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# Evolution of the number of communicative civilizations in the Galaxy: implications on Fermi paradox

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## Abstract

It has been recently proposed DeVito (2019) that a *minimal* number of contacts with alien radio-communicative civilizations could be justified by their logarithmically slow rate of growth in the Galaxy. Here we further develop this approach to the Fermi paradox, with the purpose of expanding the ensemble of the possible styles of growth that are consistent with the hypothesis of a minimal number of contacts. Generalizing the approach in DeVito (2019), we show that a logarithmic style of growth is still found. We also find that a style of growth following a power law would be admissible, however characterized by an exponent less than one, hence describing a sublinear increase in the number of communicative civilizations, still qualitatively in agreement with DeVito (2019). No solutions are found indicating a superlinear increase in the number of communicative civilizations, following for example an exponentially diverging law, which would cause, in the long run, an unsustainable proliferation. Although largely speculative, our findings corroborate the idea that a sublinear rate of increase in the number of communicative civilizations in the Galaxy could constitute a further resolution of Fermi paradox, implying a constant and minimal - but not zero - number of contacts.

*Keywords:* Fermi paradox, Alien civilizations, Population dynamics

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1 **1. Introduction**

2 DeVito DeVito (2019) has recently considered some new aspects of the  
3 “Fermi paradox”, *i.e.*, the apparent contradiction between the lack of evidence  
4 for extraterrestrial civilizations existing in the Galaxy and their high probabili-  
5 ty Hart (1975); Webb (2002); DeVito (2013), suggested by the Drake equa-  
6 tion Drake (2014); Forgan (2009). Assuming that the Galaxy is explored with  
7 the only purpose of detecting signals from alien radio-communicative civiliza-  
8 tions, DeVito has argued that the rate  $R$  at which they are detected should de-  
9 pend on their number  $n(t)$  but also on their rate of increase (or decrease),  $\dot{n}(t)$ .  
10 Note that here  $n(t)$  represents the left-hand side of Drake’s equation Burchell  
11 (2006); Sandberg et al. (2018), denoted by  $N$  and customarily assumed to be  
12 constant. A functional dependency like  $R = R(n, \dot{n})$  appears to be justified,  
13 assuming an ideal scenario in which the Galaxy has been continuously explored  
14 during a significantly long period of time, taking note of the contacts with alien  
15 societies and continuing the search. Apart such idealized experiment, it seems  
16 clear that an explicit mathematical expression for the rate of detection can  
17 hardly be conjectured, although it seems reasonable to assume that  $R$  would be  
18 increasing with  $n(t)$  and  $\dot{n}(t)$ . In general, the rate of successful detections shall  
19 depend upon the SETI strategy adopted, on the resources deployed, as well as  
20 on a number of other factors - also involving socio-political aspects - that can  
21 be hardly quantified lacking observational constraints.

22 Following DeVito, we make the hypothesis that  $n$  is large enough to be  
23 effectively treated as a continuous variable and that its time derivative  $\dot{n}(t)$   
24 can be evaluated for all values of  $t$ . Furthermore, assuming the functional  
25 dependency  $R = R(n, \dot{n})$ , the quantity

$$26 \quad N^d = \int_0^T R(n, \dot{n}) dt \quad (1)$$

27 represents the number of societies effectively detected over the exploration time  
28 interval  $0 \leq t \leq T$ . The argument in DeVito (2019) is that  $N^d$  cannot be a

29 large number, otherwise some contact would have occurred by now. Since in the  
 30 environment we have still not found evidence for such contacts (though search  
 31 strategies for alien footprints have been suggested, see (Davies, 2012)), the De-  
 32 Vito’s hypothesis is that  $N^d$  is small and *minimal*. This essential - although not  
 33 verifiable - assumption, is the requisite for a quantitative approach to the prob-  
 34 lem, which otherwise would not be possible. Indeed, from functional analysis  
 35 Kot (2014), for  $N^d$  being an extremum,  $R(n, \dot{n})$  must obey the Euler-Lagrange  
 36 (E-L) partial differential equation

$$37 \quad \frac{\partial R}{\partial n} - \frac{d}{dt} \frac{\partial R}{\partial \dot{n}} = 0, \quad (2)$$

38 where henceforth we can assume  $R \geq 0$  since  $R$  represents a rate of detection.  
 39 Furthermore, a *necessary* condition for  $R$  being a minimum is

$$40 \quad \frac{\partial^2 R}{\partial \dot{n}^2} \geq 0, \quad (3)$$

41 where  $\dot{n}(t)$  is the time-derivative of the solution of Eq. (2). We note however  
 42 that this constraint, known as “Legendre condition” in the calculus of variations  
 43 (see *e.g.*, Gelfand and Fomin (1963)), has not been exploited in DeVito (2019).  
 44 It is noteworthy that in the context of classical population dynamics, the in-  
 45 troduction of variational principles dates back to the work of Volterra (1939),  
 46 who considered the problem of minimizing an appropriate functional,  
 47 leading to an E-L equation that is satisfied by the Verhulst (logistic) equation.  
 48 The idea of Volterra proved to be fecund, being later reevaluated in Leitmann  
 49 (1972) and Gatto et al. (1988).

50 Searching for a particular solution of the E-L equation (2) in the factorized  
 51 form

$$52 \quad R(n, \dot{n}) = G(n)H(\dot{n}), \quad (4)$$

53 where Lagrangian  $R$  is not explicitly time-dependent and the unknown functions  
 54  $G(n)$  and  $H(\dot{n})$  depend upon  $n(t)$  and  $\dot{n}(t)$  separately, DeVito (2019) has  
 55 determined a *simple* solution of the problem, in which  $H(\dot{n}) \approx \dot{n}^2$  (henceforth

56  $\approx$  is used to denote proportionality). With this choice, the minimum rate of  
57 detection turns out to be a constant, *i.e.*,

$$58 \quad \dot{R}(n, \dot{n}) = 0, \quad (5)$$

59 a condition that, by Occam's razor, appears to be reasonable and valid for any  
60 other acceptable solution of the E-L equation. According to DeVito, the solution  
61  $n(t)$  slowly increases with time following an unbounded logarithmic growth<sup>1</sup>  
62 (details shall be given in Section 2 below). Intriguingly, from this result DeVito  
63 has suggested a further possible resolution of Fermi paradox Webb (2002), *i.e.*,  
64 that the lack of contacts with alien communicative civilizations is hampered by  
65 their limited rate of growth in the Galaxy.

66 As emphasized in DeVito (2019), the solution of the E-L equation is, from a  
67 mathematical standpoint, highly non-unique. Furthermore, any solution could  
68 be hardly tested against experimental observations, at least until SETI shall  
69 succeed. Nevertheless, we think that searching and classifying other possible and  
70 yet unknown solutions of the DeVito's problem may constitute an interesting  
71 intellectual exercise. Indeed, their nature could provide new resolutions of Fermi  
72 paradox, either supporting or challenging that proposed in DeVito (2019). For  
73 instance, solutions characterized by a marked growth in time like  $\sim e^t$  or  $\sim t^\alpha$   
74 ( $\alpha > 1$ ) would undermine DeVito's argument; *vice versa*, weakly increasing  
75 ( $\sim t^\alpha$ ,  $\alpha < 1$ ) or decaying solutions (as  $\sim e^{-t}$  or  $t^{-\alpha}$  with  $\alpha > 0$ ) would  
76 strengthen it. In this work we explore such possibilities, conventionally defining  
77 as *viable solutions* those for which Eqs. (2), (3) and (5) are simultaneously  
78 valid, as they are valid for DeVito's original logarithmic solution. Obviously, of  
79 particular interest are those viable solutions that can be expressed in terms of  
80 elementary functions, thus having a value similar to the *simple* solution sought  
81 (and found) in DeVito (2019). As far as we know, such alternatives have not  
82 been systematically explored so far. It is certain, however, that assuming for

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<sup>1</sup>To avoid confusion, it is worth to remark that in population ecology the term *logarithmic growth* is used to indicate the phase of population growth during which the number of cells increases exponentially, in conditions of unlimited resources (see *e.g.*, Berryman (2003)).

83  $H(\dot{n})$  a degree three polynomial is not leading to viable solutions (see Appendix  
 84 of DeVito (2019)).

85 This brief communication is organized as follows. In Section 2 we review  
 86 and complement the DeVito’s solution. In Section 3, we extend DeVito’s solu-  
 87 tion scheme, obtaining a class of viable solutions characterized by logarithmic  
 88 growth. Section 4 proposes a further viable and simple solution exhibiting a  
 89 power law style of growth. Section 5 discusses the various styles of growth sug-  
 90 gested by our results, which are compared with basic styles of growth known in  
 91 the literature of population dynamics. Our conclusions are drawn in Section 6.

## 92 2. Extending DeVito’s solution

93 DeVito DeVito (2019) relied upon the factorized form (4), in which  $H(\dot{n})$   
 94 is the lowest-degree monomial expression for which a “simple” solution can be  
 95 easily determined. Note that with respect to DeVito (2019), here we use a  
 96 slightly different notation. Assuming

$$97 \quad H(\dot{n}) = (c\dot{n})^2, \quad (6)$$

98 where  $c$  is a constant, and solving the E-L equation (2) by separating the vari-  
 99 ables we obtain

$$100 \quad -2 \frac{\ddot{n}}{\dot{n}^2} = \frac{G'(n)}{G(n)} = k^2, \quad (7)$$

101 where we have defined  $G'(n) = \frac{dG}{dn}$  and  $k^2$  is a dimensionless separation constant.  
 102 Henceforth we assume, without loss of generality, that functions  $G$  and  $H$  are  
 103 positive. The second of the two equalities in Eq. (7) gives  $G(n) = G_0 e^{k^2(n-n_0)}$ ,  
 104 where  $G_0 > 0$  is a constant and  $n_0 = n(0)$  is the initial number of communica-  
 105 tive civilizations, while from the first we obtain the following linear ordinary  
 106 differential equation

$$107 \quad \dot{n} = \frac{\dot{n}_0}{1 + \frac{\dot{n}_0 k^2}{2} t}, \quad (8)$$

108 where  $\dot{n}_0$  is the initial rate of change of  $n(t)$ . Here we depart slightly from DeVito  
 109 (2019), since we consider separately two cases that differ for the sign of the initial

110 rate  $\dot{n}_0$ . Of course, according to (8), in the particular case  $\dot{n}_0 = 0$ ,  $n(t)$  would  
 111 remain constant to  $n_0$  during the whole observation period. By integrating (8)  
 112 for  $\dot{n}_0 \neq 0$ , and defining a time constant  $\tau$  such that  $\tau^{-1} = |\dot{n}_0|k^2$ , we obtain  
 113 the time evolution of communicative civilizations that ensures an extremum for  
 114  $N^d$ , namely

$$115 \quad n_{\pm}(t) = n_0 + 2\tau |\dot{n}_0| \log \left| \frac{t}{2\tau} \pm 1 \right|, \quad (9)$$

116 where  $n_+(t)$  and  $n_-(t)$  correspond to the two mutually excluding conditions  
 117  $\dot{n}_0 > 0$  and  $\dot{n}_0 < 0$ , respectively.

118 In Figure 1, solutions (9) are qualitatively depicted for some particular values F1  
 119 of the free parameters; details are given in the caption. We note that solution  
 120  $n_+(t)$  (red curve) corresponds to the one found in DeVito (2019). It is charac-  
 121 terized by a slow unbounded growth and by a rate of change decreasing like  $t^{-1}$ ,  
 122 hence approaching zero for  $t \mapsto \infty$ . Although  $n_-(t)$  (blue curve) is matching  
 123  $n_+(t)$  for sufficiently long times ( $t \gg \tau$ ), it appears that the sign of  $\dot{n}_0$  has a  
 124 significant role in shaping the solution for times  $t \approx \tau$ . Remarkably, Figure 1  
 125 shows that the condition of minimum for  $N^d$  (see Eq. 1) could be compatible  
 126 with an initial decline and a subsequent recovery of the number of communi-  
 127 cative civilizations, as indicated by solution  $n_-(t)$ . It should be observed, however,  
 128 that according to our assumptions,  $n(t)$  should be enough large to be consid-  
 129 ered as a real (and differentiable) variable, so that close to the singularity of  
 130 Figure 1 the solution found has merely a formal character. It is straightforward  
 131 to verify that the Legendre condition (3) is met for both  $n_+(t)$  and  $n_-(t)$ , in-  
 132 dicated that they could effectively correspond to a minimum of  $N^d$ . Note that  
 133 the constraints represented by the Legendre condition has not been taken into  
 134 consideration in DeVito (2019). In addition, the minimum rate of detection,  
 135 *i.e.*, the value of  $R(n, \dot{n})$  evaluated using for  $n(t)$  the expressions of  $n_{\pm}(t)$ , is  
 136 a constant (see 5). Hence, according to our definition of viable solution given  
 137 above, the DeVito's solution and its extension (9) are both viable, being at the  
 138 same time mathematically simple.

139 **3. Generalizing DeVito's scheme**

140 To better explore the range of possibilities existing, with the aid of the  
 141 algebraic manipulator `Mathematica`® Research (2010), we have been searching  
 142 for other viable and mathematically simple solutions of the E-L equation. In this  
 143 section, we consider a few examples in which a factorized form (4) for  $R(n, \dot{n})$   
 144 is preserved.

145 First, we have found that a straightforward generalization of DeVito's solu-  
 146 tion (9) is possible by making the particular choice

$$147 \quad H(\dot{n}) = (c\dot{n})^p, \quad (10)$$

148 where  $c$  is an inessential constant and  $p \geq 2$  is an integer (for  $p = 2$ , Eq. 10  
 149 reduces to 6). In this case, imposing the validity of the E-L equation (2), after  
 150 some algebra we still find a logarithmic law

$$151 \quad n_{\pm}(t) = n_0 + p\tau |\dot{n}_0| \log \left| \frac{t}{p\tau} \pm 1 \right|, \quad (11)$$

152 where constant  $\tau$  and the meaning of  $n_{\pm}(t)$  are the same of Eq. (9). It is easily  
 153 verified that for even values of  $p$  the Legendre condition is satisfied, hence  $N^d$   
 154 could effectively have minimum for  $n(t) = n_{\pm}(t)$ . Conversely, for odd values of  
 155  $p$ , the Legendre condition only holds for  $\dot{n} > 0$ , hence, for  $\dot{n} < 0$  the solution  
 156 certainly does not correspond to a minimum. Note that similar to DeVito's  
 157 solution, for  $n = n_{\pm}(t)$  the rate of detection  $R(n, \dot{n})$  is a constant. Hence, for  
 158 even values of  $p$ , solution (11) is viable and characterized by the same level of  
 159 mathematical complexity of (9). Figure 2 shows  $n_+(t)$  for some even values F2  
 160 of  $p$ , using log-log axes. All the curves are similar to curve  $n_+(t)$  in Figure  
 161 (1), and regardless the  $p$  value adopted their trends become distinguishable only  
 162 for  $t \geq \tau$ . This example clearly supports the DeVito's argument about the  
 163 logarithmic nature of the growth of  $n(t)$ . For  $p \mapsto \infty$ , it is easily verified that  
 164  $n_+(t)$  approaches asymptotically the linear growth model  $n(t) = n_0 + (\dot{n}_0\tau)(t/\tau)$ ,  
 165 which is plotted by the purple curve in Figure 2.

166 By algebraic manipulation, we have found other interesting analytical so-  
 167 lutions of the E-L equation. To provide a few examples, here we consider the

168 three characterized by the simplest structure, namely  $H(\dot{n})=(c_1\dot{n}) \log(c_2\dot{n})$ ,  
 169  $H(\dot{n})=c_1\dot{n} + (c_2\dot{n})^{-1}$  and  $H(\dot{n})= e^{c\dot{n}}$ , where  $c_1, c_2$  and  $c$  are positive constants.  
 170 In the first case, for the time evolution of the number of communicative civi-  
 171 lization we find

$$172 \quad n_{\pm}(t) = n_0 + |\dot{n}_0|\tau \log \left| \frac{t}{\tau} \pm 1 \right|, \quad (12)$$

173 where constant  $\tau$  and the meaning of  $n_{\pm}(t)$  are the same as in Eq. (9). In the  
 174 second case, after some algebra, we still find a solution that varies logarithmi-  
 175 cally with time, namely

$$176 \quad n_{\pm}(t) = n_0 - |\dot{n}_0|\tau \log \left| \frac{t}{\tau} \mp 1 \right|, \quad (13)$$

177 whereas in the third case, we obtain

$$178 \quad n(t) = n_0 + \frac{t}{\tau_1} + \tau_2 \left( \dot{n}_0 - \frac{1}{\tau_1} \right) \left( 1 - e^{-t/\tau_2} \right), \quad (14)$$

179 where  $\tau_1 > 0$  and  $\tau_2 > 0$  are two independent time constants. We note that (12)  
 180 and (13) confirm qualitatively the character of the original DeVito's solution (9).  
 181 However, a qualitatively different style of growth is implied by (14), which shows,  
 182 for sufficiently long times ( $t \gg \tau_2$ ), a constant rate of increase, with  $\dot{n}(t) \approx$   
 183  $\tau_1^{-1}$ . It is easy to establish, however, that all the three solutions considered  
 184 above imply a time-varying minimum rate of detection ( $\dot{R} \neq 0$ ), contrary to the  
 185 original DeVito's solution (9) and to its extension (11). Hence, according to our  
 186 conventions, they cannot be considered viable solutions.

#### 187 4. More solutions

188 From the results so far, it appears that DeVito's hypothesis of a minimal  
 189 number of detected civilizations suggests a logarithmic evolution for  $n(t)$ . As  
 190 pointed out in DeVito (2019), it is of course impossible to scrutinize all the  
 191 possible particular solutions of the E-L equation. However, either using an al-  
 192gebraic manipulator or by trial and error, we have made efforts to determine  
 193 viable alternatives to the logarithmic growth that we have often encountered,  
 194 hoping that in this way the *zoo* of possible solutions can be better explored.

195 Since the style growth (or decline) of a time-dependent function are commonly  
 196 expressed terms of logarithms ( $\log t$ ), exponentials ( $e^{\alpha t}$ ) and powers ( $t^\alpha$ ), we  
 197 have first searched for exponential solutions, but we have not been success-  
 198 ful. Indeed, finding a solution characterized by a diverging exponential increase  
 199 could be important, since this would challenge the results achieved in DeVito  
 200 (2019) about the slowly growing number of radio-communicative civilizations in  
 201 the Galaxy, assuming that the rate of detection is minimal. Similarly, for the  
 202 same reason, the existence of a solution that grows according to a power law like  
 203  $t^\alpha$  with  $\alpha > 1$  would be engrossing, since it would influence the interpretation of  
 204 Fermi paradox. We have not found viable solutions having a periodic character.

205 In our exploration, an interesting and surprisingly simple power-law solution  
 206 for  $n(t)$  has been found by trial and error assuming a rate of detection

$$207 \quad R(n, \dot{n}) \approx n^p \dot{n}^q, \quad (15)$$

208 where  $p \geq 2$  and  $q \geq 2$  are free parameters. The form (15) appears meaningful,  
 209 since it predicts a rate of detection that, for a given value of the number of  
 210 societies  $n(t)$ , increases with their rate of change  $\dot{n}(t)$ , and *viceversa*; the values  
 211 of  $p$  and  $q$  determine which of the two functional dependencies is stronger. We  
 212 note, however, that Eq. (15) implies  $R = 0$  if  $n(t)$  is constant. Of course,  $p$   
 213 and  $q$  are *a priori* unconstrained, since we do not dispose of any experimental  
 214 observation of  $R$  yet. Imposing the validity of the E-L equation (2), after some  
 215 algebra we obtain a non-linear, autonomous ordinary differential equation in the  
 216 unknown  $n(t)$  that reads

$$217 \quad p \dot{n}^2 + q n \ddot{n} = 0. \quad (16)$$

218 By direct substitution, it can be verified that (16) has a particular solution in  
 219 the form of a power law

$$220 \quad n(t) \approx \left(\frac{t}{\tau}\right)^\beta, \quad (17)$$

221 consistent with the initial condition  $n_0 = 0$ , where  $\tau$  is a time constant, and  
 222 where the exponent is

$$223 \quad \beta = \frac{q}{p+q}. \quad (18)$$

224 We note that since  $\beta < 1$  for any value of  $p$  and  $q$ , the growth of  $n(t)$  is relatively  
225 slow and its rate is decreasing with time, never exceeding a linear trend. We  
226 remark that, based on our criteria, solution (17) is viable since *i*) it obeys the  
227 Legendre condition (3), and *ii*) the minimum rate of detection corresponding to  
228 the solution in Eq. (17) is a constant, according to (5).

## 229 5. Discussion

230 The existence of viable alternatives to the logarithmic model of growth,  
231 suggested by result (17), justifies a short discussion, in a broad perspective,  
232 about the significance of styles of growth encountered or simply mentioned in  
233 this work. It is convenient to classify them into two families, *i.e.*, superlinear and  
234 sublinear, according to the trend that they show in the long run, in comparison  
235 to a linear growth.

236 Some examples of superlinear styles of growth are shown in the plot of Fig- F3  
237 ure 3, where they are compared to the linear growth  $n_{lin}(t) = t/\tau$  depicted by  
238 the dashed line. They are the exponential growth  $e^{+\frac{t}{\tau}}$  (*i*, black curve), which  
239 exemplifies the Malthusian law of uninhibited growth known in population dy-  
240 namics (Berryman, 2003), and two power laws with exponent  $\alpha > 1$ , *i.e.*, the  
241 quadratic (*ii*,  $\alpha = 2$ ) and the cubic (*iii*,  $\alpha = 3$ ) displayed in orange and red,  
242 respectively. In our exploration of the possible solutions of the E-L equation  
243 obeying the DeVito’s hypothesis of a minimal number of detected civilizations,  
244 we have never encountered superlinear growth models like those considered in  
245 Figure 3. Of course, since our search cannot be exhaustive, the existence of  
246 admissible superlinear models is not ruled out. However, it seems unlikely that  
247 an exponentially diverging number of communicative civilizations may be com-  
248 patible with the minimum (and constant) detection rate hypothesized in DeVito  
249 (2019). A common tenet in population dynamics is that an exponentially di-  
250 verging growth would eventually become unsustainable and cause a collapse,  
251 analogous to the well known Malthusian catastrophe Malthus (1872). Along  
252 these lines, it is interesting to note that a “sustainability solution” to the Fermi

253 paradox has been proposed in Haqq-Misra and Baum (2009), in which the ab-  
254 sence of contacts is explained by the possible non sustainability of exponential  
255 (or faster) growth patterns of hypothetical intelligent civilizations.

256 As possible examples of sublinear styles of growth, in Figure 4 we have F4  
257 considered the (shifted) logarithm  $\log(1+t/\tau)$  (*i*, green curve), and two samples  
258 of power laws with exponent  $0 < \beta < 1$ , namely  $(t/\tau)^{0.2}$  (*ii*, orange) and  $(t/\tau)^{0.5}$   
259 (*iii*, red). The dashed line still indicates the linear growth  $n_{lin}(t) = t/\tau$ . In  
260 Section 2, logarithmic solutions like (*i*), qualitatively similar to the one originally  
261 proposed by DeVito (DeVito, 2019) and encountered in this study, have been  
262 found to be in agreement with the E-L equation. Comparing the dashed curve  
263 with the green one, the sublinear character of the logarithmic growth is apparent  
264 although for times  $t \ll \tau$  the two curves are matching. Similarly, in Section 3,  
265 we have shown that power-like styles of growth similar to those exemplified by  
266 (*ii*) and (*iii*) are admissible solution of the E-L equation (see, in particular,  
267 Eq. 17). We note that depending upon the value of exponent  $\beta$ , power-like  
268 sublinear growths can exceed the logarithmic one, as it is indeed the case in  
269 Figure 4 for  $\beta = 0.5$  (*iii*). Both, however, remain strictly sublinear for  $t \geq \tau$   
270 and, *a fortiori*, sub-exponential.

271 It is worth to remark that, in our search of possible solutions to the DeVito's  
272 problem, we have not found examples of self-limiting patterns of growth that  
273 would eventually evolve to a constant value of  $n(t)$ , hence ultimately turning  
274 to sublinear and bounded styles of growth. This is characteristic of the very  
275 well known law in population ecology expressed by the logistic function first  
276 found by Verhulst (Berryman, 2003), and of other qualitatively similar models  
277 encountered in various fields like those of Gompertz (Zwietering et al., 1990),  
278 von Bertalanffy (Fabens et al., 1965), Beverton-Holt (Beverton and Holt, 2012)  
279 or Liquori and Tripiciano Liquori and Tripiciano (1980). All these sigmoidal  
280 growth models are characterized by a horizontal asymptote for long times, hence  
281 they are bounded (for a review, see Buis (2017)). As far as we now, a purely  
282 logarithmic unbounded growth like the one consistent with the DeVito's hypoth-  
283 esis of a minimal number of contacts, has never been proposed in the framework

284 of population dynamics. Indeed, this could be partly due to the limited time  
285 period covered by the observations available (see *e.g.*, Bre), which hinders a  
286 precise assessment of a possible long-term asymptote. However, we note that  
287 Tanaka Tanaka (1982) has proposed a complex growth law of logarithmic nature  
288 to explain the life-lasting development of the size of certain mollusks (see also  
289 Ebert et al. (1999)). Similarly, we are not aware of the existence of theoretical  
290 growth models based on unbounded power laws with exponent less than one,  
291 which according to our results may constitute a solution of the DeVito’s prob-  
292 lem as well. It should be noted, however, that an unlimited growth resembling  
293 a power law has been observed in nature for certain secular trees Buis (2017).

## 294 6. Conclusions

295 Following DeVito’s DeVito (2019) hypothesis of a constant and minimal rate  
296 of detection of communicative societies in the Galaxy, we have studied the gen-  
297 eral style of growth of such societies. Our results confirm that the logarithmic  
298 style of growth already proposed by DeVito (2019) would constitute a viable  
299 solution of the E-L equations. However, in this work, we have shown that a log-  
300 arithmic solution would be also viable starting from more general Lagrangians  
301 DeVito (2019). Furthermore, by exploring the range of possible “simple” so-  
302 lutions of the E-L equations, we have found that styles of growth following a  
303 power law could be also compatible with DeVito’s hypothesis, but only if char-  
304 acterized by an exponent less than one, hence by a decreasing rate of variation.  
305 Such possibility was not previously considered in DeVito (2019). No periodic,  
306 sigmoidal (*i.e.*, logistic) or exponentially diverging solutions seem to be compat-  
307 ible with DeVito’s hypothesis. As proposed in Haqq-Misra and Baum (2009) in  
308 the context of Fermi paradox, these latter would be not sustainable in the long  
309 run.

310 Expanding the main result in DeVito (2019), our work suggests that a possi-  
311 ble resolution of Fermi paradox is the slow, *sublinear growth* of the number of  
312 communicative civilizations in the Galaxy.

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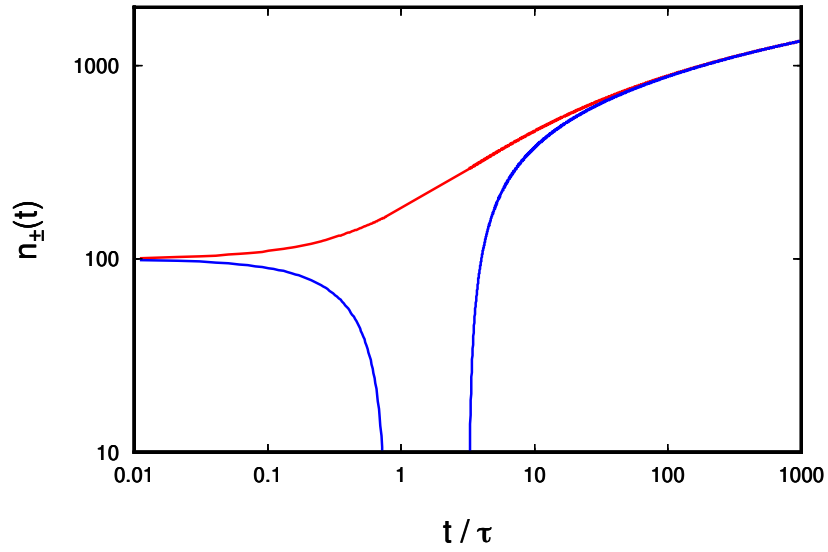


Figure 1: Solutions of the DeVito's problem, given by Eq. (9), for  $n_0 = 100$  and  $\dot{n}_0\tau = 1$ , as a function of the non-dimensional time  $t/\tau$ , in a log-log plot. Red and blue curves correspond to solutions  $n_+(t)$  and  $n_-(t)$ , respectively.

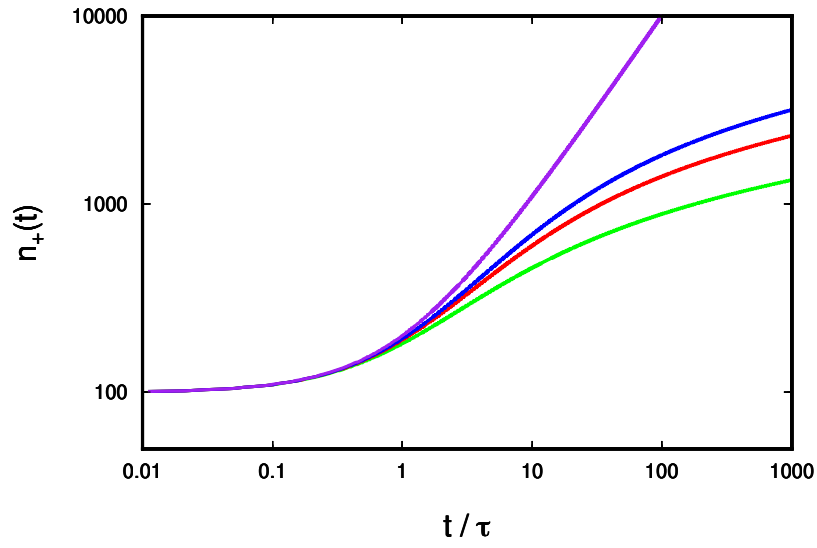


Figure 2: Plots of  $n_+(t)$  according to Eq. (11), for  $n_0 = 100$  and  $\dot{n}_0\tau = 100$ , as a function of  $t/\tau$ , in a log-log plot. Green, red, blue, and purple curves correspond to values  $p = 2, 4, 6$ , and  $p \mapsto \infty$ , respectively.

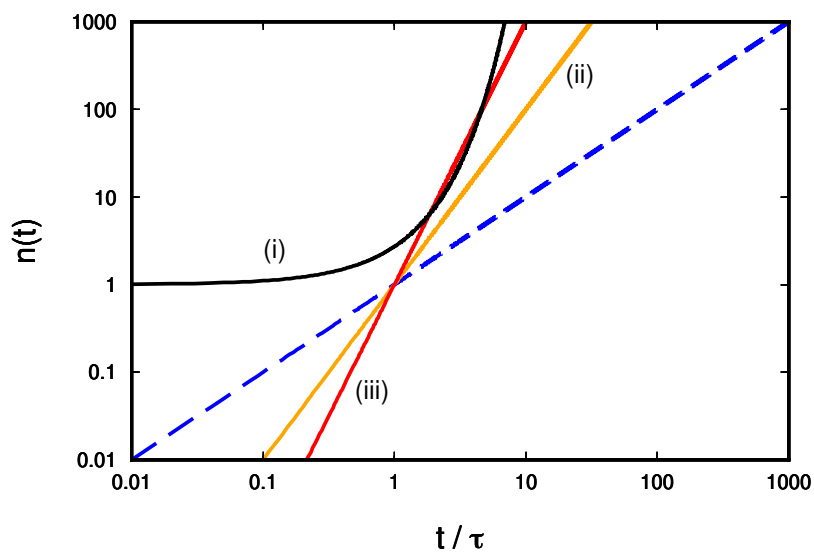


Figure 3: Number of communicative civilizations  $n(t)$  according to a few hypothetical super-linear growth models. These include the positive exponential (*i*, black) and two samples of power-laws  $(t/\tau)^\alpha$  with exponent  $\alpha > 1$ ,  $\alpha = 2$  (*ii*, orange) and  $\alpha = 3$  (*iii*, red). The blue dashed curve shows, for reference, the linear growth. Since we are adopting a log-log scale, the power laws appear as lines with slopes increasing with  $\alpha$ .

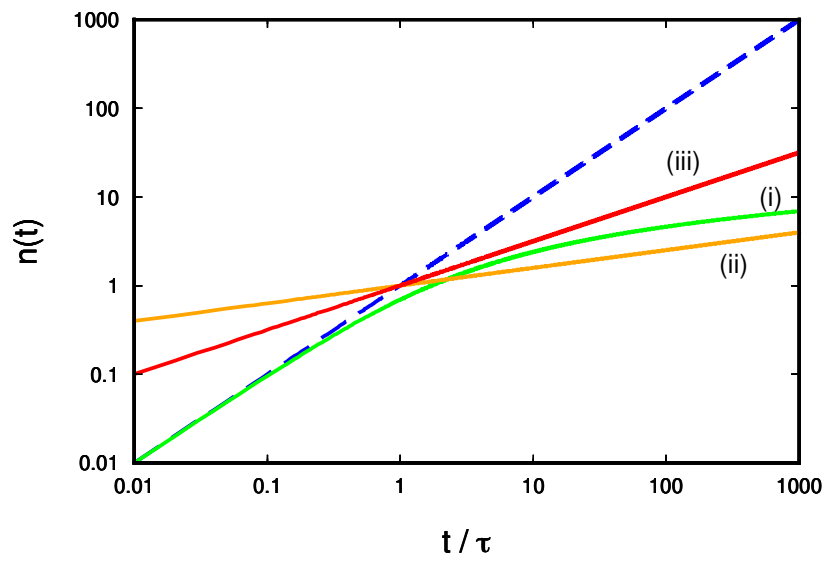


Figure 4: Number of communicative civilizations  $n(t)$  according to various sublinear growth models of interest in this work. These include the logarithmic law (i, green), the power-laws  $(t/\tau)^\beta$  with exponents  $\beta = 0.2$  (ii, orange) and  $\beta = 0.5$  (iii, red). The linear model is shown for reference by a dashed curve. Power laws appear as linear trends in this log-log plot.