} \quad (26)$$

195 where u_b is the bulk velocity, ρ is the fluid density and c_p the specific heat capacity
 196 at constant pressure. The energy equation for the problem is therefore

$$u \left(\frac{q' + \kappa_0 E_z^2 A_c}{u_b A_c} \right) = \kappa \nabla^2 T + \kappa_0 E_z^2 \quad (27)$$

197 with κ thermal conductivity of the fluid. The non-dimensional form of Eq.(27) is
 198 obtained after definition of the non-dimensional temperature and of M_z

$$T^* = \frac{\kappa (T - T_w)}{q'} \quad M_z = \frac{\kappa_0 E_z^2 D_h^2}{q'} \quad (28)$$

199 with T_w temperature at the heated perimeter. M_z relates Joule heating over a
 200 cross-section to the heat flux along its perimeter, and is analogous to the Brinkman
 201 number, which compares viscous dissipation and heat transfer between fluid and walls.
 202 Equation(27) becomes

$$\frac{u^* U}{u_b} \left(\frac{1}{A_c^*} + M_z \right) = \nabla^{2*} T^* + M_z \quad (29)$$

203 The bulk velocity u_b can be related to U , which was used as a normalisation variable
 204 through the definition of bulk velocity itself, and yields

$$u_b = \frac{U}{A_c^*} \int_{A_c^*} u^* dS^* \quad (30)$$

205 To solve Eq. (29) boundary conditions must be specified; for the present study,
 206 the so-called H1 (i.e. uniform heat flux along the channel's axis, q' , and uniform tem-
 207 perature over the heated perimeter of the cross-section) condition was chosen. The
 208 constraint on temperature at the walls in non-dimensional form is

$$T^* \Big|_{P_h^*} = 0 \quad \text{and} \quad \left(\frac{\partial T}{\partial \hat{n}} \right)_{P^* - P_h^*} \quad (31)$$

for the heated and the unheated portion of the perimeter, P^* , respectively. The condition on the non-dimensional line heat flux is given by specifying a value for M_z . In view of the above discussion, the temperature field is dependent on the flow field and on the heat flux over the cross-section, so that one can express this as

$$T^* = T^*(geometry, \zeta^*, \kappa D_h, M_z)$$

It is to be remarked that the dependence on the non-dimensional external electric field E_z^* has disappeared; this is due to the presence of the u^*/u_b ratio: in the momentum transport equation the term E_z^* introduces but a scaling factor in the solution, which appears both in u^* and in u_b .

Knowledge of the velocity and temperature fields allow the computation of the Poiseuille and Nusselt numbers, which are related to the momentum and heat transfer characteristics of the problem. It can be demonstrated that Poiseuille number is given by

$$Po = -\frac{2U}{P^*u_b} \int_{P^*} \frac{\partial u^*}{\partial \hat{n}^*} dP^* \quad (32)$$

Similarly, the Nusselt number is given by Eq. (33):

$$Nu = -\frac{1}{P_h^*} \frac{\int_{A_c^*} u^* dA_c^*}{\int_{A_c^*} T^* u^* dA_c^*} \quad (33)$$

where the ratio U/u_b has been expressed by means of Eq.(30). It is to be noted that the definition of the Poiseuille and Nusselt numbers make the introduction of the bulk velocity u_b necessary.

2.5. Geometry investigated

The starting geometry is rectangular with sharp corners, which are progressively rounded, up to the maximum allowable radius of curvature for a given aspect ratio, β . With reference to Fig.1, the non-dimensional radius R_c and β are defined as:

$$\beta = \frac{2a}{2b} \quad R_c = \frac{r_c}{a} \quad (34)$$

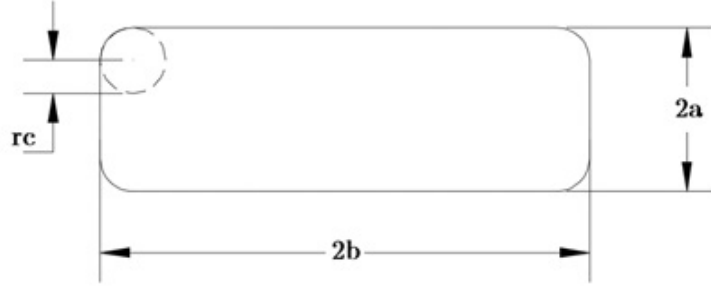


Figure 1: Rectangular cross-section with rounded corners.

The cross-sectional area A_c , the perimeter P and the hydraulic diameter, D_h , the latter used as reference dimension together with the channel length L , are given by:

$$A_c = a^2 \left[\frac{4}{\beta} - R_c^2 (4 - \pi) \right] \quad (35)$$

$$P = 4a \left[1 + \frac{1}{\beta} - 2R_c \left(1 - \frac{\pi}{4} \right) \right] \quad (36)$$

$$D_h = \frac{4A_c}{P} = a \frac{\frac{4}{\beta} - R_c^2 (4 - \pi)}{1 + \frac{1}{\beta} - 2R_c \left(1 - \frac{\pi}{4} \right)} \quad (37)$$

When the optimal radius of curvature of the cross-section is sought in terms of e.g. Performance Evaluation Criteria [41] or Entropy Generation Minimisation [43] one further geometry constraint must be applied [40, 44], either on the reference side, cross-section, heated perimeter or hydraulic diameter, this is not the case for the computation of the Poiseuille and Nusselt numbers.

2.6. Methodology for the Numerical Solution

In this paper the fully-developed electro-osmotic flow of an aqueous solution subject to joule heating and heat transfer through a microchannel of rectangular cross-section is modelled and numerically investigated. The starting configuration was that with sharp corners, which were progressively smoothed until the maximum value of the non-dimensional radius of curvature, i.e. $R_c = 1.0$ in steps of 0.1, for four aspect ratios, namely $\beta = [0.1, 0.25, 0.5, 1]$, which cover the range from square channels ($\beta = 1$) to

239 a shape approaching the parallel plate configuration ($\beta = 0.1$), where it is expected
 240 that the actual shape of the corners loses its influence on the velocity and temperature
 241 distributions. The ratio of Joule heating to heat transfer from the walls was also
 242 varied, from negligible to quite significant, $M_z = [10^{-3}, 10^{-2}, 10^{-1}, 0.3, 0.6, 1]$. Two
 243 different hydraulic diameters, $D_h = 3 \mu m$ and $D_h = 24 \mu m$ were considered with the
 244 same channel length, namely $L = 0.01 m$. The fluid is de-ionised, ultra-filtered water
 245 (DIUF) the electrolyte concentration, Stern potential and electrical field intensity are
 246 those already employed in [24, 25, 44] and, in non-dimensional form, correspond to
 247 $\lambda_D D_h = 9.85$ and $\lambda_D D_h = 78.40$, $\zeta^* = 7.92$, $E_z^* = 5000$.
 248 Equations (16), (21) and (29) were solved with the boundary conditions specified by
 249 Eqs. (17), (22) and (31); for the latter, the whole perimeter is heated. A commercial
 250 finite-element solver, *Comsol Multiphysics*[®] was used for the computations. The two-
 251 dimensional domain was discretised with a triangular, unstructured grid employing
 252 boundary layer at the walls to have a satisfactory resolution where gradients are the
 253 steepest, whilst a much wider mesh sufficed in the central region, since contrary to the
 254 case studied in [45] the EDLs do not overlap.
 255 Grid independence was checked against the values of the Poiseuille number, because the
 256 Nusselt number was found to reach a constant value at a lower number of elements: the
 257 difference in Po for a rectangular section with sharp corners between a mesh with 9624
 258 nodes and one with 45220 nodes was 0.6%, whereas between 45220 and 193800 nodes it
 259 was 0.1%. Therefore, the intermediate value was chosen as a satisfactory compromise
 260 between accuracy and computational time. For the cases investigated in this work, the
 261 maximum number of cells used was $\approx 5 \cdot 10^4$. The simulation of each configuration took
 262 a negligible computational time (a few minutes in the worst cases).
 263 Numerical verification of the volumetric flowrate for the case of a triangular cross-section
 264 was carried out against the results of [21], with results analogous to the ones reported
 265 in [23]: discrepancies between simulation and benchmark data fell within $\pm 0.7\%$.

266 3. Results and discussion

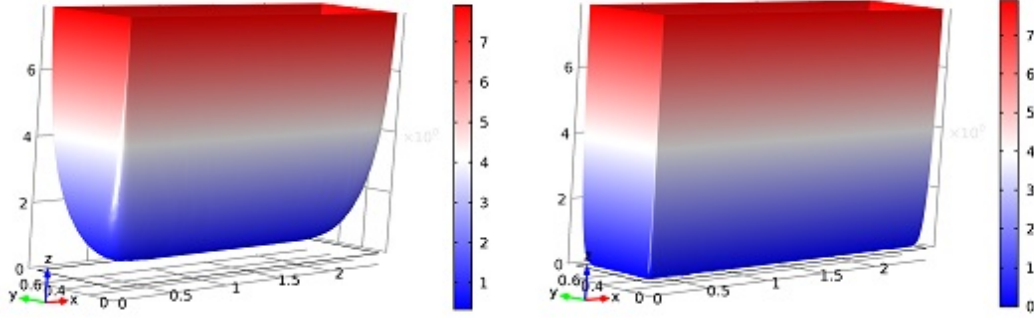


Figure 2: Electrical potential for aspect ratio $\beta = 0.25$, sharp corners and $\lambda_D D_h = 9.85$ (left) and $\lambda_D D_h = 78.40$ (right).

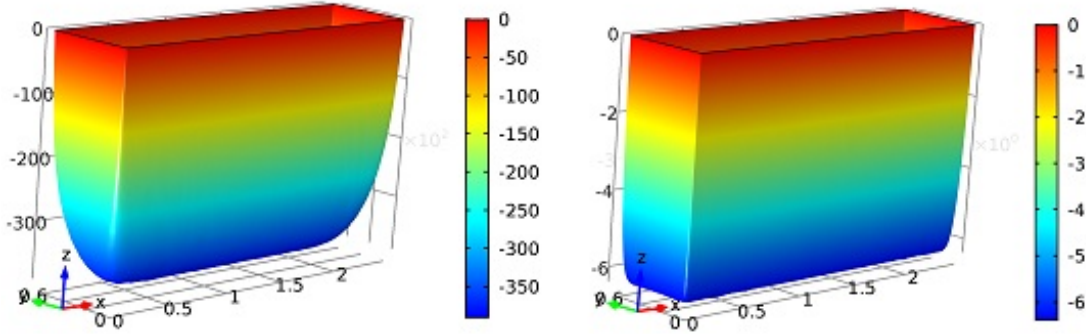


Figure 3: Velocity field for aspect ratio $\beta = 0.25$, sharp corners and $\lambda_D D_h = 9.85$ (left) and $\lambda_D D_h = 78.40$ (right).

267 The electrical potential distribution, velocity field and temperature field for $\beta = 0.25$
 268 and sharp corers are shown in Figs.2, 3 and 4 respectively for $\lambda_D D_h = 9.85$ (left) and
 269 $\lambda_D D_h = 78.40$ (right). It can be appreciated how the velocity profile is much flatter for
 270 the higher value of $\lambda_D D_h$: for higher hydraulic diameters and unchanged Debye length,
 271 as is the case here, the EDL occupies a much smaller portion of the channel, which

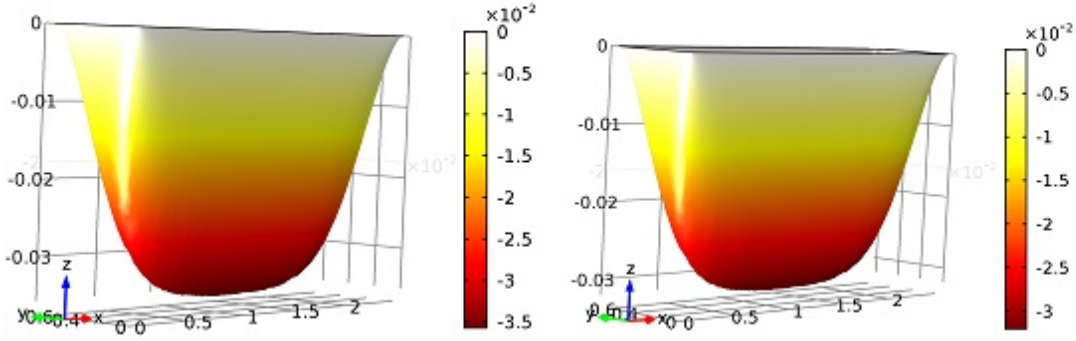


Figure 4: Temperature field for aspect ratio $\beta = 0.25$, sharp corners and $\lambda_D D_h = 9.85$ (left) and $\lambda_D D_h = 78.40$ (right).

272 results in smaller velocities and smaller flowrates, because most of the electrolyte is
 273 dragged along, hence the flat velocity profile in the central portion of the cross-section.
 274 Although the temperature profiles appear similar, an analysis of the numerical values
 275 reveals higher temperature gradients at the wall for the larger hydraulic diameter,
 276 which is reflected in the values of the Nusselt numbers. The considerations above are
 277 qualitatively valid for all values of the remaining parameters, M_z , β and R_c .

β	Po	Nu
0.10	166.74	8.60
0.25	164.65	7.28
0.50	162.63	6.09
1.00	161.78	5.52

Table 1: Values of Po and Nu for cross-sections with sharp corners, negligible joule heating, $\lambda_D D_h = 9.85$, $M_z = 10^{-3}$.

278

279 Both Poiseuille and Nusselt numbers increase with $\lambda_D D_h$, i.e., all other quantities
 280 being equal, with the hydraulic diameter. As an example, for $\beta = 0.25$, $M_z = 10^{-3}$,

281 $Po(D_h = 3 \mu m) \approx 165$, for $Po(D_h = 24 \mu m) \approx 956$; for heat transfer under the same
 282 conditions, $Nu(D_h = 3 \mu m) \approx 7.3$, $Nu(D_h = 24 \mu m) \approx 8.7$.
 283 Since the analysis of the transport coefficients as represented by the Poiseuille and
 284 Nusselt numbers yields the same qualitative trends for both hydraulic diameters, the
 285 considerations to follow will be made based on $D_h = 3 \mu m$, i.e. $\lambda_D D_h = 9.85$. Table
 286 1 shows the Poiseuille and Nusselt numbers for sharp corners, $M_z = 0.001$ and all the
 287 aspect ratios analysed in this study: it is clearly seen that both quantities decrease as
 β increases.

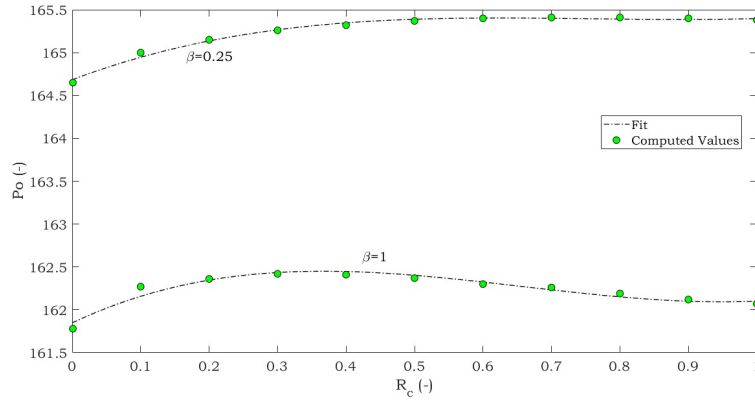


Figure 5: Poiseuille number versus non-dimensional radius of curvature for two aspect ratios, $\beta = 0.25$ and $\beta = 1$.

288

289 The trend of Po as a function of the radius of curvature is presented in Fig.5, where
 290 both the results of computations (black-edged green dots) and the fitting curve have
 291 been plotted. In this case, the value of Joule heating does not affect the Poiseuille
 292 number, as all properties are independent of temperature within the bonds of this
 293 study. Therefore, only the aspect ratio has been considered as a parameter and two
 294 different cases ($\beta = 0.25$ and $\beta = 1$) have been plotted. It is clear that the frictional
 295 losses are almost unaffected by the smoothing of the corners and over all the cases
 296 considered the maximum percentage increase experienced is 0.3%. As a consequence,
 297 the frictional losses can be said to remain unaffected by smoothing of the corners.

298

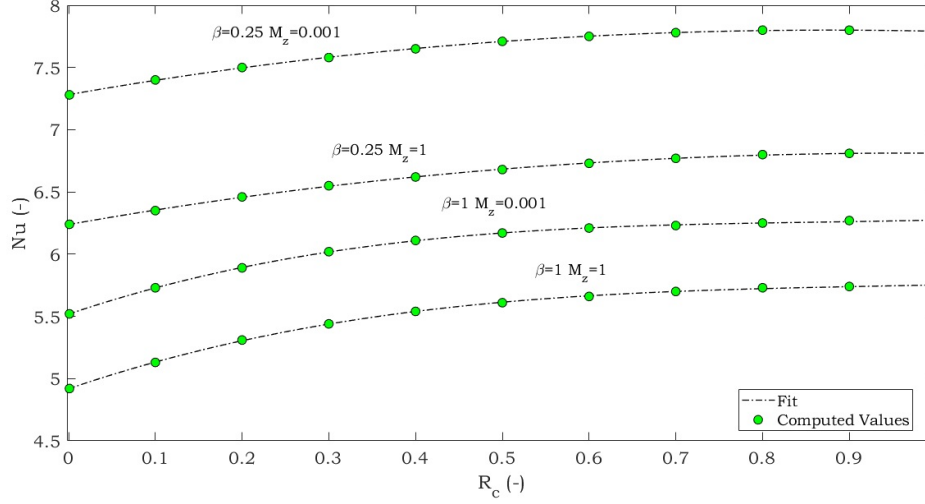


Figure 6: Nusselt number versus non-dimensional radius of curvature for two aspect ratios, $\beta = 0.25$ and $\beta = 1$ and $M_z = 0.001$ and $M_z = 1$.

299 For the thermal problem, the Nusselt number is also affected by the value of M_z :
 300 this is why Fig.6 shows four plots, for two aspect ratios ($\beta = 1$ and $\beta = 0.25$) and
 301 for $M_z = 0.001$ (negligible Joule heating compared to heat transfer) and $M_z = 1$.
 302 As was to be expected, the Nusselt number increases with the non-dimensional radius
 303 of curvature, the more significantly the smaller the aspect ratio: for $\beta = 0.25$ the
 304 maximum increase is 7%, whilst for $\beta = 1$ the increase is around 13%. This trend is
 305 also slightly increasing with M_z : for $M_z = 1$ the maximum increases in the Nusselt
 306 number are around 16.9% for $\beta = 1$ and 9.0% for $\beta = 0.25$. As was to be expected from
 307 its definition, Eq.(28), the value of M_z has a negative influence on the heat transfer.
 308 Indeed, increasing M_z while keeping E_z and D_h fixed means to decrease the heat flux
 309 per unit length, which affect the Nusselt number consequently. The decreasing trend
 310 of Nu versus M_z is reported in Fig.7: it can be seen that it is suitably approximated
 311 by a linear fit and that varying the aspect ratio only results in a change of the slope,
 312 which becomes steeper as β decreases. The trend is also unaffected by the value of R_c
 313 which only changes the constant and the slope coefficient, if slightly.
 314 This behaviour is readily explained considering the definition of M_z , Eq.(28): if the

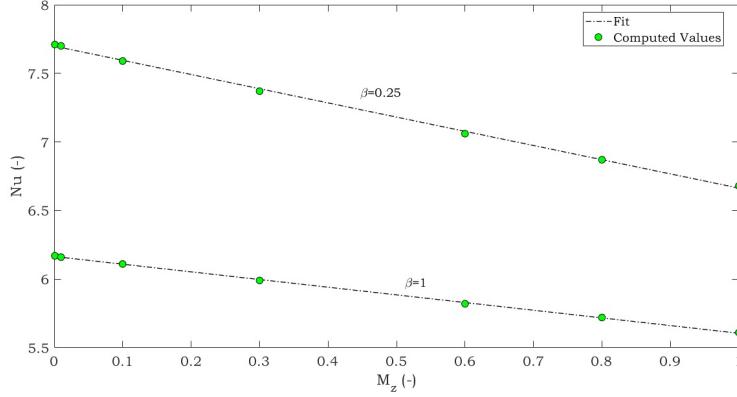


Figure 7: Nusselt number versus M_z for two aspect ratios, $\beta = 0.25$ and $\beta = 1$ and $R_c = 0.5$.

velocity field is to remain unaffected and therefore its influence on the temperature profile unchanged, M_z can only vary due to a change in the heat flux at the wall, that is directly related to the temperature gradient normal to the wall itself, to which, in its non dimensional form, the Nusselt number is proportional. As a consequence there is a linear dependence between Nu and M_z , the former decreasing as the latter increases.

3.1. Correlations for Po and Nu

In order to supply the designer with a handy tool for the determination of the Poiseuille and Nusselt numbers for the case investigated in this work, suitable correlations are suggested. In [38, 39, 46, 47], correlations were given as a function of the non-dimensional radius of curvature R_c with coefficients depending on the aspect ratio β in the absence of viscous dissipation; when the latter was present, the Nusselt number took a more involved, fractional form which could also account for entry effects through the Peclet and Graetz numbers. For the case of electro-osmotic flow, the results of the investigation allow to express the Poiseuille number in the form

$$Po(\beta, R_c) = \sum_{i=0}^3 d(\beta)_i \cdot R_c^i \quad (38)$$

with the values of d reported in Table 2.

β	d_3	d_2	d_1	d_0
0.10	1.0021	-2.2436	1.7481	166.7673
0.25	1.8031	-4.0977	3.0084	164.6838
0.50	3.1482	-6.7290	4.2784	162.7916
1.00	3.6206	-7.1418	3.7754	161.8478

Table 2: Coefficients for the Poiseuille number, Eq.(38).

For the Nusselt number, the corresponding polynomial form

$$Nu(\beta, R_c) = \sum_{i=0}^n d(\beta)_i \cdot R_c^i \quad (39)$$

would not account for the influence of M_z , which basically brings a linear decrease in Nu, as discussed above. It was therefore chosen to express Nu as a linear relationship

$$Nu(\beta, R_c, M_z) = Nu_0(\beta, R_c) - |C_{M_z}|M_z \quad (40)$$

with coefficients given in Table 3. It must be noted that Nu_0 is very close to the value obtained for a given geometry (in terms of radius of curvature and aspect ratio) but not exactly matching: this is due to Eq.(40) resulting from a best fit of the computed data; deviations are negligible in all cases.

4. Conclusions

The electro-osmotic flow in microchannels of rectangular cross-section with progressively smoothed corners has been studied for fully-developed flow and H1 thermal boundary conditions. The main findings of the work are summarised below:

- The potential, velocity and temperature field are influenced by the non-dimensional hydraulic diameter $\lambda_D D_h$: the larger its value, the flatter the profile and the lower the velocities in the microchannel;

	β							
	1		0.5		0.25		0.10	
R_c	Nu_0	C_{M_z}	Nu_0	C_{M_z}	Nu_0	C_{M_z}	Nu_0	C_{M_z}
0	5.514	0.601	6.090	0.710	7.269	1.052	8.557	2.017
0.1	5.724	0.601	6.242	0.702	7.388	1.059	8.627	2.017
0.2	5.890	0.586	6.382	0.702	7.488	1.051	8.681	2.015
0.3	6.015	0.581	6.492	0.702	7.571	1.042	8.731	2.015
0.4	6.105	0.572	6.578	0.692	7.641	1.038	8.769	2.009
0.5	6.165	0.560	6.642	0.681	7.699	1.035	8.804	2.007
0.6	6.204	0.551	6.688	0.668	7.741	1.026	8.828	1.993
0.7	6.230	0.537	6.715	0.660	7.771	1.021	8.845	1.989
0.8	6.251	0.530	6.734	0.654	7.788	1.012	8.860	1.984
0.9	6.264	0.530	6.734	0.646	7.789	0.998	8.864	1.977
1	6.271	0.526	6.720	0.638	7.781	0.987	8.865	1.966

Table 3: Coefficients for the Nusselt number, Eq.(40).

- 343 - Both the Nusselt and Poiseuille numbers increase as the aspect ratio β of the
344 channel decreases;
- 345 - The Poiseuille number is almost unaffected by the non-dimensional radius of cur-
346 vature of the corners, it does increase with R_c but negligibly so (below 0.4%).
347 This means that rounding the corners does not increase friction losses nor, sub-
348 sequently, pumping power.
- 349 - The Nusselt number is affected in a far more significant way by the value of R_c ,
350 with the effect waning as the aspect ratio β decreases; this is to be expected, since
351 for low β the section resembles more and more closely that of parallel plates, and
352 the side walls and corners lose in significance.

- 353 - The increase of Nu is monotonical, although the trend tends to flatten out for R_c
354 higher than 0.7.
- 355 - The maximum increase in the Nusselt number is about 13% for negligible Joule
356 heating, and increases with M_z , almost reaching 17% for $M_z = 1$.
- 357 - Polynomial correlations have been obtained for the Poiseuille and Nusselt number,
358 which cover the cases considered in this study.

359 References

- 360 [1] K. Schubert, J. Brandner, M. Fichtner, G. Linder, U. Schygulla, A. Wenka, Mi-
361 crostructure devices for applications in thermal and chemical process engineering,
362 Microscale Thermophysical Engineering 5 (1) (2001) 17–39.
- 363 [2] A. Rostami, A. Mujumdar, N. Saniei, Flow and heat transfer for gas flowing in
364 microchannels: A review, Heat and Mass Transfer/Waerme- und Stoffuebertragung
365 38 (4-5) (2002) 359–367.
- 366 [3] S. Venkatesan, J. Jerald, P. Asokan, R. Prabakaran, A Comprehensive Review
367 on Microfluidics Technology and its Applications, Lecture Notes in Mechanical
368 Engineering (2020) 235–245.
- 369 [4] N. Gilmore, V. Timchenko, C. Menictas, Microchannel cooling of concentrator
370 photovoltaics: A review, Renewable and Sustainable Energy Reviews 90 (2018)
371 1041–1059.
- 372 [5] M. Hossan, D. Dutta, N. Islam, P. Dutta, Review: Electric field driven pumping
373 in microfluidic device, Electrophoresis 39 (5-6) (2018) 702–731.
- 374 [6] V. Kuznetsov, Fundamental Issues Related to Flow Boiling and Two-Phase
375 Flow Patterns in Microchannels Experimental Challenges and Opportunities, Heat
376 Transfer Engineering 40 (9-10) (2019) 711–724.

- [7] G. L. Morini, M. Lorenzini, S. Colin, S. Geoffroy, Experimental investigation of the compressibility effects on the friction factor of gas flows in microtubes, in: Proceedings of the 4th International Conference on Nanochannels, Microchannels and Minichannels, ICNMM2006, vol. 2006 A, 411–418, 2006.
- [8] G. Morini, Y. Yang, M. Lorenzini, Experimental analysis of gas micro-convection through commercial microtubes, *Experimental Heat Transfer* 25 (3) (2012) 151–171.
- [9] Y. Yang, H. Chalabi, M. Lorenzini, G. Morini, The effect on the nusselt number of the nonlinear axial temperature distribution of gas flows through microtubes, *Heat Transfer Engineering* 35 (2) (2014) 159–170.
- [10] N. Kockmann, C. Holvey, D. Roberge, Transitional flow and related transport phenomena in complex microchannels, in: Proceedings of the 7th International Conference on Nanochannels, Microchannels, and Minichannels 2009, ICNMM2009, vol. PART B, 1301–1312, 2009.
- [11] M. Lorenzini, I. Daprá, G. Scarpi, Heat Transfer for a Giesekus Fluid in a Rotating Concentric Annulus, *Applied Thermal Engineering* 122 (2017) 118–125.
- [12] M. Ohadi, K. Choo, S. Dessiatoun, E. Cetegen, *Next Generation Microchannel Heat Exchangers*, Springer, NY, 2013.
- [13] Y. Han, Y. Liu, M. Li, J. Huang, A review of development of micro-channel heat exchanger applied in air-conditioning system, vol. 14, 148–153, 2012.
- [14] P. Kew, D. Reay, Compact/micro-heat exchangers - Their role in heat pumping equipment, *Applied Thermal Engineering* 31 (5) (2011) 594–601.
- [15] G. Morini, M. Spiga, The role of the viscous dissipation in heated microchannels, *Journal of Heat Transfer* 129 (3) (2007) 308–318.

- 401 [16] B. Kirby, Micro- and Nanoscale Fluid Mechanics, Cambridge University Press,
402 Cambridge, UK, 2010.
- 403 [17] H. Bruus, Theoretical microfluidics, OUP, Oxford, 2010.
- 404 [18] M. Al-Rjoub, A. Roy, S. Ganguli, R. Banerjee, Assessment of an active-cooling
405 micro-channel heat sink device, using electro-osmotic flow, International Journal
406 of Heat and Mass Transfer 54 (21-22) (2011) 4560–4569.
- 407 [19] C. Rice, R. Whitehead, Electrokinetic flow in a narrow cylindrical capillary, Journal
408 of Physical Chemistry 69 (11) (1965) 4017–4024.
- 409 [20] G. Mala, D. Li, C. Werner, H.-J. Jacobasch, Y. Ning, Flow characteristics of water
410 through a microchannel between two parallel plates with electrokinetic effects,
411 International Journal of Heat and Fluid Flow 18 (5) (1997) 489–496.
- 412 [21] C.-Y. Wang, C.-C. Chang, Electro-osmotic flow in polygonal ducts, ELEC-
413 TROPHORESIS 32 (11) (2011) 1268–1272.
- 414 [22] P. Vocale, M. Geri, L. Cattani, G. Morini, M. Spiga, Electro-osmotic heat transfer
415 in elliptical microchannels under H1 boundary condition, International Journal of
416 Thermal Sciences 72 (2013) 92–101.
- 417 [23] P. Vocale, M. Geri, G. Morini, M. Spiga, Electro-osmotic flows inside triangular
418 microchannels, Journal of Physics: Conference Series 501 (1).
- 419 [24] G. Morini, M. Lorenzini, S. Salvigni, M. Spiga, Thermal performance of silicon mi-
420 cro heat-sinks with electrokinetically-driven flows., International Journal of Ther-
421 mal Sciences 45 (10) (2006) 955.
- 422 [25] M. Geri, M. Lorenzini, G. Morini, Effects of the Channel Geometry and of the
423 Fluid Composition on the Performance of DC Electro-osmotic Pumps, Interna-
424 tional Journal of Thermal Sciences 55 (2012) 114–121.

- 425 [26] M. Al-Rjoub, A. Roy, S. Ganguli, R. Banerjee, Improved flow rate in electro-
426 osmotic micropumps for combinations of substrates and different liquids with and
427 without nanoparticles, *Journal of Electronic Packaging, Transactions of the ASME*
428 137 (2).
- 429 [27] G. L. Morini, M. Lorenzini, S. Salvigni, M. Spiga, Thermal performance of silicon
430 micro heat-sinks with electrokinetically- driven flows, in: *Proceedings of the 3rd*
431 *International Conference on Microchannels and Minichannels*, 2005, vol. PART B,
432 231–236, 2005.
- 433 [28] M. Al-Rjoub, A. Roy, S. Ganguli, R. Banerjee, Enhanced electro-osmotic flow
434 pump for micro-scale heat exchangers, *ASME 2012 3rd International Conference*
435 *on Micro/Nanoscale Heat and Mass Transfer, MNHMT 2012* (2012) 829–833.
- 436 [29] K. Pramod, A. Sen, Flow and heat transfer analysis of an electro-osmotic flow
437 micropump for chip cooling, *Journal of Electronic Packaging, Transactions of the*
438 *ASME* 136 (3) (2014) 03101201–03201214.
- 439 [30] M. Shamshiri, R. Khazaeli, M. Ashrafizaadeh, S. Mortazavi, Electroviscous and
440 thermal effects on non-Newtonian liquid flows through microchannels, *Journal of*
441 *Non-Newtonian Fluid Mechanics* 173-174 (2012) 1–12.
- 442 [31] G. Shit, A. Mondal, A. Sinha, P. Kundu, Electro-osmotic flow of power-law fluid
443 and heat transfer in a micro-channel with effects of Joule heating and thermal
444 radiation, *Physica A: Statistical Mechanics and its Applications* 462 (2016) 1040–
445 1057.
- 446 [32] M. Al-Rjoub, A. Roy, S. Ganguli, R. Banerjee, Enhanced heat transfer in a micro-
447 scale heat exchanger using nano-particle laden electro-osmotic flow, *International*
448 *Communications in Heat and Mass Transfer* 68 (2015) 228–235.
- 449 [33] D. Maynes, B. Webb, Fully developed electro-osmotic heat transfer in microchan-
450 nels, *International Journal of Heat and Mass Transfer* 46 (8) (2003) 1359–1369.

- [34] D. Maynes, B. Webb, Fully-developed thermal transport in combined pressure and electro-osmotically driven flow in microchannels, *Journal of Heat Transfer* 125 (5) (2003) 889–895.
- [35] A. Moghadam, Thermally Developing Flow Induced by Electro-Osmosis in a Circular Micro-Channel, *Arabian Journal for Science and Engineering* 39 (2) (2014) 1261–1270.
- [36] S. Ray, D. Misra, Laminar fully developed flow through square and equilateral triangular ducts with rounded corners subjected to H1 and H2 boundary conditions, *International Journal of Thermal Sciences* 49 (9) (2010) 1763–1775.
- [37] S. Chakraborty, S. Ray, Performance optimisation of laminar fully developed flow through square ducts with rounded corners, *International Journal of Thermal Sciences* 50 (12) (2011) 2522–2535.
- [38] M. Lorenzini, G. Morini, Single-phase, Laminar Forced Convection in Microchannels with Rounded Corners, *Heat Transfer Engineering* 32 (13-14) (2011) 1108–1116.
- [39] M. Lorenzini, The Influence of Viscous Dissipation on Thermal Performance of Microchannels with Rounded Corners, *Houille Blanche* (4) (2013) 64–71.
- [40] M. Lorenzini, N. Suzzi, The Influence of Geometry on the Thermal Performance of Microchannel Heat Sinks with Viscous Dissipation, *Heat Transfer Engineering* 37 (13-14) (2016) 1096–1104.
- [41] R. Webb, *Principles of Enhanced Heat Transfer*, Wiley, New York, 1984.
- [42] P. Tabeling, *Introduction to microfluidics*, Oxford University Press, Oxford, UK, 2010.
- [43] A. Bejan, *Entropy generation through heat and fluid flow*, Wiley, New York, 1982.

- 475 [44] M. Lorenzini, Electro-osmotic Flow in Rectangular Microchannels: Geometry Op-
476 timisation, *Journal of Physics: Conference Series* 923 (2017) 1–8.
- 477 [45] G. Shit, A. Mondal, A. Sinha, P. Kundu, Two-layer electro-osmotic flow and heat
478 transfer in a hydrophobic micro-channel with fluid-solid interfacial slip and zeta
479 potential difference, *Colloids and Surfaces A: Physicochemical and Engineering*
480 *Aspects* 506 (2016) 535–549.
- 481 [46] N. Suzzi, M. Lorenzini, Viscous heating of a laminar flow in the thermal entrance
482 region of a rectangular channel with rounded corners and uniform wall tempera-
483 ture, *International Journal of Thermal Sciences* 145 (2019) 10603201–10603210.
- 484 [47] M. Lorenzini, N. Suzzi, Thermal performance optimization of microchannels with
485 smoothed corners assuming laminar flow and non-negligible viscous heating, *AIP*
486 *Conference Proceedings* 2191 (2019) 02010001–02010011.