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# Three-dimensional discrete element modelling of rubble masonry structures from dense point clouds

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## Abstract

This paper presents a framework for the three-dimensional structural analysis of full scale, geometrically complex rubble masonry structures from point clouds generated from Structure-from-Motion photogrammetry or terrestrial laser scanning. According to the method, a point-based voxelization algorithm was adopted, whereby a dense point cloud was down-sampled into equidistant points, bypassing the need for conventional intensive processes, such as watertight mesh conversion, to obtain the geometric model of the rubble masonry for structural analysis. The geometry of the rubble masonry structure was represented by a sum of hexahedral rigid blocks (voxels). The proposed “*point cloud to structural analysis*” framework was implemented to assess the structural stability of the southwest leaning tower of Caerphilly Castle in Wales, UK. Simulations were performed with the three-dimensional computational software 3DEC, based on the Discrete Element Method (DEM) of analysis. Each voxel of the rubble masonry was represented as a rigid, distinct block while mortar joints were modelled as zero thickness interfaces which can open and close depending on the magnitude and direction of the stresses applied to them. The potential of the automated procedure herein proposed has been demonstrated to quantitatively assess the three-dimensional mechanical behaviour rubble masonry structures and provide valuable information to asset owners in relation to the structural health condition of assets in their care.

**Keywords:** Point cloud, rubble masonry, Discrete Element Method (DEM), terrestrial laser scanning, structure-from-motion photogrammetry

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## Nomenclature

$x_{min}$	Minimum x-axis spatial coordinates	$EVC$	Empty voxelized point cloud
$x_{max}$	Maximum x-axis spatial coordinates	$VAC$	Volume adjustment coefficient
$y_{min}$	Minimum y-axis spatial coordinates	$C$	Joint cohesive strength
$y_{max}$	Maximum y-axis spatial coordinates	$T$	Joint tensile strength
$z_{min}$	Minimum z-axis spatial coordinates	$\varphi$	Joint friction angle
$z_{max}$	Maximum z-axis spatial coordinates	$K_n$	Joint normal stiffness
$Grid$	Voxel size	$K_s$	Joint shear stiffness

$N_x$	Number of voxels, x-axis	$T_{max}$	Maximum tensile force
$N_y$	Number of voxels, y-axis	$F_{max}^s$	Maximum shear force
$N_z$	Number of voxels, z-axis	$A_c$	Sub-contact area
$\Delta_x$	Voxel dimension, x-axis	$\theta_t$	Theoretical inclination angle
$\Delta_y$	Voxel dimension, y-axis	$g_h$	Horizontal gravitational acceleration
$\Delta_z$	Voxel dimension, z-axis	$g$	Vertical gravitational acceleration
$P$	Dense point cloud	$\lambda_h$	Horizontal inclination angle multiplier
$P_x$	Dense point cloud, x-axis	$\lambda_v$	Vertical inclination angle multiplier
$P_y$	Dense point cloud, y-axis	$g_{hx}$	Gravitational acceleration of, x-axis
$P_z$	Dense point cloud, z-axis	$g_{hy}$	Gravitational acceleration of, y-axis
$Dim_x$	Rounded point cloud x-axis	$g_{vz}$	Gravitational acceleration of, z-axis
$Dim_y$	Rounded point cloud y-axis	$\psi$	Azimuth of inclination
$Dim_z$	Rounded point cloud z-axis	$\lambda_{h,max}$	Critical inclination angle multiplier
$Dim$	Rounded point cloud	$U_{h,max}$	Critical horizontal displacement
$DVC$	Dimensionless voxelized point cloud		

## 1 Introduction

Despite advancements in computational mechanics and the substantial number of numerical techniques available, computational modelling of rubble historical masonry structures remains a complex task. This could be due to the anisotropic mechanical nature, lack of material characterisation and/or the complexity of geometry, which characterise many of these structures. Of the modern structural analysis tools available [1], the Discrete Element Method (DEM) has been demonstrated as highly effective in capturing the discrete nature of masonry structures subjected to quasi-static and dynamic loads [1-8]. For example, in [9], DEM models of rubble stone masonry walls with different section morphology have been developed to evaluate their out-of-plane structural capacity. Real masonry sections of walls surveyed from historical buildings were used. The geometries of the walls were generated using image-based computer-aided-design (CAD) files which were later imported in the discrete element model for their structural analysis. From the results, it was shown that the morphology of the stones in the wall significantly influences the stiffness and load-carrying capacity of the walls. However, the influence of the wall cross-section becomes more pronounced when assessing the displacement (the deformation capacity). Also, two dimensional (2D) numerical models based on the DEM were developed to study the structural stability of a historical rubble fortification [10]. The geometrical shapes used to represent the rubble masonry comprised of circular and polygonal elements randomly assembled. From this study, the suitability of the DEM for simulating the brittle behaviour of masonry and the need to accurately represent the geometric variability of the wall was also highlighted. In a further study, 2D models based on the DEM were used to investigate the out-of-plane capacity of rubble masonry walls with different cross-sections [11]. A drop in ultimate load-bearing capacity of rubble masonry walls with internal cavities and irregularly shaped blocks, as compared to panels of regular-shaped blocks, was found. From the above studies, it is evident that DEM is an effective modelling approach, able to account for the discrete and heterogeneous nature of masonry, yet detailed metrical geometric information is essential in order to perform accurate numerical analysis.

1 However, acquiring geometric features of rubble masonry using traditional manual surveying tech-  
2 niques and inputting them in a computational model for their structural analysis is a tedious process.  
3 With the development of remote sensing technology, such as terrestrial laser scanning (TLS) and Struc-  
4 ture-from-Motion (SfM) photogrammetry, the metrical characteristics of irregular-shaped rubble ma-  
5 sonry structures can be rapidly and reliably obtained [12]. Over the last decade, there has been some  
6 effort to capture geometrical characteristics from masonry structures and input them into numerical  
7 models for their structural analyses. For example, Riveiro et al. [13] used point clouds derived from  
8 TLS to develop a geometric model for the structural analysis of a stone masonry arch bridge (MAB)  
9 using thrust line analysis. The procedure involved manually converting raster images of the bridge  
10 into a geometric model. The same year, Lubowiecka et al. [14] generated a point cloud from a photo-  
11 grammetric survey for the 3D documentation of a multi-span MAB. Subsequently, the geometric  
12 model developed was implemented into a 3D Finite Element Model (FEM) to assess the structural  
13 capacity of the arch. A methodology of automated point cloud segmentation of the arches, spandrels,  
14 and pavement was presented [15]. In [16], the same methodology was utilised to assess the structural  
15 capacity of a multi-span damaged historical MAB using geometric data from TLS and a structural anal-  
16 ysis model based on the FEM.

17 Moreover, a method that automated the modelling of multi-story masonry building façades was developed  
18 by [17]. The procedure included the down-sampling of the original point cloud by means of voxelization  
19 and subsequent geometric model development. The investigation's main novelty lay in boundary feature  
20 detection. Thus, window and opening boundaries were detected by means of geometric criteria and imple-  
21 mented in a known-nearest-neighbours (KNN) algorithm. Geometrical validation of the proposed ap-  
22 proached followed by comparing automatically developed models with manually developed CAD-based  
23 geometric models. The generated geometric models of the façades were analysed using commercial FEM  
24 software. The approach was later extended by using the so-called FacadeVoxel algorithm [18]. Also, a  
25 semi-automated numerical modelling methodology, named Cloud2FEM, was developed in [19] by ap-  
26 plying a semi-automatic slicing algorithm to the point cloud. In this way, a geometric model was pro-  
27 duced by joining each sliced segment and directly converting it into a finite element model avoiding  
28 any segmentation. This procedure has also been employed for the structural analysis of a historic ma-  
29 sonry castle [20]. Moreover, within a general methodology employing automated 3D reconstruction for  
30 structural analysis, including the Cloud2FEM procedure also, the stability analysis of a leaning irregular  
31 tower has been performed in [21]. This was by means of automatically converting a point cloud gener-  
32 ated from TLS into a triangulated mesh and then developing both FEM and Limit States models. The  
33 mechanical results between both applied methods showed good agreement. A non-uniform rational basis  
34 spline (NURBS) approach was followed and reported higher accuracy and flexibility compared to respec-  
35 tive CAD geometric modelling. The geometric models were directly usable for FE analysis. What is more,  
36 structural analysis using FEM was performed by [22] to investigate the mechanical behaviour of a tower  
37 using models derived from point cloud data. Geometric model development was achieved by converting  
38 point clouds to meshes and using third-party software to convert the meshes into geometric models for  
39 subsequent FEM analysis. A "*Slicing Method*" was proposed in [23] which was used to convert the geom-  
40 etry of the façade into a structural analysis software based on FEM. A procedure for updating existing  
41 (CAD) geometric models with meshes from point clouds by means of computer vision algorithms was  
42 developed in [24]. Though this procedure was effective in adding details, it was not able to automati-  
43 cally develop geometric models; thus, not eliminating the need for initial CAD-based geometric model  
44 development. A methodology of automated FEM analysis from point clouds was developed in [25].  
45 Point clouds were converted initially into triangular meshes and then into quad-meshes which were  
46 more manageable computationally and suitable for conversion into NURBS. The resulting geometric  
47 models were geometrically evaluated and found to be within 1 mm from the initial mesh.

1 From the above studies, whilst all these reported approaches provide an excellent platform for the struc-  
2 tural analysis with the FEM using point cloud data, there are limited studies focusing on the automation  
3 of structural analysis of rubble masonry using DEM directly from point clouds. The aim of this paper  
4 is to present the development of an automated framework for the 3D discrete element modelling of  
5 rubble masonry directly from point clouds. The proposed framework implemented to assess the struc-  
6 tural stability of the southwest leaning tower of Caerphilly Castle located in Wales, UK. The stability  
7 of the tower was structurally assessed using the discrete element method (DEM). Within DEM, the  
8 structure can be divided into an assemblage of discrete bodies that can move independently from each  
9 other. Also, within DEM, large rotations and displacements of blocks can be allowed and new contacts  
10 and loss of existing contacts between the elements are automatically recognised and updated as the  
11 calculation progresses. This paper is organized as follows: Section 2 describes the case study under  
12 consideration; Section 3 details the proposed “point cloud to structural analysis” framework; Section 4  
13 presents the implementation of the proposed framework on the case study and the results obtained; and  
14 Section 5 outlines the conclusions and recommendations for future work.

## 15 **2 The southwest leaning tower of Caerphilly Castle**

16 The case study used to evaluate the proposed framework is the leaning tower of the Caerphilly tower  
17 located in South Wales, UK. **Errore. L'origine riferimento non è stata trovata.** Constructed in the  
18 13<sup>th</sup> Century [26], Caerphilly is the second largest castle in the UK and one of the largest in Europe  
19 [27]. The southwest tower shown in Figure 1 is 17 m tall and 9 m in diameter [27]. It is reported to have  
20 been leaning for several centuries and stands at a current angle of approximately 10 degrees off vertical.  
21 The tower was constructed of rubble masonry, with a rough texture and indefinable joints. The most  
22 probable cause of leaning of the Caerphilly tower is attributed to the lack of foundation strength and  
23 stiffness which was induced by dewatering in the 18th Century.



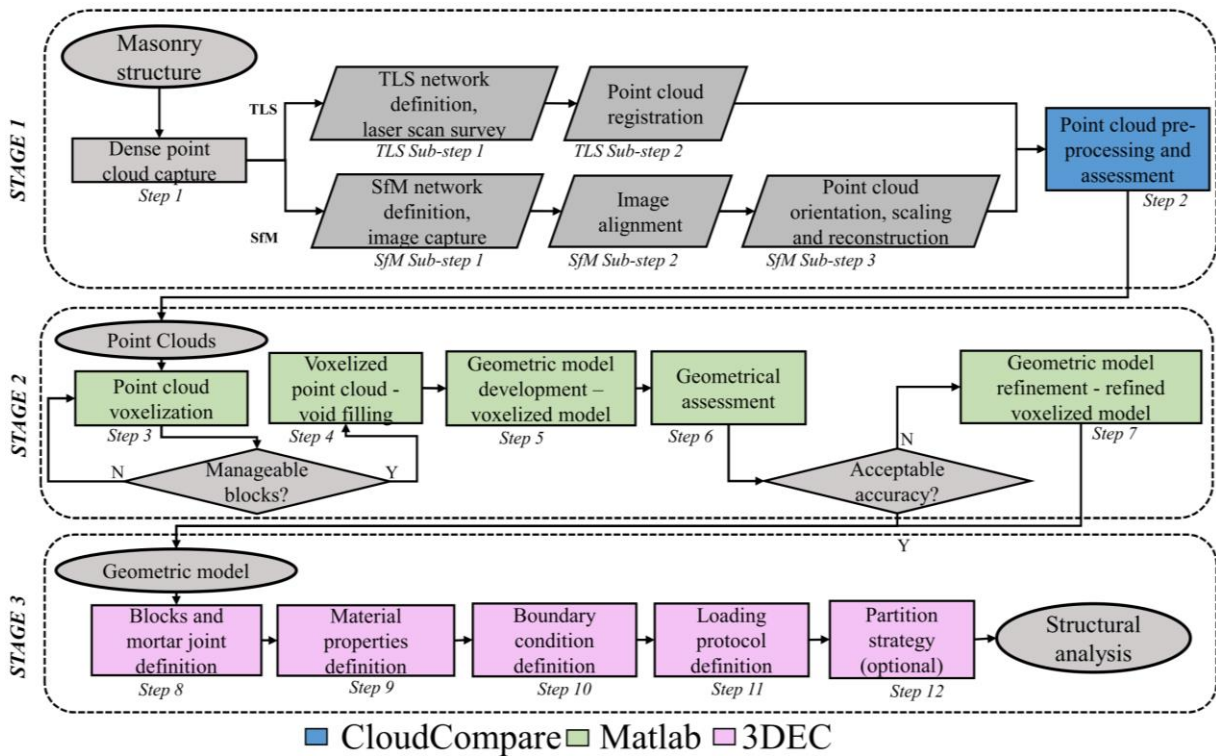
24  
25 *Figure 1. Caerphilly Castle [27]. View of the face of the southeast leaning tower.*

1 Over the last two decades, the numerical modelling of leaning towers has been investigated by many  
 2 researchers [28-31]. There are also analytical approaches to investigate the safety of leaning towers.  
 3 Notably, in [32], insight was provided into the critical inclination angle of leaning towers which were  
 4 relied upon oversimplified material assumptions (rigid masonry without tensile strength and regular  
 5 geometries). Such approaches cannot be applied to the present case study since the tower is highly  
 6 irregular in shape, with openings, voids and a non-rectangular base. Recently, FEM and Limit Analysis  
 7 have been successfully applied to perform structural analysis on leaning towers with complex geometry  
 8 [20,21]. From such studies, it was shown that the FEM cannot always accurately describe the discon-  
 9 tinuous nature of masonry [33], while Limit Analysis can, but still relies upon simplified assumptions  
 10 and is not able to provide information about the in-service condition of the structure under considera-  
 11 tion.

### 12 3 The proposed “point cloud to structural analysis” framework

13 In this section, the three-stage framework of the proposed automatic procedure for converting point  
 14 clouds into 3D numerical models based on the DEM is described. The three-stage framework involved:  
 15 Stage 1 - 3D documentation; Stage 2 - geometric model development; and Stage 3 -structural analysis.  
 16 A flowchart of the proposed framework is shown in Figure 2 and a detailed description of the steps at  
 17 each stage is provided below.

18



19

20 Figure 2. The “point cloud to structural analysis” framework.

#### 21 3.1 3D Documentation – Stage 1

##### 22 3.1.1 Step 1: Dense point cloud capture

23 The first step consists of capturing and processing the dense 3D point clouds from either an SfM pho-  
 24 togrammetry or TLS campaign. The main processes of capturing photogrammetric point clouds are: (a)  
 25 SfM photogrammetry network definition and image-capture; (b) image alignment and coarse recon-  
 26 struction; and (c) orientation, scaling as well as dense reconstruction. The main processes involved in  
 27 capturing TLS point clouds are: (a) TLS network definition and laser scan survey; and (b) scan registra-  
 28 tion. The term “acceptable density” refers to a density control after dense point cloud capture



1 whereby the surface point spacing must be smaller than the required voxel grid size. If this is not the  
2 case, then voids will appear after voxelization. The completeness of the surface survey refers to achiev-  
3 ing an accurate description of each portion of the structure. Multiple scan positions are suggested to  
4 ensure a full structure surface estimation. Figure 3 **Errore. L'origine riferimento non è stata trovata.**  
5 shows the dense point cloud obtained from a survey to document the structural health condition of the  
6 tower in 2014 [26]. In this instance, a FARO Focus 3D  $\times 130$  terrestrial laser scanner was used to acquire  
7 27 scans of the entire castle. The main challenges related to the 3D documentation of the castle were  
8 the foreign objects (e.g. scaffolding, non-structural artefacts, and statues); and pedestrians since the site  
9 is a significant tourist attraction. Twelve spherical targets were used to complete the registration process  
10 for the entire survey.

### 11 **3.1.2 Step 2: Point cloud pre-processing and assessment**

12 A sampling procedure was first carried out to homogenise the spatial description of the point cloud. The  
13 density of the point cloud can be selected by the user, taking into consideration the architectural details  
14 of the structure. The choice of the point cloud density is important as it relates to the next step, namely  
15 the discretization by voxels. Typically, up-sampling of the point cloud is required for cases where the  
16 point cloud is poorly defined (e.g. roofs, openings, etc.). Cleaning and cropping of the point cloud were  
17 also carried out to remove noise and foreign objects/irrelevant points (e.g. non-structural elements such  
18 as vegetation, etc.) This is an important task since only the points relating to the structure being assessed  
19 should be considered in structural analysis. The open-source software CloudCompare [34] was used to  
20 clean and crop the point cloud. While cleaning and cropping, relative care was taken to retain the struc-  
21 ture subject to structural analysis inside the point cloud, solely.

## 22 **3.2 Geometric model development – Stage 2**

### 23 **3.2.1 Step 3: Point cloud voxelization**

24 The developed voxelization algorithm herein presented was a point-based type, similar to the one pro-  
25 posed in [21] to [35]. It involved the down-sampling of the point cloud into a sum of equidistant points  
26 that had a common global axis orientation, as demonstrated in Figure 3b. The first process of point  
27 cloud voxelization consisted of the selection of the voxel size in meters (m), *Grid*. This voxel size  
28 defined the actual voxel dimension and appropriate choice for the correct accuracy and manageability  
29 of the structural analysis. The next process of voxelization consisted of finding the bounding box of the  
30 dense point cloud. The bounding box was composed of the minimum and maximum spatial coordinates  
31  $(x_{max}, x_{min})$ ,  $(y_{max}, y_{min})$  and  $(z_{max}, z_{min})$  of the dense point cloud, in meters. Then, the bounding  
32 box was subdivided into a grid with the user-defined voxel size equal to *Grid* for the x, y, and z-axis  
33 respectively. The number of voxels for each axis ( $N_x$ ,  $N_y$ ,  $N_z$ ) were defined by the following equations  
34 (1), (2) and (3). Ceil is the ceiling function used and *Grid*, the voxel size in meters:

$$35 \quad N_x = \text{ceil}\left(\frac{(x_{max}-x_{min})}{Grid}\right) \quad (1)$$

$$36 \quad N_y = \text{ceil}\left(\frac{(y_{max}-y_{min})}{Grid}\right) \quad (2)$$

$$37 \quad N_z = \text{ceil}\left(\frac{(z_{max}-z_{min})}{Grid}\right) \quad (3)$$

38 The number of voxels is an integer due to the rounding of the ceiling function in equations (1), (2) and  
39 (3). This means that the voxel size and actual voxel dimensions cannot coincide. Thus, the actual voxel  
40 dimensions for the x, y and z axes were  $\Delta_x$ ,  $\Delta_y$  and  $\Delta_z$  respectively and defined by the equations (4),  
41 (5) and (6):

$$42 \quad \Delta_x = \frac{(x_{max}-x_{min})}{N_x} \quad (4)$$

$$1 \quad \Delta_y = \frac{(y_{max}-y_{min})}{N_y} \quad (5)$$

$$2 \quad \Delta_z = \frac{(z_{max}-z_{min})}{N_z} \quad (6)$$

3 According to [36], the dense point cloud ( $P$ ) was defined as an unordered collection of  $n$  points  $\{P_i\}_{i=1}^n$   
4 in 3D Euclidean space, resulting from the scanning of an object and representing the surface of that  
5 object. The columns representing the  $x$ ,  $y$  and  $z$  axes of this dense point cloud  $P_x$ ,  $P_y$  and  $P_z$  were di-  
6 vided by their corresponding actual voxel dimension and rounded using the round function shown in  
7 (7), (8) and (9).  $Dim_x$ ,  $Dim_y$ , and  $Dim_z$  were then horizontally concatenated and composed the  
8 rounded point cloud  $Dim$ , as in (10). This was effectively a dimensionless array that indicated which  
9 voxel grid each point belonged to with an integer index for axes  $x$ ,  $y$  and  $z$ . As there were multiple  
10 points for each voxel, the recurring points were removed. This was done by finding the unique rows of  
11 the rounded point cloud  $Dim$  using the unique function. The result is the dimensionless voxelized point  
12 cloud  $DVC$ , with only one occasion of each voxel as in (11). The empty voxelized point cloud  $EVC$ , is  
13 the dimensionless voxelized point cloud multiplied by the respective voxel dimensions  $\Delta_x$ ,  $\Delta_y$ ,  $\Delta_z$ , as  
14 in (12).

$$15 \quad Dim_x = \text{round}\left(\frac{(P_x)}{\Delta_x}\right) \quad (7)$$

$$16 \quad Dim_y = \text{round}\left(\frac{(P_y)}{\Delta_y}\right) \quad (8)$$

$$17 \quad Dim_z = \text{round}\left(\frac{(P_z)}{\Delta_z}\right) \quad (9)$$

18 The rounded point cloud was composed of the results of (7), (8) and (9) horizontally concatenated:

$$19 \quad Dim = [Dim_x, Dim_y, Dim_z] \quad (10)$$

20 The dimensionless voxelized point cloud was found from the following equation:

$$21 \quad DVC = \text{unique} [Dim_x, Dim_y, Dim_z] \quad (11)$$

22 The empty voxelized point cloud ( $EVC$ ) was equal to the dimensionless voxelized point cloud ( $DVC$ )  
23 multiplied by the corresponding actual voxel dimension as in the following equation:

$$24 \quad EVC = [Dim_x \times \Delta_x, Dim_y \times \Delta_y, Dim_z \times \Delta_z] \quad (12)$$

25 The empty voxelized point cloud,  $EVC$  was the final product of voxelization. Essentially, this is a point  
26 cloud which describes the dense point cloud as a sum of the active voxels. With the term active voxel,  
27 one means that it is occupied by at least one point of the dense point cloud. Figure 3**Errore. L'origine**  
28 **referimento non è stata trovata.**c shows the empty voxelized point cloud.

29 As previously stated, if the voxel size (or *Grid*) were smaller than the available point cloud surface  
30 density, then there would be voids where the gridline was not occupied by active points. The mean  
31 surface densities of the dense point cloud were measured to find the smallest permissible voxel size  
32 with the given dense point cloud. This was determined by measuring the population of the point clouds  
33 within an area of  $1 \text{ m}^2$ . This was done in CloudCompare with the density measure function using a  
34 circular radius of 0.564, corresponding to an area of calculation of  $1 \text{ m}^2$ . The mean surface densities of  
35 the empty voxelized point cloud and the dense point cloud were 25 and 855 points/ $\text{m}^2$  (as shown in  
36 Figure 3**Errore. L'origine riferimento non è stata trovata.**a,c). Based upon experimentation with the  
37 above dense point cloud, the smallest voxel size permissible for the above dense point would be equal  
38 to 1 cm.

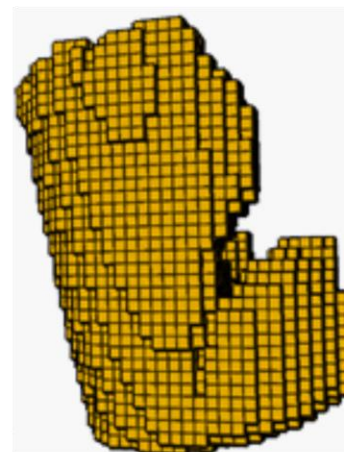
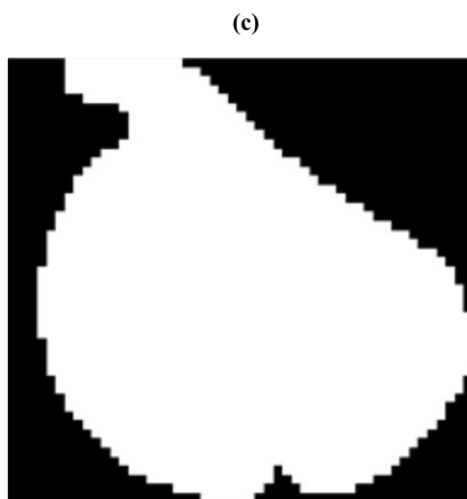
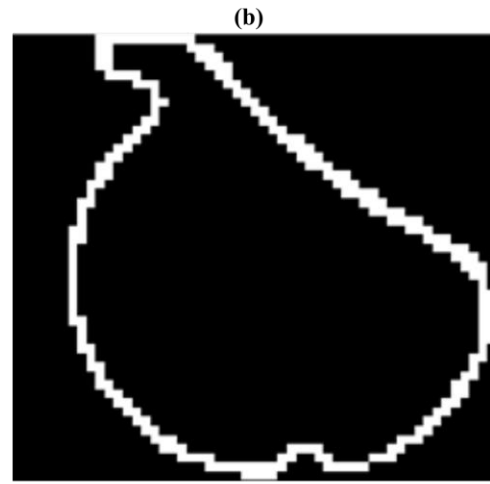
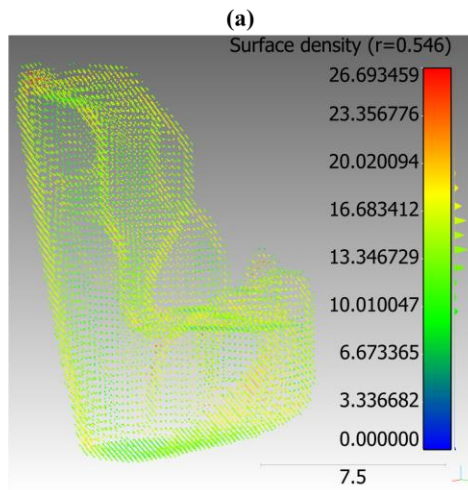
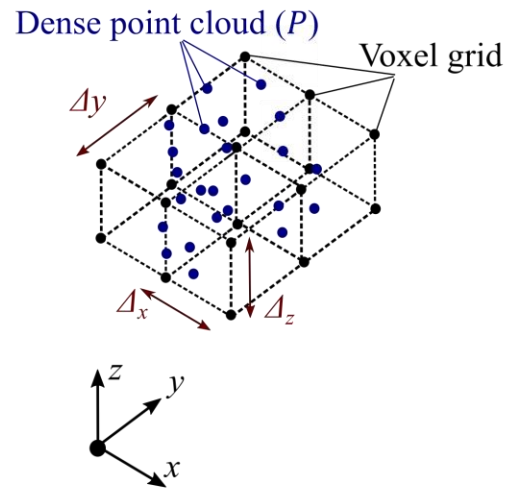
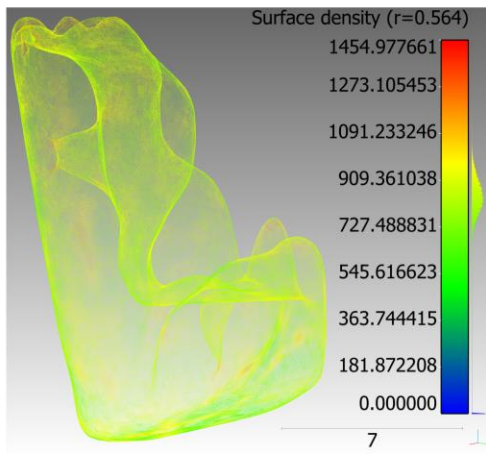


1 **3.2.2 Step 4: Voxelized point cloud - void filling**

2 This was a key step of the procedure since the geometrical domain occupied by the 3D structure was  
3 hollow and thus it was carried out by means of a multiple filling of the voxelized point cloud surfaces.  
4 Similar to [19], the voxelized point cloud was treated as a stack of raster images with common pixel  
5 size and dimension and characterized by a specific height,  $z$ . **Errore. L'origine riferimento non è stata**  
6 **trovata.**Figure 3**Errore. L'origine riferimento non è stata trovata.**d-e shows the raster images of the  
7 empty and filled voxelized point clouds for a horizontal section of the tower. This section was at an  
8 arbitrary height equal to 3.75 m. In specific, the active voxels are shown in white colour. To fill the  
9 whole domain, the raster image corresponding to each of the voxelized point cloud heights was mor-  
10 phologically opened and closed. The perimeter of the empty voxel cloud needed to be continuous, so  
11 that its' contained area may later be filled. If the perimeter of the empty voxel were not continuous, it  
12 could have been modified and closed. All the inactive pixels which were found to be contained inside  
13 the perimeter of the tower were converted into active pixels, thereby resulting in the filled voxelized  
14 point cloud. These functions can be applied to any type of point cloud. All these procedures have been  
15 fully automated and incorporated into the voxelization algorithm.

16 **3.2.3 Step 5: Geometric model development - voxelized model**

17 In step five, the geometric model development was carried out. For the case of the tower investigated  
18 in this study, DEM numerical modelling undertaken using the software 3DEC [37]. Each block of the  
19 geometric model was defined as an 8-noded polyhedron. The polyhedron development was based upon  
20 assigning the polyhedron nodes in clockwise order, for two parallel faces of each voxel of the cloud.  
21 This was an automatic procedure within the voxelization algorithm developed in this study. Figure 3f  
22 shows the 50 cm model which consists of 9,407 blocks.



1  
2  
3  
4

Figure 3. Voxelization: (a) dense point cloud; (b) voxelization process; (c) empty voxelized point cloud. Void filling: (d) empty; and (e) filled raster image for  $z$  equal to 3.75 m of the voxelized point cloud. Geometric model development (f)

### 3.2.4 Step 6: Geometrical assessment

The geometrical accuracy of the geometric models was assessed by comparing their properties with that of a watertight mesh. This was a watertight mesh constructed using the Poisson Surface Reconstruction algorithm of the dense point cloud prior to voxelization (Figure 4a). For the watertight mesh generation, the octree depth was 12 (the term octrees refers to the partitioning of the 3D space by recursively subdividing it into eight octants-octrees which are a 3D analogy of quadtrees), samples per node were 1.5, the full depth was 5, the point weight was 4.0 and the boundary was free. The mesh was made with a plugin of CloudCompare [34] based on the well-known Poisson Reconstruction algorithm [38]. Initially, a volumetric comparison was carried out to assess the error in the volume of the geometric models. In addition, the absolute cloud-to-mesh (C2M) distances were calculated between the empty voxelized point clouds and the reference mesh (Figure 4b) to assess the surface errors of the geometric model. This was again carried out in CloudCompare. During the voxelization, there was no displacement or rotation of the point cloud involved. Thus, for comparison between the voxelized and dense point clouds, alignment was not required.

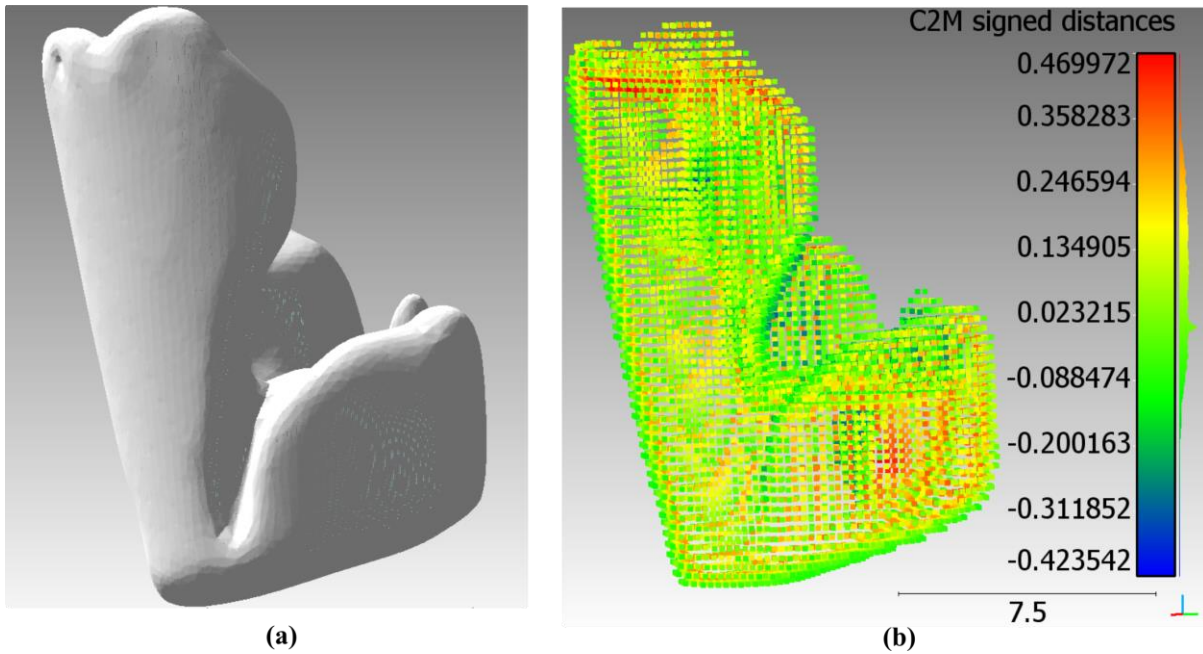


Figure 4. Geometric assessment: (a) watertight mesh; and (b) cloud-to-mesh distance of the 50 cm voxelized model with the watertight mesh.

### 3.2.5 Step 7: Geometric model refinement - refined voxelized model

A procedure of geometric model refinement was developed to increase the volumetric accuracy of the voxelized models. This was done by using the same dimensionless point cloud of the voxelized model and an altered voxel dimension prior to geometric model development. The coefficient for adjusting the voxel dimension, termed the volume adjustment coefficient (*VAC*), was obtained from equation (13) below. The refined voxelized model volumes were obtained from (14).

$$VAC = \sqrt[3]{\frac{\text{Watertight mesh volume}}{\text{Voxelized model volume}}} \quad (13)$$

$$\text{Refined voxelized model volume} = (\text{Voxelized model volume}) \times VAC^3 \quad (14)$$

## 3.3 Structural analysis with the discrete element method – Stage 3

### 3.3.1 Step 8: Block and mortar joint definition

DEM is an approach that has been widely used to simulate the static and dynamic behaviour of blocky structures. Within DEM, masonry units (i.e. blocks) are represented as rigid or deformable blocks,

1 which may form any arbitrary geometry. Interactions between blocks are governed by appropriate  
 2 stress-displacement constitutive laws at point contacts at the edges of the blocks [37]. The motion of  
 3 the blocks is simulated throughout a series of small but finite time-steps, numerically integrating the  
 4 Newtonian equations of motion. Contacts in blocks can be face-to-face, vertex-to-face or edge-to-edge  
 5 type. The seldom case of edge-to-edge is shown in Figure 5a. Finite displacements of the discrete bodies  
 6 and rotations are allowed which includes the complete detachment of blocks and new contact generation  
 7 as the calculation proceeds. Forces are considered as linear functions of the actual penetration in the  
 8 shear and normal directions [1].

9  
 10 Figure 5b-c show the adopted Mohr-Coulomb joint constitutive model implemented in 3DEC. The in-  
 11 elastic material properties used within these models were the joint cohesive strength ( $C$ ), the joint tensile  
 12 strength ( $T$ ), and the joint friction angle ( $\varphi$ ). According to the adopted joint constitutive model, the  
 13 structure's behaviour is governed by the joint normal and shear stiffnesses,  $K_n$  and  $K_s$  in the normal and  
 14 shear elastic range accordingly.

15 In the model, the tensile normal force is limited to  $T_{max}$  and the shear force is limited to  $F_{max}^s$ ; see  
 16 equations (15) and (16); where  $T$  is joint tensile strength,  $A_c$  is the sub-contact area,  $C$  is joint cohesive  
 17 strength and  $\varphi$  is the joint friction angle.

18  
 19 
$$T_{max} = -T \times A_c \tag{15}$$

20 
$$F_{max}^s = c \times A_c + F^n \times \tan \varphi \tag{16}$$

21  
 22

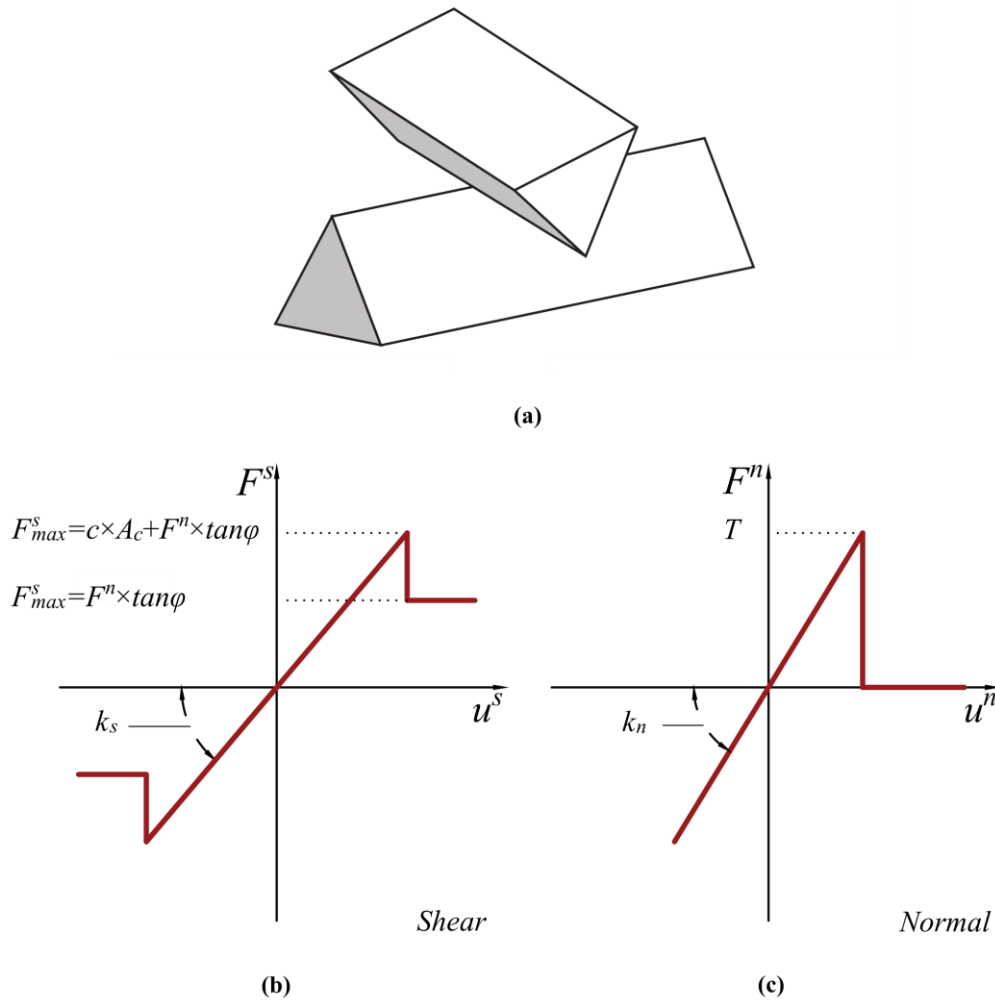


Figure 5. Contact between two blocks: the seldom edge-to-edge type contact (a) [37]. Force-displacement relationship of the adopted joint constitutive model for: (b) shear; and (c) normal direction.

### 3.3.2 Step 9: Selection of material properties for the DEM model

In general, when modelling periodic in texture masonry structures with DEM, the masonry units are represented as an assemblage of distinct blocks separated by zero thickness interfaces at each mortar joints. However, in this study, since we are dealing with rubble masonry, the tower was discretised into equal in shape blocks/voxels. In a rigorous way, a homogenization procedure such as the one proposed in [39] should be used to set up the material properties which characterise the mechanical interaction between blocks. However, since the present study mainly focuses on the “point cloud to structural analysis” framework, for simplicity, material properties were obtained from the literature. The material properties used in this study satisfied at least one of the following three criteria: (a) a proximity to the actual physical properties of historic masonry structures; (b) a proximity to values previously used in numerical modelling with the same numerical method (DEM); and (c) an overall structural capacity estimation in general agreement with previous research on historic masonry structures, e.g. [21]. Blocks were modelled as rigid elements having a density equal to 1,900 kg/m<sup>3</sup>, obtained from a previous investigation on the specific tower [21]. The joint normal ( $K_n$ ) and shear ( $K_s$ ) stiffnesses were taken equal to 20 and 15 GPa/m, respectively. The joint friction angle ( $\varphi$ ) between blocks was taken as 25 degrees, while the joint tensile ( $T$ ) and cohesive ( $C$ ) strength at the joints were both taken equal to 0.25 MPa; to represent old and deteriorated low bond strength masonry.

1 *Table 1. Mechanical properties of the zero-thickness interface in the numerical models.*

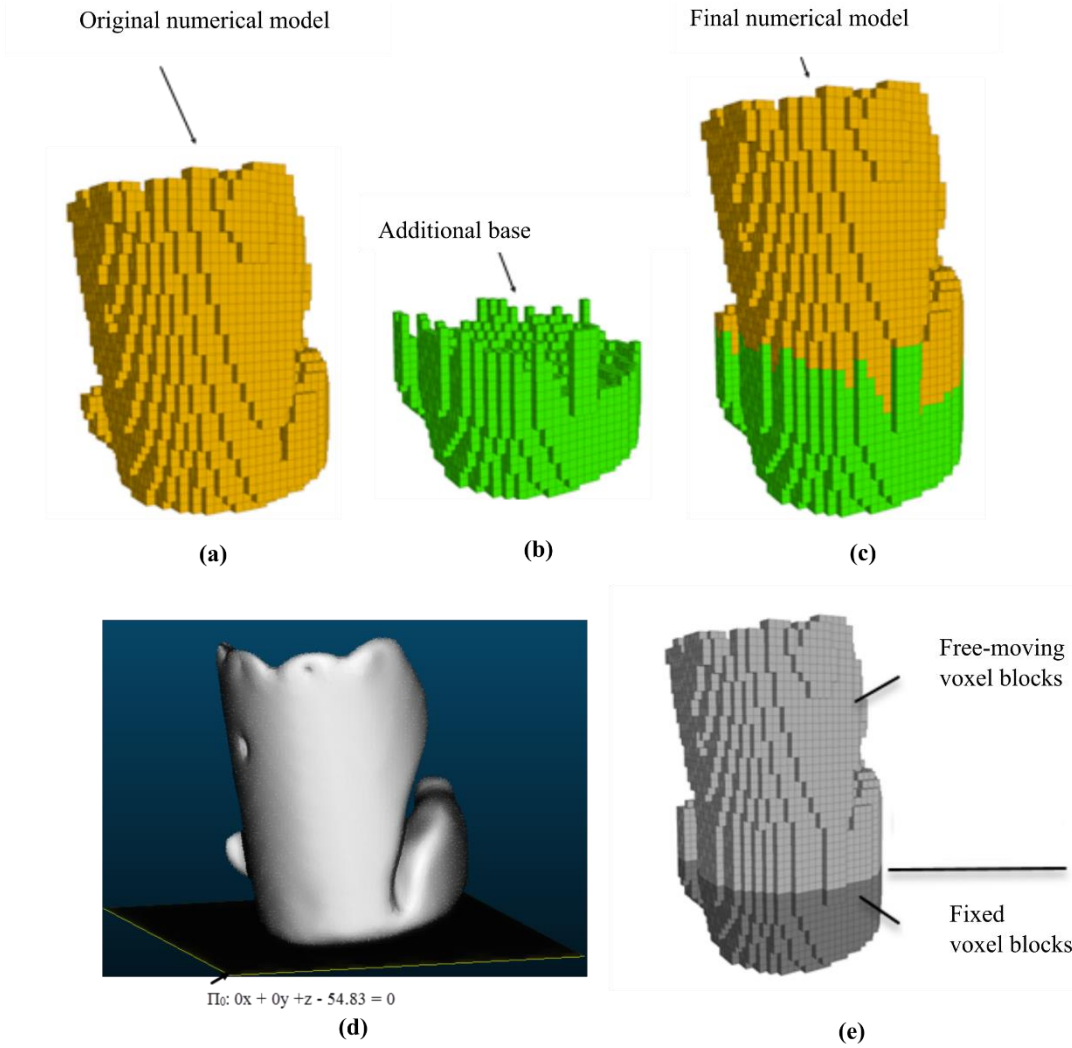
Parameter	Symbol	Unit	Model Values
Joint Normal Stiffness	$K_n$	GPa/m	20
Joint Shear Stiffness	$K_s$	GPa/m	15
Joint Cohesive Strength	$C$	MPa	0.25
Joint Tensile Strength	$T$	MPa	0.25
Joint Friction	$\varphi$	°	25

2

### 3 **3.3.3 Step 10: Boundary condition definition**

4 As the base of the voxelized models resulting from the given point cloud of the tower was not flat, an  
 5 additional base was generated. This was in order to define a level, planar support to the tower. This  
 6 process was carried out fully automatically within the voxelization algorithm by means of extending  
 7 the dimensionless point cloud in the direction of the z-axis. Figure 6a-c shows the original geometric  
 8 model, the additional base and the final numerical model used for structural analysis.

9 After adding the base, a level plane ( $\Pi_0$ ) was defined as the boundary of fixed and free-moving blocks.  
 10 This is shown in Figure 6d (at the lowest course of the blocks of the original numerical model) and  
 11 defines the boundary between the fixed and free-moving blocks. Below this level, blocks were fixed  
 12 against movement in all directions (the dark grey zone shown in Figure 6e), while above this level the  
 13 blocks were considered to represent the rubble masonry and were free to move (the silver zone shown  
 14 in Figure 6e). By adding this additional base and ensuring planar support, failure of the structure enabled  
 15 by means of material degradation only, and not support failure.



1

2 *Figure 6. View of the numerical model of the tower developed using DEM: (a) original geometric model derived from point*  
 3 *clouds; (b) base added to assist with the numerical simulations; (c) final numerical model (note that green colour refers to an*  
 4 *additional base while gold colour relates to the original 50 cm voxelized model). Boundary conditions of tower visualized in:*  
 5 *(d) point cloud; and (e) numerical model.*

### 6 **3.3.4 Step 11: Loading protocol definition**

7 In the numerical model, a tilt plane analysis was performed with the aim of quantifying the maximum  
 8 theoretical inclination angle ( $\theta_t$ ) of the tower, if it were situated on a tilted plane. This value was effec-  
 9 tively the measure of the tower's structural capacity. The inclination angle was estimated by applying  
 10 a horizontal acceleration ( $g_h$ ) equal to  $\lambda_h \times g$ , see equation (17) and altering the vertical acceleration  
 11 of gravity from  $g$  to  $g_{vz}$ , equal to  $\lambda_v \times g$ , see equation (18). The horizontal and vertical inclination  
 12 angle multipliers  $\lambda_h$  and  $\lambda_v$  were obtained from the equations (17) and (18). Figure 7 **Error: L'origine**  
 13 **riferimento non è stata trovata.** a shows a view of the tower with the gravitational acceleration compo-  
 14 nents annotated.

$$15 \quad \lambda_h = \sin(\theta_t) \quad (17)$$

$$16 \quad \lambda_v = \cos(\theta_t) \quad (18)$$

17 Figure 7 **Error: L'origine riferimento non è stata trovata.** b shows the plan of the tower base and the  
 18 azimuth ( $\psi$ ) of inclination (i.e. horizontal direction in which the inclination takes place). In the numer-  
 19 ical model, 3D loading due to the theoretical inclination was achieved by assigning gravitational accel-  
 20 eration components as per equations (19) to (21).



1  $g_{hx} = g \cdot \lambda_h \cdot \cos \psi$  (19)

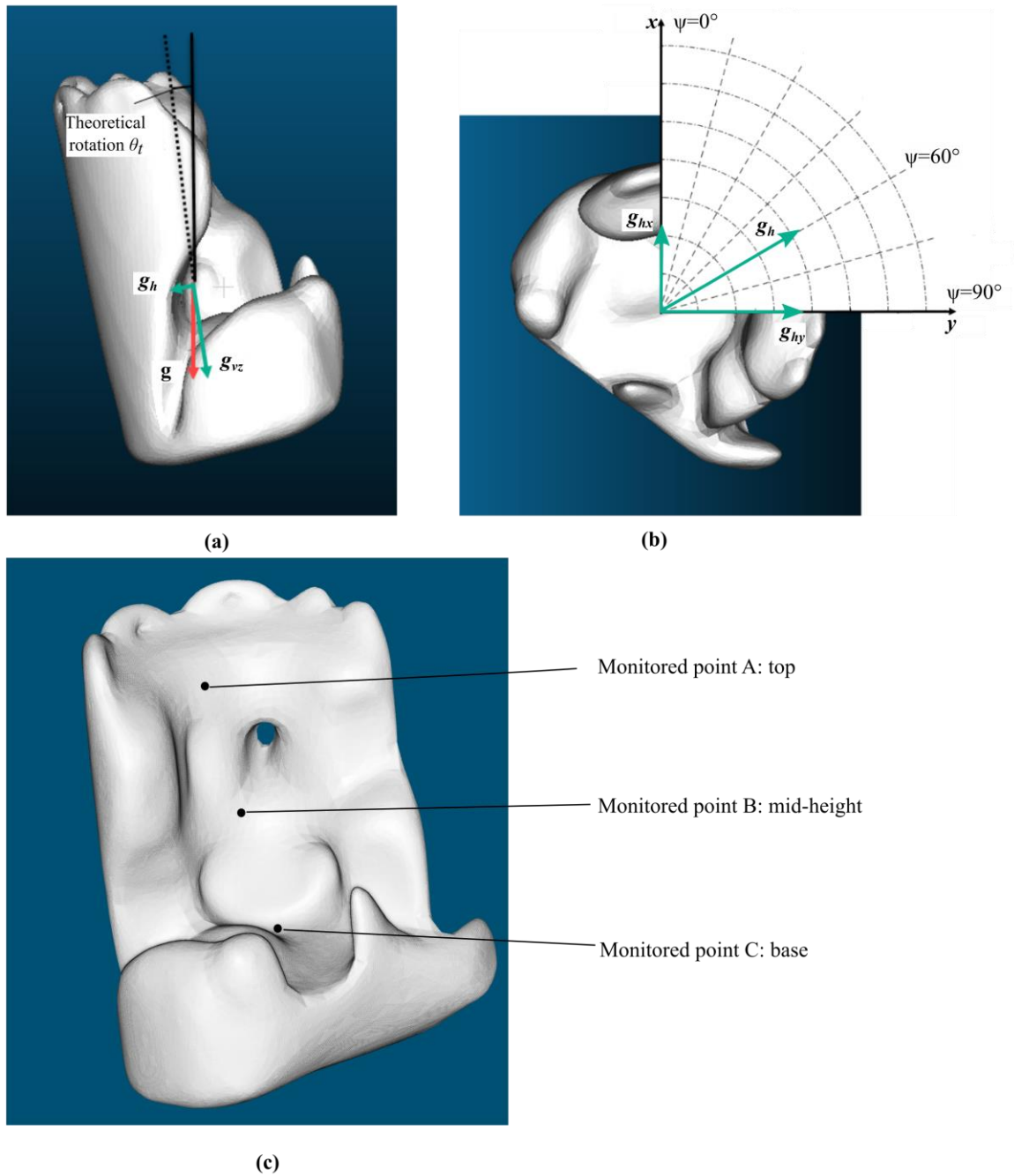
2  $g_{hy} = g \cdot \lambda_h \cdot \sin \psi$  (20)

3  $g_{vz} = g \cdot \lambda_v$  (21)

4 In particular, equations (19), (20) and (21) describe the x-axis horizontal component, y-axis horizontal  
5 component, and vertical component of gravity, respectively (as shown in Figure 7**Errore. L'origine**  
6 **riferimento non è stata trovata.a-b**). So, for any given azimuth of inclination ( $\psi$ ), the theoretical in-  
7 clination angle ( $\theta_t$ ) is proportional to the horizontal component of gravity applied to the structure. The  
8 resulting destabilization is common with that of a tilt-table, parallel to the azimuth of inclination ( $\psi$ ).  
9 The structural analysis of this investigation was carried out for an azimuth of  $\psi$  equal to  $60^\circ$  ( Figure  
10 **7Errore. L'origine riferimento non è stata trovata.b**). This was due to the structure's pre-existent  
11 inclination being most significant in this direction [21].

12 Starting from a value of  $\theta_t$  equal to 0 (no inclination), the theoretical inclination angle  $\theta_t$  was increased  
13 incrementally. During the simulation, the inclination angle multiplier  $\lambda_h$  (corresponding to  $\theta_t$ ) was rec-  
14 orded. The critical inclination angle  $\lambda_{h,max}$  was employed to assess the load-bearing capacity of the  
15 structure, equal to the inclination angle multiplier at which the structure could not arrive at equilibrium  
16 at the end of a given loading cycle. This was calculated by monitoring both the total unbalanced force  
17 of the model and the so-called inclination angle multiplier-displacement curves of strategically selected  
18 monitored points. The unbalanced force [37] in specific is a metric employed to evaluate the mechanical  
19 equilibrium state of the model (and subsequent occurrence of the joint slip or plastic flow), during struc-  
20 tural analysis. Equilibrium of the model is achieved when either the net nodal force vectors at each  
21 block centroid or gridpoint are equal to zero and this is monitored in form of: a) the maximum nodal  
22 force vector termed the “unbalanced” or “out-of-balance” force; or alternatively b) the ratio of the un-  
23 balanced force towards the representative forces of the system, termed “unbalanced force ratio”. During  
24 the structural analysis of this investigation, an unbalanced force ratio equal to  $1e-4$  was employed. This  
25 means that during the loading, increments were added as long as the unbalanced force ratio was smaller  
26 to or equal to  $1e-4$ . The monitored points at Points A, B and C, shown in Figure 7**Errore. L'origine**  
27 **riferimento non è stata trovata.c** were strategically selected, being situated: a) on the azimuth of the-  
28 oretical rotation of  $\psi$  equal to  $60^\circ$ ; and b) at various heights (top, mid-height and bottom). This selection  
29 of the monitored points ensured reliable information about the structure's behaviour was provided for  
30 global and local failure, in the principal direction of loading. It's noteworthy that further than the critical  
31 inclination angle multiplier  $\lambda_{h,max}$ , the critical horizontal displacements  $U_{h,max}^A$ ,  $U_{h,max}^B$  and  
32  $U_{h,max}^C$  of the monitored points A, B and C were employed as a metric of quantifying the tower's de-  
33 formation capacity.

34



2

3 *Figure 7. The tower with gravitational acceleration components annotated (the green vertices denote the gravitational accel-*  
 4 *eration components for a theoretical inclination angle of  $\theta_t$ ): (a) view; and (b) plan of the tower base with the azimuth of*  
 5 *inclination ( $\psi$ ). Monitored points A, B and C at the top, mid-height and base of the tower (c).*

### 6 3.3.5 Step 12: Partitioning strategy (optional)

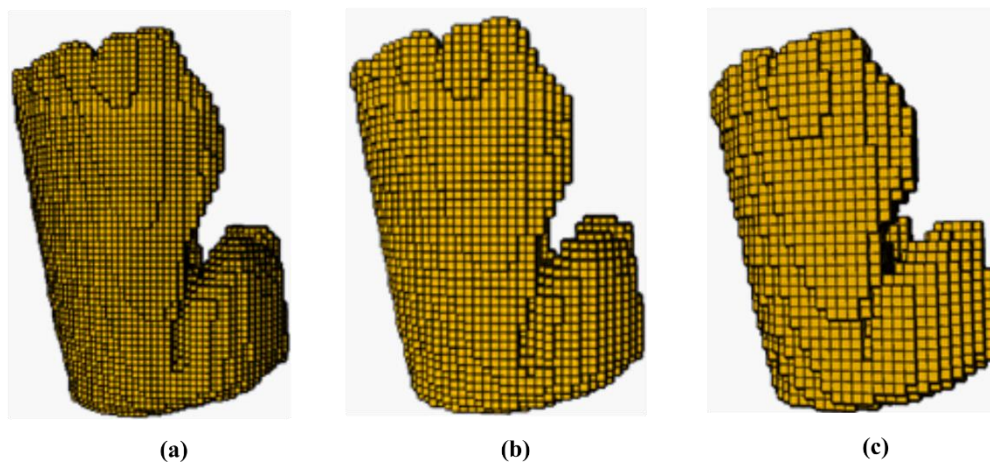
7 In the case that the numerical model was either non-executable or computationally unmanageable due to  
 8 the number of contacts, then a partitioning strategy should be adopted. Historic masonry structures  
 9 are characterised by low bond strength and failure due to shear at the mortar joints or by hinge forma-  
 10 tion. In specific, the formation of the hinges in the tower allows the rotation of some of the blocks  
 11 while restricting the rotation of others. In this case, we can have areas of high strength concentration  
 12 Displacements due to external load applied to the tower are accompanied by the opening and shear  
 13 sliding of the blocks. As inclination angle increases in the tower, hinges are formed as bricks slide and  
 14 rotate against each other. According to Mifsud [40], the development of hinge formation depends on

1 the material properties of masonry, the geometric characteristics and confinement of blocks as well as  
2 the load distribution in the structure. This effectively means that partitions of structure that are not prone  
3 to failure can be considered rigid. In the numerical model, such partitions were defined by joining the  
4 elements that were desired to form rigid bodies in the structural analysis model. This strategy requires  
5 that the analyst has *a priori* knowledge to where the hinge formation is most likely to occur,  
6 the herein termed failure zone of the structure.

7 The employment of partition strategy is particularly advantageous in terms of optimization of compu-  
8 tational resources and increasing geometrical accuracy. Since the number of free-moving blocks of the  
9 calculation cycle is reduced, computational resources are optimized for a given voxel size. Furthermore,  
10 as will be demonstrated in the forthcoming paragraphs, voxel size reduction significantly increases  
11 block numbers yet also increases geometrical accuracy. Thus, owing to the optimization of computa-  
12 tional resources, numerical models can be executed with a larger number of voxels (of a decreased  
13 voxel size) with improved geometrical accuracy.

#### 14 4 Application of the proposed framework in the case study

15 Various geometric models of the Caerphilly tower with different voxel sizes have been developed. Fig-  
16 ure 8 shows three numerical models with a voxel size equal to 50, 40 and 30 cm. Though another three  
17 models with a voxel size of 25, 20 and 10 cm were also developed, structural analysis was not under-  
18 taken since they were found to be computationally unmanageable with the available computational re-  
19 sources. This inability to simulate smaller voxel size models was a result of the unmanageable number  
20 of free-moving blocks and total contacts. As will be demonstrated further on, the best compromise  
21 between structural capacity estimation, geometric accuracy and computational time was represented  
22 with the 30 cm voxel size. Thus, for the voxel size of 30 cm, the sensitivity of the voxel orientation was  
23 also investigated on the structural capacity.



24  
25 Figure 8. Voxelized models of the tower developed using different voxel sizes: (a) 30 cm; (b) 40 cm;  
26 (c) 50 cm

##### 27 4.1 Influence of the voxel size on the characteristics and structural capacity of the tower

28 **Errore. L'origine riferimento non è stata trovata.** Table 2 shows the characteristics of the developed  
29 voxelized models, including: a) geometric accuracy (volumetric differences and cloud-to-mesh dis-  
30 tances); and b) geometric model characteristics (block and contact numbers). From Table 2**Errore.**  
31 **L'origine riferimento non è stata trovata.**, the volumetric error exponentially decreases as the voxel  
32 size in the model decreases. Also, for voxel sizes below and equal to 25 cm (that correspond to an error  
33 of equal to 5.3 %), voxelized models without refinement could be considered acceptable; all voxel sizes  
34 above 25 cm needed numerical model refinement. The difference in mean cloud-to-mesh distance of  
35 the voxelized models and refined voxelized models shows that the model refinement can induce error  
36 to the external surfaces of the structure. Concerning the geometric model properties and computational

times, all the quantities were inversely and exponentially proportional to the voxel size. From Table 2 **Errore. L'origine riferimento non è stata trovata.**, it also appears that the larger the size of the voxel, the faster the numerical simulation is. Furthermore, the best compromise between structural capacity estimation, geometric accuracy and computational time was represented with the 30 cm voxel size. Considering simulation times with the available resources (i.e. an Intel(R) Xeon(R) CPU @ 3.00 GHz processor and 64 GB memory RAM), a model containing 40,000 free-moving blocks and 240,000 total contacts (voxel size of 30 cm) was developed and adopted in this study

Table 2. Characteristics of the voxelized models.

Voxel size (cm)	Geometrical accuracy					Geometric model properties		
	VAC	Volume (m <sup>3</sup> )	Volume error %	Mean C2M (cm)	St. dev. C2M (cm)	Blocks	Total Contacts	
Voxelized models	50 cm	1	1028	19.5	0.1	20.4	13,385	155,556
	40 cm	1	980	13.9	0	16.4	22,532	266,296
	30 cm	1	933	8.5	0.1	12.1	47,827	576,449
	25 cm	1	911	5.9	0.2	10.1	74,969	912,491
	20 cm	1	886	3.0	0.1	8	111,821	2,552,214
	10 cm	1	836	-2.8	0	3.7	844,343	10,645,763
Refined Voxelized	50 cm	0.943	862	0.2	8.2	27.3	13,385	155,556
	40 cm	0.958	862	0.2	11.5	20.6	22,532	266,296
	30 cm	0.974	862	0.2	9	13.9	47,827	576,449
	25 cm	0.981	860	0.0	6.2	11.2	74,969	912,491
	20 cm	0.990	859	-0.1	3.3	8.4	111,821	2,552,214
	10 cm	1.009	859	-0.2	3.5	4.8	844,343	10,645,763

For the voxel sizes of 50, 40 and 30 cm, the influence of the voxel size upon the structures' load-bearing capacity, deformation capacity and failure mode was investigated. Table 3 reports the results of the structural analysis of the 50, 40 and 30 cm voxelized models for an azimuth of inclination ( $\psi$ ) equal to 60°. From Table 3, it's evident that the decrease of voxel size is associated with: a) decrease in load-bearing capacity (i.e. the critical inclination angle multiplier); b) decrease in the deformation capacity (i.e. the critical horizontal displacements of monitored points A, B and C). This is in agreement with a previous investigation on the out-of-plane loading of masonry structures with the DEM [41], that block size significantly influences the structural capacity. The reasoning behind this is that effectively, the joints form planes of weakness in the structure. As the voxel size decreases, the number of joints significantly increases, resulting in a consequent reduction of the structural capacity. It is hypothesized that this reduction would become negligible for a voxel size lower than a certain threshold, yet this should be examined in further investigation.

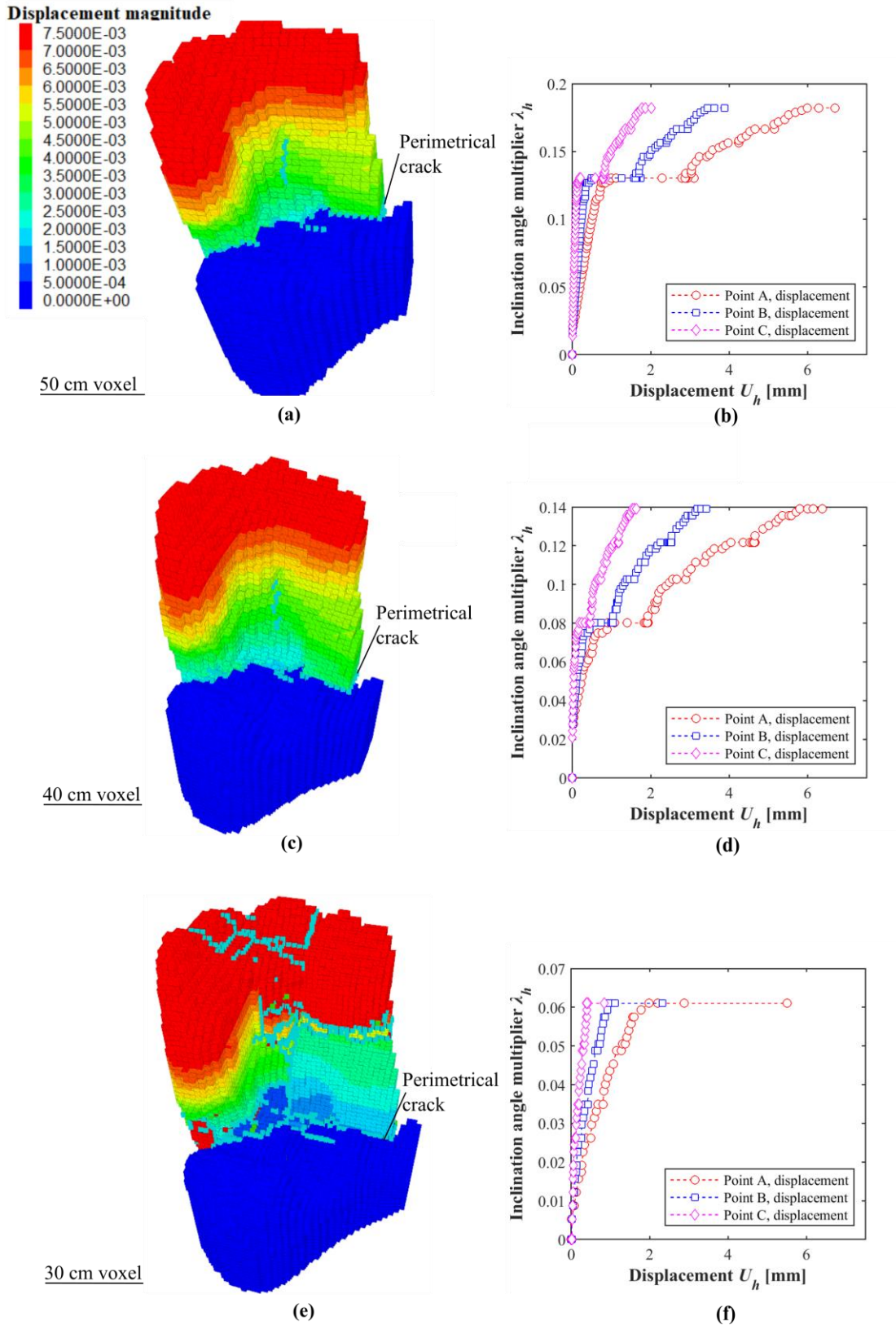
Further than results of Table 3, the accompanying failure modes and inclination angle multiplier-displacement curves are shown in Figure 9a-b, Figure 9c-d and Figure 9e-f respectively. It's noteworthy that the blue, green and red cuboids located on the joints signify joint tensile failure, current joint slipping and past slipping respectively. Furthermore, the displacement contours of the blocks are plotted demonstrating the magnitude of displacement due to inclination. From Figure 9a, c, e, it's evident that failure modes of the models were common and consisted of perimetrical cracks developed at the junc-

1 tion of the remaining up-right body and base of the tower progressing towards the base. With the appli-  
 2 cation of any further inclination, the remaining up-right body detached and began to rotate freely (sim-  
 3 ultaneously breaking up into pieces. Similar failure modes were observed in a previous study [21].

4 *Table 3. Influence of voxel size on structural capacity.*

Voxel size (cm)	$\lambda_{h,max}$	$U_{h,max}^A$ (mm)	$U_{h,max}^B$ (mm)	$U_{h,max}^C$ (mm)
50 cm	0.18	6	3.8	1.8
40 cm	0.14	5.6	3.0	1.4
30 cm	0.06	2.0	1.0	0.3

5



1

2 *Figure 9. Failure modes and inclination angle multiplier-displacement curves (different scale) of: (a-b) 50 cm; (c-d) 40 cm*  
 3 *(e-f) 30 cm voxel size models (azimuth of inclination  $\psi$  equal to  $60^\circ$ ). The blue markers denote joint tensile failure while the*  
 4 *displacement contour is common and in meters.*

1 **4.2 Influence of the voxel orientation on the structural capacity of the tower**

2 For the voxel size of 30 cm, the influence of the voxel orientation upon the structures' load-bearing  
 3 capacity, deformation capacity and failure mode was investigated. With respect to Figure 9**Errore. L'origine riferimento non è stata trovata.**,  
 4 **L'origine riferimento non è stata trovata.**, the horizontal voxel principal directions were altered by  
 5 rotation of the dense point cloud, prior to voxelization. Two simulations were performed whereby the  
 6 dense point cloud was rotated by 30 and 60 degrees prior to voxelization around the z-axis and loaded  
 7 for  $\psi$  equal to 60°.

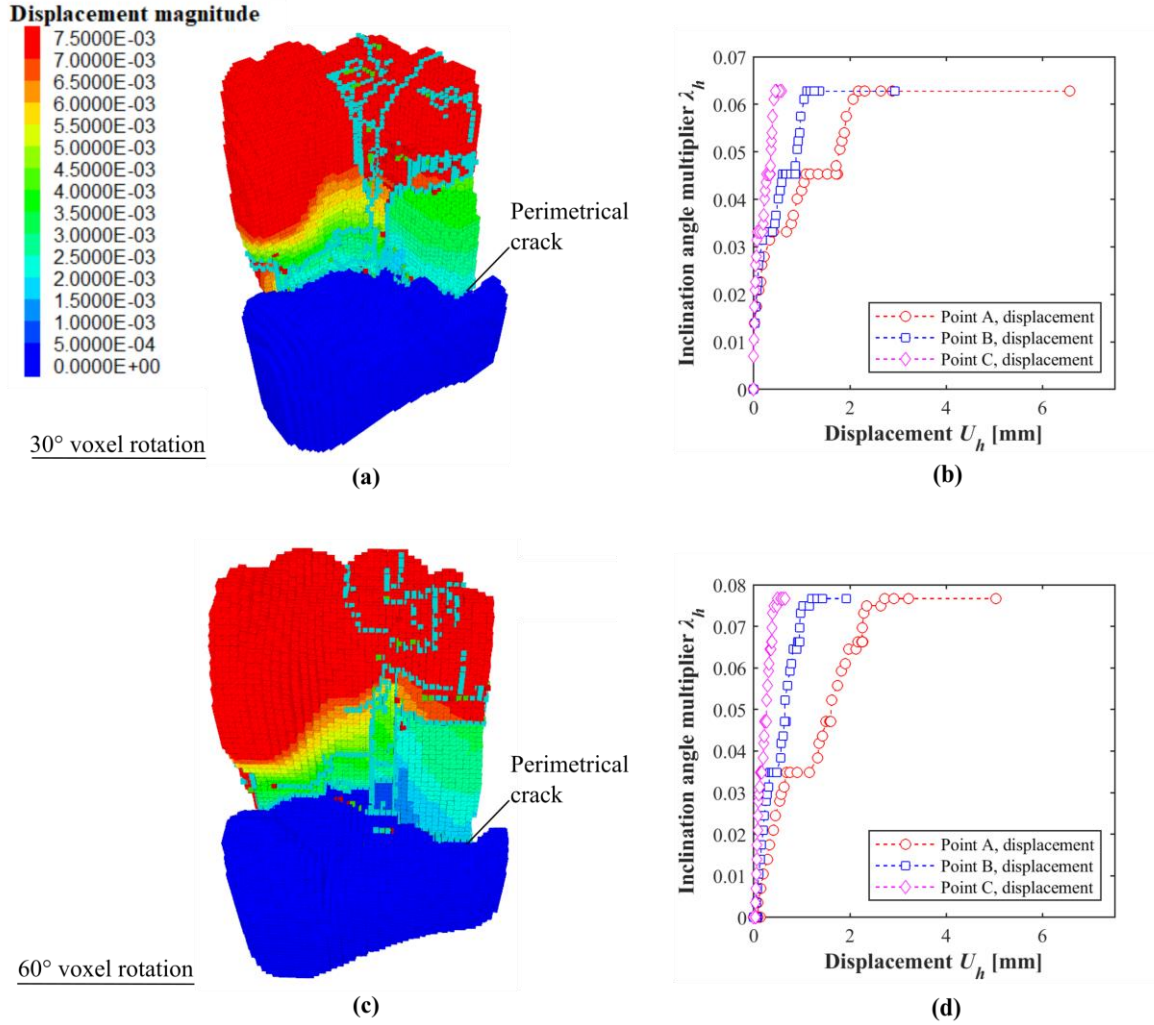
8 Table 4 reports the results of the structural analysis of the models with a voxel rotation. From Table 4,  
 9 it's evident that, in comparison with the original 30 cm voxel size model of Table 3, the models with a  
 10 voxel rotation demonstrated: a) comparable load-bearing capacities (i.e. the critical inclination angle  
 11 multiplier); and b) comparable deformation capacities (i.e. the critical horizontal displacements of mon-  
 12 itored points A, B and C). Furthermore, the failure modes and inclination angle multiplier-displacement  
 13 curves of the models with the voxel rotation are shown in Figure 10a-b and Figure 10c-d respectively.  
 14 Comparing the differences between Figure 9e-f and Figure 10**Errore. L'origine riferimento non è**  
 15 **stata trovata.**, it is evident that the failure modes and inclination angle multiplier-displacements curves  
 16 between the models with voxel rotation and original 30 cm voxel size model show a good agreement.

17 *Table 4. Influence of voxel orientation on structural capacity.*

Voxel ro- tation	$\lambda_{h,max}$	$U_{h,max}^A$ (mm)	$U_{h,max}^B$ (mm)	$U_{h,max}^C$ (mm)
30°	0.06	2.0	1.0	0.4
60°	0.08	2.4	1.0	0.4

18





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Figure 10. Failure modes and inclination angle multiplier-displacement curves of 30 cm voxel size with: (a-b) 30° voxel rotation; and (c-d) 60° voxel rotation. The blue markers denote joint tensile failure while the displacement contours are common and in meters.

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## 5 Conclusions

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Predicting the mechanical behaviour of rubble masonry structures is a complex task due to their highly nonlinear behaviour. Numerical methods such as the DEM have been demonstrated as highly effective for accurately capturing the in-service, collapse and post-collapse behaviour of masonry structures. Due to its numerical formulation which was specifically developed for rock mechanics (i.e. specific problem of sliding of rocks), the DEM permits the discrete and non-linear behaviour of the tower to be better represented than other numerical methods such as the FEM, yet it requires detailed metrical geometric information of the structure subject to structural analysis. So far, the geometry of rubble masonry structures is captured with traditional geospatial techniques (e.g. visual inspection and CAD-based design methods) which are labour intensive and error-prone. Over the last 10 years, advances in terrestrial laser scanning and SfM photogrammetry have drastically changed the building industry since such techniques are able to rapidly and remotely capture millions of points of the entire scene, resulting in point clouds. This paper presents a novel framework for the expeditious and automatic modelling of rubble masonry, directly from a dense point cloud, and without the need for mesh development. The proposed “point cloud to structural analysis” framework consists of three stages and nine steps. The three-stage framework involved: Stage 1 - 3D documentation; Stage 2 - geometric model development; and Stage 3 - structural analysis. The methodological workflow proposed here has been demonstrated on the south-

1 west leaning tower in Caerphilly castle. A quantitative structural assessment of the specific rubble ma-  
2 sonry tower undertaken. The geometry of the tower was represented by the sum of hexahedral blocks  
3 (voxels) defined by the user. The main findings of this investigation are summarized below:

- 4 • The potential of the proposed “point cloud to structural analysis” framework has been demon-  
5 strated to quantitatively assess the three-dimensional mechanical behaviour of complex in ge-  
6 ometry rubble masonry structures such as towers.
- 7 • As the voxel size decreases, the load-bearing and deformation capacity decreases. This is in  
8 agreement with a previous investigation on the out-of-plane loading of masonry structures with  
9 the DEM [41]. Furthermore, due to voxel size decrease, the computational time required to  
10 perform structural analysis increases dramatically and could lead to models that cannot be han-  
11 dled with standard workstations.
- 12 • For the case study investigated herein (i.e. the Caerphilly tower), for a coarse voxel size of 50  
13 cm, structural analysis was carried out in a manageable computational time of 71 minutes for a  
14 geometric model with 9,000 blocks, 100,000 contacts.
- 15 • Finally, the best compromise between, geometric accuracy and computational time was  
16 achieved with a voxel size of 30 cm. This was very close to the size of the masonry stones  
17 observed on the structure. However, such model resulted in a less manageable computational  
18 time of approximately 238 minutes.

19 The above findings suggest that with the proposed procedure, it is possible to perform unprecedented  
20 structural analyses of rubble masonry structures with high-level structural analysis methods such as the  
21 DEM in a manageable time. This contribution paves the way for their automated structural analysis of  
22 large, complex in geometry and discontinuous in nature masonry structures.

23 To increase the reliability of the structural analysis of the proposed approach, further investigation  
24 would be beneficial on the effect of the mechanical properties of the interface, the size of the block and  
25 orientation of the blocks, and computation resource optimization techniques. Furthermore, since rubble  
26 masonry possesses voids and flaws, a future investigation should also be carried out to simulate this by  
27 removing voxels in critical locations. Finally, due to the orthogonal nature of the voxels, analysing  
28 curve forms, such as domes, arches, can be cumbersome leading to problems such as shear-locking that  
29 does not permit, hinging failure mechanism. To overcome this, the development of automated ap-  
30 proaches employing non-orthogonal blocks such as those in [42,43] is suggested.

### 31 **Acknowledgements**

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34 cloud of the southwest tower of Caerphilly Castle.

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