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Robust Optimization Models for Integrated Train Stop Planning and Timetabling with Passenger Demand Uncertainty

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Abstract

In this work, we consider the problem of scheduling a set of trains (i.e., determining their departure and arrival times at the visited stations) and simultaneously deciding their stopping patterns (i.e., determining at which stations the trains should stop) with constraints on passenger demand, given as the number of passengers that travel between an origin station and a destination station. In particular, we face the setting in which demand can be uncertain, and propose Mixed Integer Linear Programming (MILP) models to derive *robust* solutions in planning, i.e., several months before operations. These models are based on the technique of Light Robustness, in which uncertainty is handled by inserting a desired *protection* level, and solution efficiency is guaranteed by limiting the worsening of the nominal objective value (i.e., the objective value of the problem in which uncertainty is neglected). In our case, the protection is against a potential increased passenger demand, and the solution efficiency is obtained by limiting the train travel time and the number of train stops. The goal is to determine robust solutions in planning so as to reduce the passenger inconvenience that may occur in real-time due to additional passenger demand. The proposed models differ in the way of inserting the protection, and show different levels of detail on the required information about passenger demand. They are tested on real-life data of the Wuhan-Guangzhou high-speed railway line under different demand scenarios, and the obtained results are compared with those found by solving the nominal problem. The comparison shows that robust solutions can handle uncertain passenger demand in a considerably more effective way.

Keywords: Train Timetabling, Train Stop Planning, Passenger Demand, Robustness, Mixed Integer Linear Program

1. Introduction

Origin-destination passenger demand is commonly utilized to determine train lines and train stopping patterns (i.e., the set of stations at which each train will stop), in order to provide an adequate service to the passengers (Schöbel, 2012). Once the train stops have been decided, the train schedules or timetables are derived, i.e., for every train, the departure and arrival times at every station it visits are determined (Cacchiani and Toth, 2012). Both problems, Train Stop

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Planning and Train Timetabling, are solved at the planning stage, i.e., several months before operations, when the origin-destination passenger demand is often only an estimate of the real demand. The decisions of the train stopping patterns and timetables have to be made before making the schedule public for the passengers, since, based on the available train service, the passengers buy their tickets. Train Stop Planning and Train Timetabling are usually solved in sequence. However, the integration of these problems in a single (more complex) one can allow finding better solutions. As shown in the recent literature, the number of works studying the integration of these two stages is increasing (see e.g., Jiang et al., 2017, Qi et al., 2018b, Yang et al., 2016 and Yue et al., 2016). In order to integrate these two problems, it is also important to handle the passenger demand requirement within the decision process: indeed, the train stop selection is strictly related to the number of passengers who want to travel between an origin station and a destination station. This requirement can be obtained, for example, by imposing a minimum number of trains stopping at each station (Yue et al., 2016), by limiting the number of skipped stops within the train scheduling process (Jiang et al., 2017) or by requiring to satisfy the cumulative passenger demand at each station (Qi et al., 2018b; Yang et al., 2016). In the latter case, constraints on the maximum train capacity are usually imposed, and even the passenger distribution on the trains can be tracked (Qi et al., 2018b).

Passenger demand highly affects train stop plans and timetables, but it varies from day to day, depending on the season, weather conditions, particular events or holidays, etc. The real passenger demand may become known only very close to the day of departure, since passengers are allowed to buy tickets right before the train departs. However, the train stop plans and the timetables must be defined much earlier than the day of departure, so as to make the schedule public and available to the passengers, and to determine the rolling stock circulation and the crew schedules, which are based on the train timetabling solution (Caprara et al., 2007). As a consequence, the fluctuation of passenger demand can lead to overcrowded trains or even unsatisfied demand, causing passenger discomfort, and can make the train transport less attractive. On the other hand, the capacity utilization of the railway network is close to saturation in many countries, and it is thus hard to increase the train frequency. In this context, we consider the integration of Train Timetabling with Stop Planning when passenger demand is uncertain, with the aim of determining *robust* solutions against this uncertainty. Our aim is to determine, at the planning stage, train stopping patterns and timetables that are resistant to the changes of the passenger demand that can occur during operations. Note that adjusting the train stop plans or the timetables during operations is highly undesirable. First of all, any change would significantly affect the passengers, who have already bought their tickets: if a stop plan is changed by skipping some stops, then some passengers might not be able to reach their destination; if, on the contrary, some stops are added, then their travel time would increase. Similarly, a timetable change would require communicating the variation to all passengers who have bought a ticket, and the new schedule might not be convenient for them. Furthermore, a modification of stopping patterns and timetables would strongly influence the rolling stock plan and the crew schedule, that have been determined based on the original timetable: a change in this timetable could require additional train units or a different duty for the crew, making the original plan infeasible. Therefore, we study the integration of train stop planning and timetabling at the planning stage and aim at deriving robust solutions that can cope with uncertain passenger demand. We call this problem Robust Train Stop Planning and Timetabling (RTSPT).

RTSPT and its nominal version are NP-hard, since they generalize the Train Timetabling problem, which calls for determining the arrival and departure times of trains in a railway network, and was proven to be NP-hard in Caprara et al. (2002) by a reduction from the well-known Max-Independent Set Problem.

We underline that the concept of robustness used in RTSPT is different from that of robustness against delays or disruptions, where the goal is to minimize the delay propagation or the number of cancelled train services, and buffer times are inserted in the train schedule to protect against this type of inconvenience (see e.g. Cacchiani and Toth, 2018, Lusby et al., 2018, Goverde, 2007, Bešinović et al., 2016, Sparing and Goverde, 2017). More details are provided in Section 2.1.

RTSPT was introduced in Qi et al. (2018a), where a MILP model, based on Light Robustness (LR), was proposed. LR is a powerful technique, introduced in Fischetti and Monaci (2009), to determine robust solutions: it consists of inserting a desired *protection level* against uncertainty, and using *slack variables* when the protection level cannot be guaranteed; the goal is to minimize the sum of the slack variables in order to obtain the maximum robustness against uncertainty, while limiting, by an additional constraint, the worsening of the objective function value of the nominal problem (i.e., the non-robust problem). Indeed, as usual in robust methods, a good trade-off between robustness and quality of the solution must be achieved. A major advantage of LR is that the complexity of the robust version of the problem remains similar to that of the nominal one, and robust solutions can be obtained in reasonable computing times for real-life instances. Indeed, LR consists of a single stage method, as opposed to two-stage approaches, often used in stochastic programming, that require a set of scenarios to be included in the formulation.

The MILP model proposed in Qi et al. (2018a) requires protection against additional passenger demand defined for each pair of stations, and limits the worsening of the nominal objective value by introducing two additional constraints: one is used to limit the train travel time and the other one to bound the total number of train stops. The model was tested on a real-world instance of the Wuhan-Guangzhou high-speed railway line, and several scenarios were generated to evaluate the robustness of the obtained solution. The results showed that the robust solution reduced the unsatisfied demand by about five times with respect to the nominal case. We will call this model *Demand based Robust Model* (DRM).

In this paper, we propose three new MILP models, all based on LR, but featuring different ways to insert robustness for passenger demand uncertainty. The first model is a new variant of DRM and considers protection on additional passenger demand to be satisfied as much as possible, the second one considers protection as a buffer on the train capacity, while the third one uses a combination of the first two models. Different levels of detail are taken into account for the passenger demand, leading to different solutions and computing times. These models feature new characteristics that were not included in DRM and are useful in practice. In particular, it is possible to limit the changes that can be applied to a given train stop plan, so as to use a stop plan similar to one chosen by the practitioners, but, at the same time, derive robust solutions. In addition, in the latter two models, it is possible to control the distribution of passengers on the trains. More precisely, in one model we reserve some seats for the additional passengers on each train, so that, when a high-demand scenario occurs, the additional passengers can find seats on many trains, i.e., more alternative options are available to accommodate the passengers and transport them to their destinations. In the other model, we limit the number of additional passengers on each train, so as to distribute them on several trains, and derive stop plans featuring additional stops at the most crowded stations, thus making trains less crowded in high-demand scenarios. Therefore, both models aim at determining more balanced robust solutions than DRM.

We observe that, for all three models, the robustness measures (protecting against additional demand, reserving seats for additional passengers on each train, limiting the number of additional passengers on each train) are embedded within the robust models in order to derive robust train stop plans and timetables. Once the proposed robust models have been solved, the railway operator can employ the obtained robust train stop plans and timetables, without any need of applying the robustness measures. Indeed, thanks to the protection inserted within the models, the robust train stop plans and timetables are characterized by the capability of effectively handling uncertain demand scenarios. In particular, since all three robust models aim at transporting additional passengers, the train stop plans they produce are usually characterized by a larger number of trains stopping at the most crowded stations compared to the solutions of the nominal model. This benefit is more evident for the latter two models, since they control the distribution of passengers on the trains, i.e., they avoid that all passengers are allocated on the same train even if the train capacity could accommodate all of them, thus forcing more trains to stop at the most crowded stations in order to transport many passengers. Note that a large number of trains stopping at a station leads to more travel choices available for the passengers traveling to/from that station, and thus to train stop plans and timetables more resilient to demand fluctuations. We observe that we are not requiring Train Operators to control actively the passenger distribution on different trains in practice: thanks to the control of passenger distribution on trains applied *in the models*, the computed robust train stop plans and timetables are characterized by more travel options available to the passengers *in practice*. I.e., it is sufficient that the Train Operators operate the computed robust train stop plans and timetables to benefit of the additional resilience to changes in the passenger demand.

The robust train stop plans and timetables have different features, depending on which of the three models has been solved to derive them, as described above. In particular, the first model allows using an available stop plan, while the two latter models allow deriving more balanced solutions. Therefore, the railway operator can choose which features are more important, and the appropriate model is solved to derive the robust train stop plans and timetables.

The three models are tested on the same data of the Wuhan-Guangzhou high-speed railway line used in Qi et al. (2018a). The obtained results are compared with those of the nominal model, showing that the robust stop plans and timetables can deal with uncertain passenger demand in a much more effective way than the nominal one.

The paper is organized as follows. Section 2 presents a brief overview of related works and highlights our contribution. Section 3 describes the RTSPT problem. In Section 4, we present the MILP models: we first report the nominal model, since it will be used for comparison purposes; then, we describe the proposed robust models. Section 5 shows the obtained computational results, and the paper is concluded with some remarks in Section 6.

2. Literature Overview

In this section, we first present an overview of works from the literature that study Robust Train Timetabling, but employ different robustness measures (Section 2.1). Then, we focus on works that are more closely related to the RTSPT problem in Section 2.2.

2.1. Other Robustness Measures in Timetabling

The problem of Robust Train Scheduling or Timetabling, deeply studied in the literature, considers robustness from a perspective that is different from the one used in RTSPT: in Robust Train Scheduling, robustness is usually against *unexpected train delays* or *disturbances*, such as uncertain travel times or uncertain departure times, that cause deviations from the nominal plan. Approaches are proposed to obtain train timetables that are able to absorb delays and reduce travel time uncertainty. In Robust Train Scheduling, several works propose to include buffer times between trains or take into account train rerouting as a way to reduce delay propagation. In Liebchen and Stiller (2009), a theoretical analysis is provided to explain how buffer times should be distributed, and heuristic algorithms are described to incorporate a certain degree of robustness while ensuring nominal efficiency in the case of periodic timetabling. By introducing buffer times, if trains have enough headway times between each other, a train delay will not impact on the following trains. We refer to Cacchiani and Toth (2018) for a complete overview on Robust Train Scheduling.

We underline that, in RTSPT, robustness has a different aim: indeed, robustness is against *unexpected passenger demand*, i.e., we derive train stop plans and timetables that are able to deal with uncertain passenger demand and thus avoid unsatisfied demand and overcrowded trains as much as possible. In RTSPT, we propose to reserve capacity on the trains in order to be able to transport additional passengers. Therefore, we apply robustness to achieve a new different goal. In the following, we give a brief overview of works that deal with robustness measures used against unexpected train delays or disturbances.

Several works on Robust Train Scheduling propose two-stage approaches. Stochastic optimization is a typical example of a two-stage method, in which the first stage requires to determine a timetable, while the second stage considers a set of uncertainty scenarios, occurring under stochastic disturbances, and applies recourse actions to make the timetable feasible. The goal usually takes into account efficiency and minimization of the expected delay. A seminal work applying stochastic optimization to Robust Train Scheduling with a Periodic Event Scheduling Problem model is Kroon et al. (2008). In Khan and Zhou (2010), a stochastic programming formulation is proposed for dealing with uncertain travel times with the goal of minimizing the total travel time and reducing the expected delay. A decomposition method is developed to reduce the problem complexity, by sequentially solving the problem for individual trains. More recently, in Liu and Dessouky (2019) the integration of freight and passenger train scheduling is considered with uncertainty in freight train departure times in complex railway networks. In this case, train rerouting is also taken into account as a way to reduce deviations, and a two-stage stochastic model is proposed: the first stage aims at optimizing passenger train schedules, while the second stage modifies the passenger train schedules according to the realization of the freight train departure times. A hybrid heuristic algorithm embedded in a branch-and-bound framework is proposed.

There are works that consider methods that combine different phases (e.g., microscopic and macroscopic levels routing and timetabling) to determine timetables robust against delays. In Bešinović et al. (2016), a hierarchical approach, that combines a microscopic model with a macroscopic one, is proposed in order to determine timetables that are robust against delays. This approach iteratively adjusts train travel times and minimum headway times, based on a feedback loop between macroscopic and microscopic levels, until a feasible and stable timetable at the microscopic level has been determined. In particular, in the macroscopic model, several solutions are derived by a randomized multi-start greedy heuristic algorithm that considers the minimization of running, dwell and transfer times and the robustness cost, represented by the weighted sum of unresolved train conflicts and delays on a given set of scenarios. In the microscopic model, train blocking times are computed for detecting potential track conflicts, and, if there are conflicts, new headways and running times are transferred to the macroscopic model. The process is iterated until timetable stability is verified. In Burggraeve and Vansteenwegen (2017), an approach that combines train routing and timetabling is proposed with the aim of optimizing passenger robustness, defined as the total travel time of all passengers in case of frequently occurring small delays. This approach, applied to complex railway stations, consists of two Integer Linear Programming models that are solved in sequence. The first model, used to determine the train routing, has the goal of minimizing the interaction between trains, achieved by minimizing the node usage over the network, and the use of long routes and detours. The second model, used to determine the train timetables at a microscopic level, has the goal of maximizing the buffer times between the trains in order to absorb delays. No feedback loop is executed, but the routing model takes the following timetabling step into account by considering the node usage, since the latter limits the maximum possible buffer time in the corresponding node.

Other two-stage approaches are based on the concept of Recoverable Robustness introduced in Liebchen et al. (2009): it combines robustness with recoverability, i.e., delay management, and uses recovery actions to recover a solution in every likely scenario. More precisely, in Robust Train Scheduling, Recoverable Robustness requires to compute timetables that include extra time to absorb delays, and to consider recovery options that can be applied during operations when delays occur: these recovery options are provided as input. In D'Angelo et al. (2009), Recoverable Robustness is applied to evaluate the effect of robustness on train scheduling in different delay scenarios occurring on a train line.

LR is also applied in Robust Train Scheduling: in contrast to Stochastic Optimization and Recoverable Robustness, LR consists of a single stage approach, and has the advantage that it does not need to include all the uncertainty scenarios in the model. In Fischetti et al. (2009), LR is applied to a Periodic Event Scheduling Problem in order to improve the robustness against delays of an existing timetable while imposing a maximum increase of the solution cost with respect to the nominal one. The robust model requires a certain protection level (buffer time) for the events (departures and arrivals of trains), so that trains travel at given time distance. The protection level is allowed to be violated and the goal is to minimize the total violation. In Liebchen et al. (2010), an extension of a LR method is proposed to compute delay resistant timetables, by taking the expected number of missed connections into account.

All the considered works aim at deriving timetables that are resistant to delays. In RTSPT we propose LR models that, differently from the existing literature, are used to handle scenarios of uncertain passenger demand, instead of train delay scenarios. Therefore, since very different contexts are considered, the models that we propose are also very different from those found in the literature. Furthermore, we include decisions on the train stop plans in our models, which are fundamental when dealing with passenger demand, while they are not considered in works on Robust Train Scheduling.

2.2. Works on Timetabling with Stop Planning and Passenger Distribution

The works that are most closely related to our consist of those that integrate Train Timetabling with Stop Planning or integrate Train Timetabling with Passenger Distribution. In the former case, passenger demand is considered in a cumulative and less detailed way, while, in the latter one, the passenger flow is taken into account more accurately.

2.2.1. Timetabling with Stop Planning

Train Timetabling has been recently combined with Stop Planning in order to perform the selection of train stops while determining the departure and arrival times of trains at stations, with the aim of reducing the train travel time or the passenger travel and waiting times and/or increasing the number of trains scheduled in the network. The option of skipping stops is frequently used to reduce passenger travel times and operating costs, for example by letting express trains avoiding some stops (Niu et al., 2015b). In addition, by stop skipping it can be possible to schedule a larger number of trains (Jiang et al., 2017). Therefore, the integration of the two problems can improve the quality of the obtained timetables.

In Yue et al. (2016), the train stop plan and schedule are determined simultaneously with the goal of maximizing the sum of the train profits, penalized by the stopping times and by the number of stops. Passenger demand is not explicitly considered, but constraints are imposed on the minimum number of trains serving passengers travelling between any pair of stations. Lagrangian relaxation is applied on the constraints for the train stopping time at the stations, and an algorithm based on column-generation is developed to solve the obtained problem. The algorithm is tested on the Beijing-Shanghai high-speed line. The problem of determining train timetables while selecting a subset of stops that can be skipped is studied in Jiang et al. (2017). Two sets of trains are given: the existing trains for which a feasible timetable is available and the additional trains for which a desired timetable is provided. The goal is to maximize the number of additional scheduled trains, while minimizing the changes to the existing timetables. Also in this case, passenger demand is not explicitly considered, but constraints are imposed on the maximum number of stops that can be skipped for each train. A Lagrangian-relaxation based algorithm is developed and tested on real-world instances of the Beijing-Shanghai high-speed line.

In Niu et al. (2015b), predetermined train skip-stop patterns are given and a time-dependent passenger demand is considered for each pair of origin and destination stations. The problem calls for determining the train timetables with the aim of minimizing the passenger waiting time, by respecting the given train skip-stop patterns, and the train capacity that takes into account the arriving and departing passengers at each station. A Non-Linear Programming model is proposed and tested on an instance of the Shanghai-Hangzhou high-speed line of China. In Yang et al. (2016), train stop planning and train scheduling problems are integrated, and the passenger demand is considered as a quantity to be satisfied at each station, while respecting train capacity adjusted according to an estimated loading factor. The goal is to minimize a weighted sum of the total dwelling time at the visited stations and of the total delay time at the origin stations. A MILP model is presented and tested on the Beijing-Shanghai high-speed line.

2.2.2. Timetabling with Passenger Distribution

Several works consider detailed information on passenger demand during the timetabling process, but neglect the integration with train stop planning. The main goal of combining train timetabling with passenger distribution on the trains is to select the departure and arrival times of the trains at the stations so as to minimize the passenger waiting times at stations and their travel times. In this case, passenger demand is usually provided as a time-dependent origin-destination matrix. Most of these works deal with a metro line or a rapid transit service system, in which the passenger waiting time is usually approximated by counting all passengers arriving between the departure of two consecutive trains and considering that each passenger arriving in this interval waits, on average, half of the interval time.

In Barrena et al. (2014a), a rapid train service system is considered, and train timetabling is studied with dynamic passenger demand, with the aim of minimizing the passenger average waiting time at the stations. In particular, the time horizon is discretized, and the passenger demand is given for each pair of stations and time interval. Three formulations, based on passenger flow variables, are proposed. These formulations are solved by a branch-and-cut algorithm, which is tested on a line of the Madrid Metropolitan Railway. The same problem is studied in Barrena et al. (2014b), where an Adaptive Large Neighborhood Search algorithm is proposed to solve larger size instances. Dynamic passenger demand, expressed as time-variant demand ratio for every pair of stations and time interval, is also considered in Yin et al. (2017) with the goal of minimizing both passenger waiting time and energy consumption represented by traction energy consumption and regenerative braking energy. Two MILP models, based on a space-time graph, and a Lagrangian-relaxation heuristic algorithm are presented. A real-world instance of Beijing metro is solved by the proposed method.

A bi-level model is presented in Zhu et al. (2017) for the train timetabling problem with passenger demands, represented by an origin-destination pair and a desired arrival time at destination. The goal is to minimize the total passenger cost, given by the travel time and by penalties due to early/late departures/arrivals and to overcrowded trains. In the bi-level model, the upper level model considers the timetabling part, that consists of determining the train headways at the start terminal, while the lower level model refers to the passengers and minimizes the total passenger cost, while satisfying train capacity constraints. A Genetic Algorithm is proposed to solve the problem, and is tested on an instance of the Yizhuang subway line in Beijing. A metro service system is also considered in Sun et al. (2014) with the aim of reducing passenger waiting time and overcrowding. A uniform distribution of passengers that arrive at a station in a specific time interval is considered, and passenger waiting profiles are given. Three mathematical models, which consider time-dependent demand, and train capacity and peak/off-peak strategy constraints, are proposed, and tested on a Mass Rapid Transit service in Singapore. In Wang et al. (2015), an urban rail transit network is studied, and passenger transfers at stations (i.e., passengers that want to transfer from one line to another one) are taken into account. Since several routes can be selected to reach the destination, passengers can choose different lines: therefore, splitting rates, representing the splitting of passenger flows at transfer stations, are events of an event-driven model proposed in Wang et al. (2015). The goal is to minimize the total travel time and the consumed energy. A Sequential Quadratic Programming approach and a genetic algorithm are developed and tested on a small network. In Yang et al. (2020), a last train timetabling is studied for metro networks, in which the aim is to improve the accessibility of passenger demand in night operations.

Not only urban rail or metro systems are studied in this context, but also railway networks. In particular, in Cordone and Redaelli (2011), the classical Cyclic Train Timetabling Problem is extended to deal with variable passenger demand that depends on the quality of the timetable, aiming at maximizing the total served demand. A Mixed Integer Non-Linear Programming (MINLP) model based on the Cycle Periodicity Formulation combined with a discrete-choice model to handle variable demand is proposed. The former computes a timetable, and the latter is used to derive the demand captured by the new obtained timetable, and a feedback between the two models leads to convergence. The MINLP model is linearized, and a branch-and-bound algorithm is used to solve the problem. The algorithm is tested on real-world instances of a regional network in the North of Italy.

Several works study the situation of overloaded trains. In Niu and Zhou (2013), a congested urban rail corridor with limited train availability is studied. A time-dependent demand is given, and not all passengers can board the desired train but some of them have to wait. An Integer Non-Linear Programming model is presented, which takes into account the number of passengers arriving, departing or waiting at each station in each time interval, and considers train capacity constraints. The objective is to minimize the weighted sum of the number of passengers waiting at a station and of the number of passengers that could not board the train. A Genetic Algorithm is proposed to solve the problem and tested on a real-world instance of the subway line in Guangzhou in China. In Shang et al. (2018), an oversaturated urban rail transit system is considered. In this system, train capacity is a scarce resource and passengers have to wait at the stations because the train is full. The possibility of skipping some stops is used to reach an equitable schedule. The problem is formulated as a multi-commodity flow model, in which passengers correspond to commodities. Lagrangian decomposition is applied to solve the problem, and an instance of the Batong line in Beijing is used for testing the algorithm.

The problem of oversaturated situations is considered also in Li et al. (2017), Shi et al. (2018), Xu et al. (2016) and Liu et al. (2020), where the focus is on improving the passenger flow for the access to platforms in metro lines. In particular, passenger flow control is applied in China to regulate the passenger arrivals and reduce the risk of accidents and the delays caused by increased train dwell times. In Li et al. (2017), the optimization of headway regularity and train commercial speeds is combined with passenger flow control strategies in a dynamic model, solved by a model predictive control algorithm and tested on a line of the Beijing metro. In Shi et al. (2018), an Integer Non-Linear Programming model for the problem of determining the train schedules together with a passenger control strategy is presented and then linearized. The model is decomposed into subproblems, and a heuristic local search algorithm is proposed to solve them. The algorithm is tested on a metro line of Beijing. In Xu et al. (2016), the concept of station service capacity is introduced, by taking into account inbound, outbound and transfer passenger flows, and a model is proposed in the case of uncertain demand. A simulation method that embeds Data Envelopment Analysis (DEA) with a Genetic Algorithm (GA) is presented. In this method, DEA is used to evaluate the quality of the solutions found by the GA.

Demand uncertainty in the train scheduling field is also faced in Yang et al. (2009) and Yin et al. (2016). In Yang et al. (2009), a train timetabling problem with fuzzy passenger demand, representing the number of passengers that board each train, is considered for a railway line. A mathematical model with two objectives, namely the minimization of the total passenger travel time and of the difference between the actual and the minimum train travel time, is presented, and a branch-and-bound algorithm is proposed. In Yin et al. (2016), uncertain demand is represented as a Poisson distribution (instead of a deterministic time-dependent value), and handled in a stochastic programming model for train rescheduling in a metro line. The objective is to minimize the delay, the travel time and the consumed energy. An approximate dynamic programming algorithm is proposed to solve the problem, and tested on an instance of the Yizhuang line in Beijing. Train rescheduling in an overcrowded scenario after a disruption occurred is also studied in Gao et al. (2016). In this case, skipping stops is a method used to minimize the passenger travel time during and after the disruption as well as the number of passengers waiting. In this problem, time-dependent passenger flows are considered, and train capacity constraints are imposed. The problem is formulated as a MILP model, and decomposed into subproblems. A heuristic algorithm is proposed to solve it, and tested on data of the Yizhuang metro line in Beijing.

In Qi et al. (2018b), the integration of the stopping pattern selection with train scheduling and passenger distribution on trains is studied. The passenger demand is given as a matrix of origin-destination pairs, and the primary goal is to minimize the sum of the train travel times, while satisfying the given demand and train capacity, and constraints on running, dwelling and headway times. The secondary goal, that is considered once the train stop plans and timetable are determined, is to minimize the total passenger travel time, which is computed by optimizing the passenger distribution over the different trains. A MILP model is proposed in Qi et al. (2018b) and tested on the Wuhan-Guangzhou high-speed railway line.

2.3. Contribution

As mentioned above, we consider robustness for handling additional passenger demand, while all the works described in Section 2.1 introduce robustness for dealing with delays or disturbances. Therefore, the robustness goal considered in our work differs from that in the literature. For what concerns the works described in Section 2.2, we observe that they consider either train stop planning and timetabling (Section 2.2.1) or passenger demand within the train timetabling process (Section 2.2.2), but there is no integration that includes train stop planning, timetabling and passenger distribution. To the best of our knowledge, only Qi et al. (2018b) studies the problem of integrating the stopping pattern selection with train scheduling and passenger distribution on the trains, but it does not consider demand uncertainty, while we consider the integration of train stop planning with timetabling and passenger distribution in a setting of uncertain passenger demand. The importance of facing demand uncertainty is shown in several papers presented in Section 2.2.2: however, no work, except from our former paper Qi et al. (2018a), considers robustness for handling passenger demand uncertainty in this integrated problem setting. In Qi et al. (2018a), the DRM model was proposed: it inserts protection against additional passenger demand defined for each pair of stations, and limits the increase of travel time and number of train stops. The goal of DRM is to minimize the unsatisfied passenger demand. As shown in Qi et al. (2018a), DRM is able to significantly reduce the unsatisfied demand compared to the nominal model.

In this paper, we propose a new variant of DRM and two novel models, aiming at achieving robust solutions with new characteristics useful in practice. With respect to DRM, the new variant uses a different way of limiting the number of additional stops: in particular, it restricts the changes to the train stop plan based on the nominal stop plan. This allows utilizing a stop plan similar to one chosen by the practitioners, and, at the same time, deriving robust timetables. In addition to this variant, we propose two novel models: in one model, the protection against uncertain demand is required not only for each pair of stations but also for each train. In the other model, the protection is required for each pair of stations as in DRM, but the number of additional passengers on each train is controlled. Both these latter models require to reserve capacity on the trains for handling additional passenger demand, and aim at minimizing the unavailable train capacity, a different goal from the unsatisfied demand minimization used in DRM. Note that the unsatisfied demand represents the number of passengers that cannot be transported, and minimizing it corresponds to minimizing the lost sales. The minimization of unavailable capacity also goes in the direction of minimizing the lost sales, because we aim at transporting additional passengers. However, the difference is that, when minimizing the unavailable capacity, we are minimizing the number of seats, on each train, which cannot be reserved for transporting additional passengers. The usefulness of reserving seats for *each* train, as is done in the first new model, is that these seats are available for the additional passengers on each train, and, thus, more options are provided to the passengers, in a high-demand scenario, to reach their destinations. Similarly, in the second new model, by controlling the number of additional passengers on each train, we avoid that all passengers are distributed without any limit, thus leading to more travel choices available to the passengers for reaching their destinations, and consequently making trains less crowded. On the contrary, in DRM, nominal and additional passengers can be distributed on any train without any limit. Therefore, the solutions of the new models can be more balanced in terms of travel options for the additional passengers and number of additional passengers traveling on each train, useful features in scenarios with high passenger demand.

To summarize, the contribution of this paper is threefold:

- we study the integrated Train Stop Planning and Timetabling Problem with passenger distribution, when passenger demand is uncertain;
- we propose three robust optimization models for the RTSPT problem, all based on Light Robustness: these models differ in the way they manage the uncertainty, and have different levels of detail for dealing with the passenger demand, as explained above;
- we perform computational experiments by using a real-world instance of the Wuhan-Guangzhou high-speed railway line, and compare the quality of robustness achieved under different demand scenarios. In addition, we perform a sensitivity analysis of the parameters used in the proposed models.

3. Problem Description

In this section, we describe the problem by using the same notation as in Qi et al. (2018a). We first describe the problem setting, and then present the *nominal* and *robust* goals.

Let T be the planning horizon (e.g. one day with time discretization in minutes), S the set of stations belonging to the considered railway line, and K the set of trains, travelling in the same direction, to be scheduled. In addition, let Q_{ij} represent the passenger demand between stations i

and j, i.e. the number of passengers who want to travel between i and j ($i, j \in S$, $i \neq j$). Passenger demand Q_{ij} is the nominal demand, i.e. the demand that occurs in the nominal (standard) scenario. This demand has to be fulfilled by appropriately selecting the train stopping patterns and the train departure and arrival times at every visited station. More precisely, for each train $k \in K$, we know the subset $S_k \subseteq S$ of stations that the train visits, which include its fixed origin station O_k and its fixed destination station D_k. For each station $s \in S_k \setminus \{O_k, D_k\}$, we have to decide if train k has to stop or not at s: clearly, if trains stop only at a small subset of the stations, the travel time for the passengers to reach their destinations will be shorter; on the other hand, train service must be guaranteed at every station. To meet these two conflicting requirements, we are given in input, for each train $k \in K$, the maximum number N_k of stops where k can stop along its trip, and for each station $i \in S$ the minimum number R_i of trains that have to stop at i.

One of the requirements for having a good quality of service for the passengers is that every passenger can have a seat on the train: thus, we have to satisfy the passenger demand Q_{ij} ($i, j \in S$, $i \neq j$) and respect the train capacity C_k ($k \in K$). Clearly, the choice of the train stopping patterns depends on the constraints on passenger demand and train capacity. More precisely, we have to keep track of the passengers getting on and off the trains at every station (i.e., the passenger distribution on the trains) to verify the train capacity constraints.

For each train $k \in K$, we are also given its desired departure time T_k from its origin station O_k, and a maximum deviation time ΔT_k that can be used to modify the departure of train k from O_k. In addition, for each $k \in K$, we are given the fixed running time t_{ki}^{run} from station i to the successive station $i+1$ ($i, i+1 \in S_K$), and the minimum dwelling time t_{ki}^{dwell} at each station $i \in S_k$, representing the minimum time that train k needs to dwell at station i, if i is chosen as a stop in its stopping pattern.

In order to avoid train conflicts, minimum headway times are required between any two trains departing from or arriving at the same station along the railway line: we call h_d the minimum departure headway time and h_a the minimum arrival headway time. Finally, since we deal with a single railway line, overtaking can only be performed at stations. All trains travel along the line in the same direction, hence train crossing cannot occur.

Given this problem setting, the goal of the *nominal* problem is the minimization of the sum of the train travel times (Qi et al., 2018b), one of the most frequent objectives in the Train Timetabling Problem. Indeed, one of the key elements that passengers look for is a short travel time to reach their destination. Therefore, train timetables and stop plan must be determined to fulfill the passenger demand and, at the same time, keep the travel time low. However, if the demand exceeds the provided service, e.g. trains become overcrowded or passengers cannot buy a ticket because trains are full, then the passengers will decide to use an alternative transportation mode. To avoid that a solution obtained for the nominal problem turns out to be infeasible or shows poor quality when critical scenarios occur, the RTSPT problem calls for determining train timetables and a stop plan that not only feature short travel times, but are capable of handling unexpected additional passenger demand. This can be achieved by developing LR models for RTSPT, in which a desired protection level is inserted to protect against unexpected uncertain demand scenarios. In particular, we consider this protection alternatively as an additional number of passengers that want to travel between every pair of stations i and j ($i, j \in S$, $i \neq j$) or as a buffer of empty capacity on every train $k \in K$ between every pair of stations i and j ($i, j \in S$, $i \neq j$). The goal of the RTSPT problem is then to minimize the unachievable protection level, i.e. the unsatisfied passenger demand or the unavailable train capacity, respectively. Obviously, the need to transport more passengers can lead to different stop plans and train timetables, since more trains than in the nominal solution might stop at some highly requested stations. Consequently, longer travel times and additional train stops might be needed in crowded scenarios. This could lead to robust solutions characterized by low efficiency, i.e., long travel times and a large number of train stops, that are not appreciated by the passengers. Therefore, the goal of the nominal problem needs to be taken into account also in the RTSPT, in order to obtain a solution that not only is robust but also efficient. To this aim, constraints are added to the RTSPT model on the maximum increase of the total train travel time and/or on the maximum increase of the total number of stops. Both these constraints limit the worsening of the solution efficiency, as it is typically done in LR approaches. Notice that, although the goal of the nominal problem is the minimization of the total train travel times, we further impose in RTSPT a constraint on the maximum number of additional stops, so as to control both these aspects that highly affect the passengers trips and the railway operators costs. Indeed, a plan with more stops can have more risk of delays, since at every stop passengers board on and get off the trains, and the required time is unknown. Furthermore, when the train stops it needs to decelerate and then to accelerate, and this implies a higher energy consumption. In summary, the RTSPT problem calls for deriving train timetables and stopping patterns with minimum unachievable protection against uncertain demand, while satisfying the following constraints:

- the nominal passenger demand must be transported;
- train capacity must be respected by taking into account the passengers that get on/off at stations where the train stops, and the passenger distribution on different trains;
- every train can stop at a given maximum number of stations, and must stop at its origin and destination stations;
- a given minimum number of trains must stop at every station;
- train deviation time at the origin station, as well as train running and dwelling times must be respected;
- overtaking is only allowed at stations;
- a given protection level must be achieved or slack variables (to be minimized in the objective function) must account for it;
- a maximum worsening of the nominal objective value must be guaranteed;
- a maximum number of additional stops with respect to the nominal solution can be inserted.

4. Mathematical Models

We first report, in Section 4.1, the MILP model for the nominal problem, since it contains most of the constraints used also in the robust models, and then present, in Sections 4.2, 4.3 and 4.4, the three new robust models. For each robust model, we highlight the differences with respect to the nominal model and the other robust ones.

4.1. Nominal Model

The MILP model for the nominal problem, proposed in Qi et al. (2018b), is reported here for sake of clarity. It contains four classes of decision variables to account for train timetables, i.e., both departure and arrival times, train ordering for avoiding overtaking outside stations, train stop plans (i.e., selection of the stations where each train has to stop) and passenger distribution over the different trains. To define the train timetables, let t^d_{ki} ($k \in K$, $i \in S_k \setminus \{D_k\}$) and t^a_{ki} ($k \in K$,

$i \in S_k \setminus \{O_k\}$) be non-negative integer variables that represent, respectively, the departure time and the arrival time of train k at station i. In addition, let y_{kli} be binary variables used to establish the order of trains between consecutive stations: in particular, y_{kli} assumes value 1 if train k departs from station i and arrives at the successive station $i+1$ before train l, and 0 otherwise ($k, l \in K$, $k < l$, $i \in S_k \setminus \{D_k\} \cap S_l \setminus \{D_l\},$). Binary variables x_{ki} are introduced for determining the train stop plans: x_{ki} ($k \in K$, $i \in S_k$) assumes value 1 if train k stops at station i, and 0 otherwise. Finally, the passenger distribution is represented by non-negative integer variables q_{ij}^k ($i, j \in S_k$, $i < j$, $k \in K$): q_{ij}^k represents the number of passengers travelling on train k from station i to station j.

The MILP model for the nominal problem reads as follows:

$$min\ TT = \sum_{k \in K} (t_{kD_k}^a - t_{kO_k}^d) \tag{1}$$

$s.t.$

$$x_{kO_k} = x_{kD_k} = 1,\ k \in K \tag{2}$$

$$\sum_{k \in K: i,j \in S_k} q_{ij}^k = Q_{ij},\ i,j \in S, i < j \tag{3}$$

$$\sum_{j \in S_k, i<j} q_{ij}^k \leq \sum_{j \in S_k, i<j} Q_{ij} x_{ki},\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{4}$$

$$\sum_{j \in S_k, i>j} q_{ji}^k \leq \sum_{j \in S_k, i>j} Q_{ji} x_{ki},\ k \in K,\ i \in S_k \setminus \{O_k\} \tag{5}$$

$$\sum_{i' \in S_k, i' \leq i} \sum_{j \in S_k, i<j} q_{i'j}^k \leq C_k,\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{6}$$

$$\sum_{i \in S_k} x_{ki} \leq N_k,\ k \in K \tag{7}$$

$$\sum_{k \in K: i \in S_k} x_{ki} \geq R_i,\ i \in S \tag{8}$$

$$T_k \leq t_{kO_k}^d \leq T_k + \Delta T_k,\ k \in K \tag{9}$$

$$t_{ki}^d - t_{ki}^a \geq t_{ki}^{dwell} x_{ki},\ k \in K,\ i \in S_k \setminus \{O_k, D_k\} \tag{10}$$

$$t_{ki+1}^a - t_{ki}^d = t_{ki}^{run},\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{11}$$

$$t_{ki}^d + h_d \leq t_{li}^d + M(1 - y_{kli}),\ i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}),\ k,l \in K, k < l \tag{12}$$

$$t_{li}^d + h_d \leq t_{ki}^d + M y_{kli},\ i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}),\ k,l \in K, k < l \tag{13}$$

$$t_{ki+1}^a + h_a \leq t_{li+1}^a + M(1 - y_{kli}),\ i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}),\ k,l \in K, k < l \tag{14}$$

$$t_{li+1}^a + h_a \leq t_{ki+1}^a + M y_{kli}, i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}), k,l \in K\, k < l \tag{15}$$

$$t_{ki}^d \geq 0,\ integer,\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{16}$$

$$t_{ki}^a \geq 0,\ integer,\ k \in K,\ i \in S_k \setminus \{O_k\} \tag{17}$$

$$x_{ki} \in \{0, 1\},\ k \in K,\ i \in S_k \tag{18}$$

$$q_{ij}^k \geq 0,\ integer,\ k \in K,\ i,j \in S_k, i < j \tag{19}$$

$$y_{kli} \in \{0, 1\},\ i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\}),\ k,l \in K, k < l. \tag{20}$$

The nominal objective function (1) calls for the minimization of the sum of the train travel times. In the following, we define this sum as TT (Travel Time). By imposing constraints (2) it is ensured that every train stops at its origin and destination stations (i.e., all trains are scheduled and the train operating zone cannot be changed). Constraints (3) guarantee that the total passenger demand is transported between every pair of stations: this demand corresponds to the number of passengers travelling in the nominal (standard) scenario.

Constraints (4) and (5) consider the passenger flow between any two stations i and j. Constraints (4) establish that, given a train k and a station i, if k does not stop at i, then no passenger can depart from i by using k. On the contrary, if k stops at i, the maximum number of passengers that can depart from i on train k is bounded by the total passenger demand from station i towards any other station j. Constraints (5) impose the same requirements, given a train k and a station i, on the passenger arrivals at i by using train k.

By constraints (6) it is ensured that the train capacity C_k is respected for every train $k \in K$: this is guaranteed by limiting the number of passengers on train k at any station i between i' ($i' \leq i$) and j ($i < j$) to be at most C_k. In particular, constraints (6) consider all passengers that boarded the train before or at station i and will get off after i. With constraints (7) and (8), two conflicting requirements are satisfied: the maximum number of stops for each train, and the minimum number of trains stopping at each station are restricted to N_k and R_i, respectively.

Constraints (9)-(11) are used to impose the requirements on the feasibility of the timetable for train k: in particular, the departure time from the origin station can be moved by at most ΔT_k (constraints (9)), the dwelling time at each station i where k stops must be at least t_{ki}^{dwell} (constraints (10)), and the running time of k from station i to the successive station $i+1$ must be t_{ki}^{run} (constraints (11)).

Constraints (12)-(15) are used to avoid train conflicts along the line. In particular, the order of trains k and l between station i and the successive station $i+1$ is determined by variables y_{kli}: if k departs from i before l ($y_{kli} = 1$), then, by constraints (12), the departure time of l from i must be at least h_d time units (minimum departure headway) after the departure time of k, while constraints (13) are inactive. The opposite occurs if l departs from i before k ($y_{kli} = 0$). Similarly, constraints (14)-(15) are imposed for the minimum arrival headway time. Finally, constraints (16)-(20) define the variable domains.

Constraints (2), (4)-(20) will be used in all the LR models proposed in the following sections. Indeed, they define the requirements for the train stop plans, timetables and passenger distribution. Constraints (3) will be replaced by new ones to deal with uncertain passenger demand in the LR models. The objective function (1) will be replaced by the minimization of the unachievable protection level against uncertain demand, and a constraint will be added to all the LR models to limit the TT worsening, where parameter α will be used to control the allowed percentage of worsening. In the following sections, we will call Δ the protection level that we desire to achieve and γ the slack variables: different indexes and meanings are associated with Δ and γ according to the considered LR model.

4.2. Nominal-Plan based Robust Model

The first model that we propose for RTSPT is a new variant of the DRM, presented in Qi et al. (2018a). We call this model *Nominal-Plan based Robust Model* (NPRM), since its peculiarity is the requirement of deriving a solution that is not too different from the nominal one in terms of the train stop plan. The motivation is that it can be useful in practice to increase the robustness of a train stop plan without building it from scratch. In this case, a limit is imposed on the number of changes that can be applied to the existing nominal plan.

In NPRM and DRM, the desired protection level is introduced for each pair of stations along the line: we define Δ_{ij} ($i, j \in S$, $i < j$) as the number of additional passengers that might need to be transported from station i to station j. To maintain the feasibility of NPRM (and DRM) when the protection level cannot be achieved, we introduce integer slack variables γ_{ij} ($i, j \in S$, $i < j$), that will assume the value of the number of passengers that cannot be transported between stations

i and j. Consequently, the sum of all slack variables corresponds to the Unsatisfied Demand (UD). The goal of both NPRM and DRM is to minimize UD, and both models impose a limit on the maximum worsening of the nominal objective value TT.

In addition, in DRM, a limit was imposed on the number of additional stops used in the train stop plan with respect to the nominal plan, by the following constraint:

$$\sum_{k \in K} \sum_{i \in S_k} x_{ki} \leq (1+\beta) NS^*, \tag{21}$$

where NS^* represents the total number of stops in the nominal solution, and β is a parameter to control the allowed increase over NS^*. However, although the number of additional stops is bounded, the train stop plan obtained by solving DRM can be very different from the nominal one, as many stops can be removed and/or replaced by other ones. In NPRM, constraint (21) is replaced by an alternative one, shown below, that limits the number of changes with respect to the nominal plan. To this aim, we define:

- x^*: the nominal stop plan, where x^*_{ki} has value 1 if train k stops at station i;
- $S_k^{stop} \subseteq S_k$: the subset of stations at which train k stops in the nominal stop plan ($x^*_{ki} = 1$ for every station $i \in S_k^{stop}$ of train k);
- N^{change}: the maximum number of changes allowed with respect to the nominal stop plan.

The proposed NPRM reads as follows:

$$min\, UD = \sum_{i,j \in S,\ i<j} \gamma_{ij} \tag{22}$$

$s.t.$

constraints (2), (4) − (20)

$$\sum_{k \in K: i,j \in S_k} q^k_{ij} \geq Q_{ij},\ \ i,j \in S, i<j \tag{23}$$

$$\sum_{k \in K: i,j \in S_k} q^k_{ij} + \gamma_{ij} = Q_{ij} + \Delta_{ij},\ \ i,j \in S, i<j \tag{24}$$

$$\sum_{k \in K} (t^a_{kD_k} - t^d_{kO_k}) \leq (1+\alpha) TT^* \tag{25}$$

$$\sum_{k \in K} \sum_{i \in S_k^{stop}} (x^*_{ki} - x_{ki}) + \sum_{k \in K} \sum_{i \in S_k \setminus S_k^{stop}} x_{ki} \leq N^{change} \tag{26}$$

$$\gamma_{ij} \geq 0,\ integer,\ i,j \in S, i<j. \tag{27}$$

The objective function (22) corresponds to the minimization of the unsatisfied demand, i.e., of the sum of the γ variables that are activated by constraints (24). Constraints (2), (4)-(20) are inherited by the nominal model, while constraints (23) replace (3), i.e., we require that the nominal demand is satisfied but we also allow for transporting additional passengers. Constraints (24) are used to insert the protection level Δ_{ij}, representing the number of passengers, in addition to the nominal demand Q_{ij}, that might want to travel between stations i and j. In these constraints, the slack variables γ_{ij} are used to keep the feasibility when the protection level cannot be reached. To guarantee that the nominal goal, i.e., the minimization of the total travel time, is also taken into account, constraint (25) requires a maximum worsening controlled by parameter α with respect to the nominal total travel time TT^*. Constraint (26) is used to restrict the changes to the nominal stop plan: the number of deleted stops summed with the number of new stops must be at most N^{change}. This constraint is used instead of (21) adopted in DRM. Finally, the slack variables are required to be non-negative integer by constraints (27).

The main advantage of NPRM is that, by constraint (26), the number of changes to the nominal stop plan is kept limited. Therefore, this model can be used if practitioners want to modify the existing plan as little as possible, while allowing an improvement of its robustness. As will be shown in the computational results (Section 5), NPRM can provide a level of robustness similar to that of DRM but with much fewer changes to the nominal stop plan.

4.3. Train-Capacity based Robust Model

In NPRM, we have inserted the protection against additional demand on each pair of stations $i, j \in S$ $(i < j)$. Therefore, the passengers can be distributed on any train, provided that the train stops at the origin and destination stations of the passengers. However, this could lead to solutions in which some trains are completely used to satisfy the nominal passenger demand. Although unbalanced solutions can happen also in the nominal case, the issue becomes more relevant in scenarios with increased passenger demand, since it is important to distribute passengers among trains to guarantee more alternative options to transport passengers to their destinations.

In this model, called Train-Capacity based Robust Model (TCRM), we introduce this type of robustness that is also associated with the specific train. We introduce a protection level Δ_{ijk} for every pair of stations $i, j \in S$ $(i < j)$ and train $k \in K$: it represents the additional number of passengers that might need to be transported between i and j on train k. The protection Δ_{ijk} can also be seen as a buffer on the train capacity, i.e., it is the empty capacity that, after satisfying the nominal demand Q_{ij}, we want to keep on each train. This buffer could be used to satisfy additional demand that occurs in critical scenarios. In other words, when determining the stop plan and train timetables, we satisfy the nominal passenger demand and, at the same time, keep, on each train, a number of empty seats, so that additional passengers can use this train, in scenarios of increased demand. We note that Δ_{ijk} can be chosen to assume the same value for every train $k \in K$ or different values for different trains. In the former case, reserving some capacity on each train allows to distribute passengers in a more uniform way, while in the latter case some trains that are expected to be more crowded than other ones can be given more protection. More precisely, Δ_{ijk} values can be used to establish that, on each train $k \in K$, some capacity must be reserved for the additional passengers, i.e., we cannot have a train that is completely used by the nominal passengers because some seats are reserved for the additional passengers. In this way, in planning, we require that every train has some available seats, and, thus, limit the risk that, when the scenario occurs, that passengers cannot board some trains. Clearly, if some trains are expected to be more crowded than other ones, i.e., Q_{ij}^k $(k \in K)$ values were known, TCRM can be used to assign different protection levels Δ_{ijk} to different trains.

To maintain the feasibility of TCRM we introduce slack variables γ_k, assuming the value of the missing buffer on the capacity of train k $(k \in K)$. Note that the values assumed by the γ_k variables after solving TCRM can provide to the railway company useful information on the rolling stock types or coupling/decoupling of train units needed to perform the considered train services, in order to guarantee a certain level of robustness.

The goal of TCRM is to minimize the sum of the γ_k variables, i.e., the Unavailable Capacity (UC). In addition, we impose a limit of the worsening of the nominal objective value and a limit on the number of additional stops that can be used in the train stop plan with respect to the nominal one. These limits are controlled, respectively, by parameters α and β. The proposed TCRM reads as follows:

$$min\ UC = \sum_{k \in K} \gamma_k \tag{28}$$

$s.t.$

constraints (2), (4) − (20)

constraints (21), (23), (25)

$$\sum_{i' \in S_k, i' \leq i} \sum_{j \in S_k, i<j} (q^k_{i'j} + \Delta_{i'jk}) \leq C_k + \gamma_k,\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{29}$$

$$\gamma_k \geq 0,\ integer,\ k \in K. \tag{30}$$

The objective function (28) asks for minimizing the unavailable train capacity. Constraints (2), (4)-(20) are in common with the nominal model. Constraint (21) is the same bound used in DRM to limit the increase of the number of stops in the train stop plan. As in NPRM, constraints (23) replace the corresponding constraints (3), i.e., we impose to satisfy the nominal passenger demand and possibly transport additional passengers. In addition, we ensure the quality of the nominal objective value with constraint (25). Constraints (29) are used to insert the desired protection level Δ_{ijk} between stations i and j on train k. In particular, given a train k and a station i, we consider all passengers $q^k_{i'j}$ that are on train k when travelling from a station i' before (or corresponding to) i towards a successive station j, and require that at least Δ_{ijk} seats are left empty on the train. I.e., either the sum of passengers $q^k_{i'j}$ and the empty seats Δ_{ijk} respects the train capacity C_k, or variable γ_k will take the value of the missing buffer.

We observe that, with respect to NPRM, TCRM requires to specify the protection for every train, thus allowing to reserve capacity for the additional passengers in a more balanced way. In addition, we note that constraint (21) can be replaced by (26). We first consider the model with constraint (21), to have more flexibility, since the requirement of the protection level (for each pair of stations and train) is more restrictive in TCRM than in NPRM. The variant of TCRM that includes constraint (26) in place of (21) is analyzed in Section 5.3.

4.4. Passenger-Distribution based Robust Model

The last model that we propose combines the advantages of NPRM and TCRM: roughly speaking, it requires the protection level only between every pair of stations as in NPRM, but, at the same time, controls the passenger distribution on the trains, i.e., it limits the number of additional passengers on each train. Thus, we call this model Passenger-Distribution based Robust Model (PDRM). To formulate PDRM, we define:

- Δ_{ij}: as in NPRM, it is the required protection level between stations i and j ($i, j \in S$, $i < j$), representing the number of additional passengers that might need to be transported between i and j;
- γ_k: as in TCRM, these variables are used to keep the model feasible, and their sum, corresponding to the unavailable capacity, is minimized in the objective function;
- p^k_{ij}: they are non-negative integer decision variables that represent the additional number of passengers travelling on train k from station i to station j. These variables are similar to the q^k_{ij} ones used in the nominal model, but they correspond to the additional demand that

might need to be satisfied. We require that this additional demand equals the protection level Δ_{ij}. Moreover, we require that either the passenger flow q^k_{ij} summed with the additional passenger flow p^k_{ij} respects the train capacity C_k or, if not, γ_k assumes the value of the unavailable capacity;

- H_k: it represents the maximum number of additional passengers that we allow to be transported on train k. These parameters are used to control the passenger distribution on the trains with the aim of obtaining a balanced load for all trains.

The proposed PDRM reads as follows:

$$min\, UC = \sum_{k \in K} \gamma_k \tag{31}$$

$$s.t.$$

$$\text{constraints} \qquad (2) - (20)$$

$$\text{constraints} \qquad (21), (25)$$

$$\sum_{k \in K} p^k_{ij} = \Delta_{ij}, \qquad i, j \in S,\ i < j \tag{32}$$

$$\sum_{i' \in S_k, i' \leq i} \sum_{j \in S_k, i < j} (q^k_{i'j} + p^k_{i'j}) \leq C_k + \gamma_k,\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{33}$$

$$\sum_{i' \in S_k, i' \leq i} \sum_{j \in S_k, i < j} p^k_{i'j} \leq H_k, \qquad k \in K, i \in S_k \setminus \{D_k\} \tag{34}$$

$$\sum_{j \in S_k, i < j} p^k_{ij} \leq \sum_{j \in S_k, i < j} Q_{ij} x_{ki},\ k \in K,\ i \in S_k \setminus \{D_k\} \tag{35}$$

$$\sum_{j \in S_k, i > j} p^k_{ji} \leq \sum_{j \in S_k, i > j} Q_{ji} x_{ki},\ k \in K,\ i \in S_k \setminus \{O_k\} \tag{36}$$

$$\text{constraints} \qquad (30)$$

$$p^k_{ij} \geq 0,\ integer,\ k \in K,\ i, j \in S_k, i < j. \tag{37}$$

The objective function asks for minimizing the unavailable capacity, i.e. it is the same objective used in TCRM. Constraints (2)-(20) are those of the nominal model: we observe that, differently from the other robust models, we here require, with constraints (3), to satisfy the nominal passenger demand with equality constraints, as in the nominal model. Indeed, the additional passenger demand is handled by variables p^k_{ij} in constraints (32)-(36). Constraints (21) and (25) limit, respectively, the number of additional stops and the increase of the total travel time through parameters β and α, as in TCRM. With constraints (32) we insert the desired protection against uncertain demand between stations i and j: the slack variables are not inserted in these constraints, but in the following ones, i.e., constraints (33), that impose to respect the train capacity. In this way, we can keep track of the additional passenger flow on each train, but do not need to specify the protection level for each train. The additional passenger flow is then limited in the following constraints (34) by parameter H_k, which specifies the accepted limit for each train $k \in K$. Similar to constraints (4) and (5), constraints (35) and (36) are used to avoid that passengers depart from or arrive at, respectively, station i if the train does not stop at i. Finally, constraints (30) and (37) define the domain of variables γ_k and p^k_{ij}.

The PDRM has the advantage of allowing to control the additional passenger distribution on each train, while inserting the desired protection directly on the demand between pairs of stations.

This can be seen as a compromise between NPRM and TCRM, since PDRM only requires protection for each pair of stations but also limits the number of additional passengers distributed on each train. As in TCRM, also in PDRM, we first use constraint (21) instead of (26) to give more freedom to the problem solution. The variant of PDRM that includes constraint (26) in place of (21) is analyzed in Section 5.3.

4.5. Comparison of the Models

This section compares the proposed robust models by considering the types and number of variables, and the number of constraints involved in each model to ensure robustness, the model objective function, and the additional constraints required to guarantee efficiency with respect to the nominal problem. Since all the proposed robust models contain the nominal variables and constraints, we first report, in Table 1, the corresponding numbers, and then indicate, in Table 2, the number of additional variables and constraints used by each robust model. We include in this comparison model DRM, presented in Qi et al. (2018a).

Table 1 reports the sets of variables (Var.) used in the nominal model and the corresponding number (#), as well as the sets of constraints (Constr.) contained in the nominal model and the corresponding number (#).

Var. or Constr.	Total number #
Positive integer variable t^d_{ki}	$\sum_{k\in K}(\vert S_k\vert - 1)$
Positive integer variable t^a_{ki}	$\sum_{k\in K}(\vert S_k\vert - 1)$
Positive integer variable q^k_{ij}	$\sum_{k\in K}\vert S_k\vert * (\vert S_k\vert - 1)/2$
Binary variable x_{ki}	$\sum_{k\in K}\vert S_k\vert$
Binary variable y_{kli}	$\sum_{k,l\in K,k<l}(\vert(S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\})\vert)$
Constraints (2)	$2 * \vert K\vert$
Constraints (3)	$\vert S\vert * (\vert S\vert - 1)/2$
Constraints (4)-(6)	$3 * \sum_{k\in K}(\vert S_k\vert - 1)$
Constraints (7)	$\vert K\vert$
Constraints (8)	$\vert S\vert$
Constraints (9)	$2 * \vert K\vert$
Constraints (10)	$\sum_{k\in K}(\vert S_k\vert - 2)$
Constraints (11)	$\sum_{k\in K}(\vert S_k\vert - 1)$
Constraints (12)-(15)	$4 * \sum_{k,l\in K,k<l}(\vert(S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\})\vert)$

Table 1: Number of variables and constraints of the nominal model.

The nominal model contains variables t^d_{ki} and t^a_{ki} to determine the departure and arrival times of each train at each station, x_{ki} to decide the train stops, q^k_{ij} to define the passenger distribution on each train between each pair of stations, and y_{kli} to establish the order of trains between consecutive stations. The nominal model contains constraints (2)-(6) to control the train operating zone and the passenger distribution, (7)-(8) to limit the maximum number of stops per train and to guarantee a minimum number of trains stopping at each station, (9)-(11) to guarantee the feasibility of the timetable for each single train in terms of departure, running and dwelling times, and (12)-(15) to avoid train conflicts.

All the variables and constraints indicated in Table 1 are included in the robust models, with the exception of constraints (3), which are replaced by (23) in DRM, NPRM and TCRM. We report, in Table 2, the name of the model, its objective function identified by UD if the model aims at minimizing the unsatisfied demand or UC if it aims at minimizing the unavailable capacity, the required protection (Prot.) against uncertain demand, the set of additional variables required to ensure robustness (Robust Var.), their number, the set of additional constraints used to ensure robustness (Robust Constr.), the corresponding number, the set of additional constraints used to guarantee efficiency (Eff. Constr.), i.e., to limit the worsening of the nominal objective value in the robust solution, and the corresponding number.

Model	Obj.	Prot.	Robust Var.	Total number #
DRM	UD	Δ_{ij}	γ_{ij}	$\lvert S\rvert * (\lvert S\rvert - 1)/2$
NPRM	UD	Δ_{ij}	γ_{ij}	$\lvert S\rvert * (\lvert S\rvert - 1)/2$
TCRM	UC	Δ_{ijk}	γ_k	$\lvert K\rvert$
PDRM	UC	Δ_{ij}	γ_k, p_{ij}^k	$\lvert K\rvert + \sum_{k\in K} \lvert S_k\rvert * (\lvert S_k - 1\rvert)/2$
Model	Robust Constr.	Total number #	Eff. Constr.	Total number #
DRM	(23)-(24)	$\lvert S\rvert * (\lvert S\rvert - 1)$	(21), (25)	2
NPRM	(23)-(24)	$\lvert S\rvert * (\lvert S\rvert - 1)$	(25), (26)	2
TCRM	(23),(29)	$\lvert S\rvert * (\lvert S\rvert - 1)/2 + \sum_{k\in K}(\lvert S_k\rvert - 1)$	(21), (25)	2
PDRM	(32)-(36)	$\lvert S\rvert * (\lvert S\rvert - 1)/2 + 4 * \sum_{k\in K}(\lvert S_k\rvert - 1)$	(21), (25)	2

Table 2: Number of additional variables and constraints of the robust models.

We note that DRM and NPRM differ only for constraint (26) that replaces constraint (21) in NPRM. By comparing NPRM and TCRM, we observe that the number of robust variables in NPRM depends on the number of stations while that of TCRM on the number of trains: this is clearly due to the two different ways used for achieving robustness in these models, i.e., either by requiring to transport additional passengers, as in NPRM, or by reserving capacity on the trains, as in TCRM. Except for the $|S| * (|S| - 1)/2$ constraints (23), that appear both in NPRM and TCRM, the number of other constraints is $\sum_{k\in K}(|S_k| - 1)$ in TCRM, while it is $|S| * (|S| - 1)/2$ in NPRM: indeed, in TCRM, the protection against uncertain demand is imposed by requiring an empty buffer on train capacity for each train and each pair of stations, while, in NPRM, the protection is imposed by requiring to transport additional passengers between each pair of stations. PDRM has the largest size, as it contains more variables and constraints than each of the other two models: indeed, this model combines the protection enforced for each pair of stations, leading to $|S| * (|S| - 1)/2$ constraints, with the control of additional passenger distribution, corresponding to $4 * \sum_{k\in K}(|S_k| - 1)$ constraints. As will be shown in Sections 5.1 and 5.2, despite its largest size, PDRM has a very good performance.

We conclude this section by summarizing advantages and disadvantages of the proposed models. An advantage of DRM and NPRM is that they have the smallest size, since usually in real-life instances the number of stations is smaller than the number of trains. In addition, NPRM contains constraint (26) that allows controlling the number of changes applied to the nominal stop plan. Note that, even though the stop plan can be changed, as we work at a planning stage, practitioners often prefer to limit the changes with respect to a stop plan used, for example, in the previous year, because they are used to it. Another characteristic of NPRM is that the protection Δ_{ij} against uncertain demand is imposed for every pair of stations i and j. This can be seen as an advantage, because it is enough to specify aggregated data on the additional passenger demand between origin and destination stations. However, the drawback is that passengers can be distributed on any train without any control. This drawback is overcome by TCRM, that imposes protection Δ_{ijk} for every pair of stations i and j and every train k, so that in each train a certain number of seats is reserved for the additional passengers. In addition, TCRM can also require different protection levels for different trains. A disadvantage of TCRM is that it requires longer computing times than NPRM, as will be shown in Section 5.1: indeed, TCRM requires to satisfy, through constrains (29), the required protection for every train. PDRM tries to overcome the drawbacks of NPRM and TCRM. It applies the required protection as in NPRM: therefore, it suffices to define the protection between pairs of stations. However, to avoid the distribution of passengers on any train without any limit, and, at the same time, the higher complexity of TCRM, PDRM controls, by using variables p_{ij}^k, the distribution of additional passengers on the trains, and limits their number to be at most H_k for each train $k \in K$.

As summarized in Table 2, the proposed robust models have different objectives and structures. In particular, DRM and NPRM aim at minimizing the unsatisfied demand, while TCRM and PDRM have the goal of minimizing the unavailable capacity. For this reason, we will computationally compare only solutions of models that share the same goal. Moreover, we will compare each robust solution with the nominal solution by evaluating their behavior with a set of scenarios of uncertain demand, in order to assess the robustness of the computed train stop plans and timetables. These computational comparisons are reported in Section 5.

4.6. Extensions to the Models

We present some additional constraints and features that can be included in the proposed models and are left as future work. The first extension is to consider station capacity constraints, i.e., limit the number of trains that can be simultaneously presented at a station. These constraints can be modelled in several ways (see e.g., Yang et al., 2014, Yue et al., 2016, Jiang et al., 2017, Gao et al., 2018). To express these constraints in the proposed models, additional variables would be required in order to count, for every station i and time instant, the number of trains that are using one of the tracks available at i.

Another extension consists in explicitly considering acceleration and deceleration times that have to be added to the train travel time between two consecutive stations when the train stops at these stations. Acceleration and deceleration times are often assumed to be directly included in the travel times. However, in some works, they are explicitly handled (see e.g., Jiang et al., 2017, Gao et al., 2018). In the proposed models, constraints (11) use a fixed travel time t_{ki}^{run} for train k between consecutive stations i and $i+1$. However, to include acceleration and deceleration times, we can add an acceleration time after k leaves i if the train stops at station i (i.e., $x_{ki} = 1$), and, a deceleration time before k arrives at $i+1$ if the train also stops at station $i+1$ (i.e., $x_{ki+1} = 1$).

Another generalization of the models would be to consider passenger transfers, i.e., the possibility of passengers to travel from their origin to their destination by using different trains. Note that, even though we consider a single line, passenger transfers could be useful for passengers to reach their destination faster. Consider the example shown in Figure 1 with a line with five stations A, B, C, D and E, and suppose that a passenger wants to travel from B to E. Consider two trains travelling on this line: train 1 goes from A to E and stops only in C, while train 2 goes from A to E and stops in every station, so that train 1 is faster than train 2. A passenger can decide to travel from B to E by using train 2, which takes a longer time, or to travel from B to C on train 2 and then from C to E on train 1, thus having overall a shorter travel time, if the waiting time in C is not too long. Another reason for considering transfers is that train 2 could be full of passengers between C and E, and thus the only way for the passenger to reach his/her destination would be to make a transfer in C and take train 1. Although this generalization would allow considering these additional possibilities for the passengers, we expect that, since we deal with a single line, transfers would not be frequently chosen: indeed, changing train implies waiting at the transfer station, and potential delays could cause missing the connection. Thus, we decided not to incorporate these constraints to keep the models more tractable. Indeed, this generalization would require including in the models information on the route followed by the passengers and on the transfer possibilities (see e.g., Niu et al., 2015a, Wang et al., 2015).

The described generalizations are characterized by different levels of complexity. The first one requires several additional variables and "big-M" constraints. In particular, new binary variables

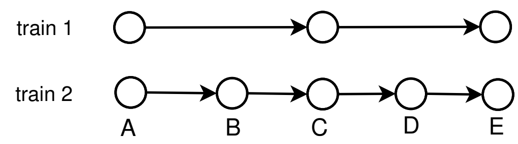

Figure 1: Example about passenger transfer on a single line with five stations.

should determine if train k arrives at station i before or at time r, and other ones if train k departs from station i after or at time r, and these variables should be linked by new "big-M" constraints with the t^a_{ki} and t^d_{ki} variables. Therefore, this generalization is not straightforward from a computational point of view. On the contrary, the second extension could be obtained without increasing the number of variables and constraints in the model, but simply modifying constraints (11) by adding acceleration and/or deceleration (constant) times multiplied by x_{ki} variables. Thus, the model would not become more difficult to solve. Finally, the last extension is the most complex one: in order to explicitly route the passengers through the network and take the transfer possibilities into account, additional flow variables should be introduced for all pairs of stations, with a consequent notable increase of the model size. Therefore, this extension could require different solution methods.

What's more, in the formulation of train timetable problem, the space-time network representation method has been applied in many literatures (Zhang et al., 2019 and Zhang et al., 2020) since this formulation can transform the timetabling problem into a routing selection problem with selecting different paths for different trains. Thus, our proposed models also can be extended by using the time-space network representation method and also taking the choice behavior of passengers into consideration.

5. Computational Results

In this section, we report the computational results obtained with the proposed models. In Section 5.1, we compare the performances of the proposed robust models, by considering the unsatisfied demand or unavailable capacity, optimality gap, computing time, and values of the total travel time and number of stops, regarded as elements of efficiency. In Section 5.2, we further analyze these models by evaluating their behavior under several scenarios of passenger demand uncertainty. Finally, in Section 5.3, we report the results obtained through an extensive sensitivity analysis of the parameters used in the proposed models.

To test the proposed models we consider the same real-world instance used in Qi et al. (2018b): it consists of the Wuhan-Guangzhou high-speed line in China, composed of 18 stations. In this line, 36 trains run in a time horizon of seven hours and 30 minutes. In particular, 30 trains are G-trains and 6 trains are D-trains, whose maximum speeds are, respectively, 300 and 250 km/h. The maximum train capacity of each train is assumed to be 650. We show in Figure 2 the layout of the considered line. The departure time window ΔT_k for each train is set to 10 minutes. As it was done in Qi et al. (2018a), we increase the nominal passenger demand by 5% so as to deal with a saturated setting. The nominal solution has an objective value of 7297 minutes, corresponding to the total travel time, and contains 138 stops in the stop plan. The nominal solution is computed by solving model (1)-(20).

All nominal and robust models were developed using GAMS and solved by CPLEX on a Windows 7 workstation with two Intel Core i3-4130M CPUs and 4 GB of RAM. We consider two alternative termination conditions: we stopped the solver either when the optimality gap becomes

Figure 2: Layout of the Wuhan-Guangzhou high-speed line.

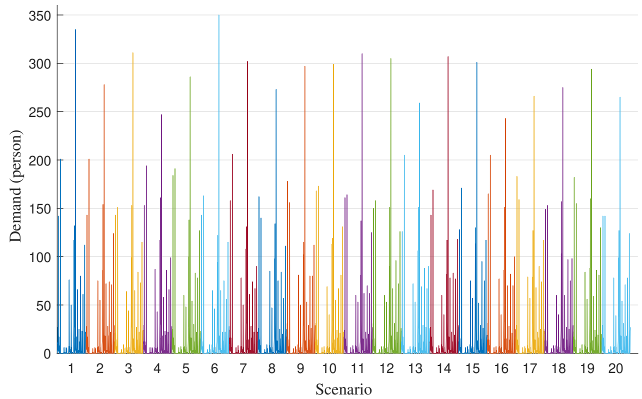

Figure 3: Representation of the considered scenarios.

less than 5% or after two hours of computation. For the results reported in Sections 5.1 and 5.2, parameters α and β were both set to 5%, and parameters N^{change} and H_k ($k \in K$) were set, respectively, to 3 and 50, while they are varied in Section 5.3.

To test the robustness of the proposed models, we define 20 scenarios characterized by different passenger demands. In particular, we use the same scenarios generated for testing DRM in Qi et al. (2018a), and set the same values of protection. More precisely, each scenario $\omega \in \Omega$ was generated by randomly increasing, for each pair of stations $i, j \in S$, the passenger demand in Qi et al. (2018b) by an integer number Δ^{ω}_{ij} between 4% and 6% of Q_{ij}. The increase of passenger demand for each pair of stations of these scenarios is not very large, because, in the considered instance, the capacity of the trains is almost saturated by the nominal passenger demand. When the train capacity is nearly reached, the possibility of reserving capacity to achieve a robust solution is very limited, and it is not possible to consider very large demand fluctuations. However, our goal is to determine train stop plans and timetables that are robust against demand fluctuation in everyday situations, and, thus, the considered scenarios are appropriate: this is similar to what is done in the literature of robustness against delays, in which small delays and disturbances are considered, rather than large disruptions. Indeed, it is very important that the derived robust solution is also efficient, as it will be used as a standard everyday plan.

In Figure 3, we show a graphical representation of the considered scenarios. On the horizontal axis we show the scenarios and, for each scenario, all the origin-destination pairs of stations, and on the vertical one the additional demand represented as number of additional passengers. The graphic shows, for each scenario and each origin-destination pair of stations, a vertical line corresponding to the number of additional passengers that want to travel between these two stations. As it can be seen, some origin-destination pairs have much larger demand increases than other ones: this is because we used a percentage of the nominal demand as the number of additional passengers, in order to have realistic demand scenarios. Clearly, the train capacity can rapidly be saturated if we consider larger demand for origin-destination pairs that have a large nominal demand. However, we think that it is more interesting and realistic to increase the demand for origin-destination pairs that are more often used by the travellers.

For NPRM and PDRM, we define the protection values Δ_{ij}, for each pair of stations $i, j \in S$, $i < j$, in the same way as for DRM. More precisely, let Ω be the set of scenarios, and Δ^{ω}_{ij} the passenger demand between stations i and j in scenario $\omega \in \Omega$, with Δ^{ω}_{ij} chosen to be in the interval [4%,6%] of the passenger demand in Qi et al. (2018b). The protection level Δ_{ij}, for every pair of stations $i, j \in S$, $i < j$, is then chosen so that $\Delta_{ij} \geq \Delta^{\omega}_{ij}$ for 90% of the scenarios in the set Ω. For TCRM, we need to define the protection level for each train: in this case, we set Δ_{ijk} so that, for every pair of stations $i, j \in S$, $i < j$, the equation $\sum_{k \in K} \Delta_{ijk} = \Delta_{ij}$ holds, and consider Δ_{ijk} either uniformly or randomly distributed on all trains $k \in K$ (with $i, j \in S_k$). More precisely, distributing Δ_{ijk} in a uniform way implies that Δ_{ijk} is the same for all trains $k \in K$ that visit both stations i and j ($k \in K$, $i, j \in S_k$). To distribute Δ_{ijk} in a random way, we randomly select Δ_{ij} times a train, among all trains that visit both stations i and j, and then set Δ_{ijk} for train k equal to the number of times that train k was randomly selected ($k \in K$, $i, j \in S_k$). In all experiments of TCRM, we use the same random values for Δ_{ijk} ($k \in K$, $i, j \in S_k$). The values Δ^{ω}_{ijk} are obtained from Δ^{ω}_{ij} by computing, for each scenario ω, them in the same way as Δ_{ijk} are obtained from Δ_{ij}, i.e., for a scenario ω, we set Δ^{ω}_{ijk} so that, for every pair of stations $i, j \in S$, $i < j$, the equation $\sum_{k \in K} \Delta^{\omega}_{ijk} = \Delta^{\omega}_{ij}$ holds, and consider Δ^{ω}_{ijk} either uniformly or randomly distributed on all trains $k \in K$ (with $i, j \in S_k$).

5.1. Performance of the Robust Models

We solved all the robust models with the parameters and protection levels set as described above. The obtained results are reported in Table 3. The first column shows the name of the robust model. The second and third ones give the objective value that corresponds to UD (unsatisfied demand) for NPRM or to UC (unavailable capacity) for TCRM (with uniformly or randomly distributed demands) and PDRM. Then, we show the optimality gap, the computing time expressed in seconds, and the total travel time and number of stops of the robust solution. We also report the results obtained by DRM presented in Qi et al. (2018a).

Model	UD	UC	Gap%	CPU time	Travel time	# Stops
DRM	268		0.0	120	7662	145
NPRM	268		0.0	840	7548	141
TCRM-un		726	4.5	180	7662	145
TCRM-rand		1020	4.3	434	7662	145
TCRM-un-2H		716	3.2	7200	7662	145
TCRM-rand-2H		995	1.9	7200	7662	145
PDRM		268	0.0	231	7662	145

Table 3: Comparison of the performance of the robust models.

As it can be seen, DRM and NPRM obtain the same UD value. However, NPRM finds a smaller increase of the total travel time and number of stops than the other model, and derives better train stop plans and timetables than those of DRM. However, we note that the solution computed for NPRM is also feasible for DRM, because constraint (26) in NPRM is more restrictive than (21) in DRM, and the CPLEX solver could have derived it for DRM as well. Both models are solved to optimality and the largest computing time is 840 seconds, which happens for NPRM, showing that it is harder to obtain a stop plan robust against uncertain demand and also close to the nominal plan. Even when the termination condition is that the optimality gap goes below 5%, for DRM and NPRM the optimal solution is obtained, because the optimality gap dropped down directly from above 5% to 0%. For TCRM, when we consider this termination condition, the solver stops with an optimality gap of more than 4% both with uniformly and randomly distributed demands: indeed, the optimality gap goes down more slowly, showing that it is harder to find the optimal solution when the protection is specified not only for every pair of stations but also for every train. Since the optimal solution was not found with this termination condition, for TCRM, we also report the results obtained within two hours: although with longer computing times the optimality gap can be reduced, the solver cannot find an optimal solution within the time limit. This confirms that it is harder to solve TCRM, due to the more specific protection requirements it includes in constraints (29). We also observe that TCRM with uniformly distributed demands shows a smaller UC value than TCRM with randomly distributed demands, and can also be solved in shorter computing times: indeed, distributing additional demands on trains in a uniform way gives to the model more flexibility. PDRM can be solved to optimality in 231 seconds, as the optimality gap directly reaches 0% from above 5%. The UC value found by PDRM is smaller than all values obtained by TCRM, confirming that it is easier to require protection only between pairs of stations, even when a limit on the number of additional passengers is imposed for each train by constraints (34). DRM, TCRM and PDRM find the same increase of the total travel time and number of stops: indeed, 7662 corresponds to an increase of 5% over the nominal travel time and 145 to an increase of 5% over the nominal number of stops. Therefore, in absence of additional constraints, all allowed flexibility is used in the robust solution.

We can conclude that NPRM and DRM have similar performances, and the former can be effectively used when utilizing a stop plan similar to one chosen by the practitioners. If we consider the UC goal, PDRM finds the solution with the smallest unavailable capacity, and is thus better than TCRM in terms of robust objective value. To assess the gain that is obtained by using the robust solutions, in the next section, we evaluate them under a set of scenarios characterized by different passenger demands.

5.2. Evaluation of the Robust Solutions under Uncertain Demand Scenarios

Beside the comparison of the robust models in terms of optimality gap and computing time, we use the scenarios in Ω to assess the robustness of the obtained solutions, and to compare the robustness quality with that of the nominal solution. The aim of this comparison is to evaluate the reduction of unsatisfied demand or unavailable train capacity that is achieved when the robust stop plan and timetables, obtained by solving the robust models, are used in place of the nominal one. This is to show that, although the travel time and number of train stops are slightly larger in the robust solutions, as shown in Table 3, the latter solutions can handle uncertain passenger demand more effectively. To perform the comparison, we compute, for every scenario $\omega \in \Omega$, the unsatisfied demand or unavailable capacity by solving a *validation model* both for the robust and the nominal solutions. More precisely, the validation model for NPRM, TCRM and PDRM is obtained by considering the same objective function and constraints of the corresponding robust model. Then, variables t^d_{ki} ($k \in K$, $i \in S_k \setminus \{D_k\}$), t^a_{ki} ($k \in K$, $i \in S_k \setminus \{O_k\}$), x_{ki} ($k \in K$, $i \in S_k$), y_{kli} ($k, l \in K$, $k < l$, $i \in (S_k \setminus \{D_k\}) \cap (S_l \setminus \{D_l\})$) are fixed, alternatively, as in the corresponding robust solution or as in the nominal one. The protection parameters Δ_{ij} (or Δ_{ijk} for TCRM) are fixed in every scenario as Δ^ω_{ij} (or Δ^ω_{ijk} for TCRM). Overall, we have thus three validation models, corresponding to NPRM, TCRM and PDRM, respectively. In each of these validation models, we first fix the robust solution and compute, for every scenario $\omega \in \Omega$, the corresponding objective value. Then, in each validation model, we fix the nominal solution and compute, for every scenario $\omega \in \Omega$, the corresponding objective value. In this way, we can compute the unsatisfied demand or unavailable capacity for every scenario $\omega \in \Omega$, and assess the quality and robustness of the robust and nominal solutions.

In the following, we report one table for each model (Tables 4, 5, 6 and 7), showing the comparison between the nominal and robust solutions. In each table, the first column is the number of the considered scenario $\omega \in \Omega$ and the second one shows the total additional demand (or capacity) required in ω. Note that for TCRM we report, in the second column, the sum of the total additional capacity already summed over all trains. Then, we show, for the nominal and robust solutions, respectively, the objective value (unsatisfied demand UD or unavailable capacity UC) and the computing time (expressed in seconds) of the validation model. In the last two rows, we report the sum and the average of the total demand and unsatisfied demand or unavailable capacity both for the nominal and robust solutions.

In Table 4, we show the comparison for NPRM. We recall that the maximum number of allowed changes N^{change} in the train stop plan with respect to the nominal one was fixed to three, obtained by rounding up 2% of the total number of stops in the nominal stop plan. The total UD value in the robust solution is about five times smaller than in the nominal one. Even though the additional demand in every scenario is rather large (close to 2000), on average the UD value is only 164.3 for the robust solution, showing that almost all passengers can be transported. In addition, the UD value is significantly reduced with respect to the nominal one (164.3 instead of 798.1). The computing time of the validation model is very short in both cases, since the train stop plans and timetable are fixed, and, thus, the solver needs only to determine, for each scenario, the passenger distribution on the trains. We wish to mention that DRM proposed in Qi et al. (2018a) has the same performance as NPRM in terms of UD value for all the considered scenarios, and, hence, the same average unsatisfied demand value 164.3. Even though the performance in the 20 scenarios is the same, the specific UD values between stations i and j ($i, j \in S, i < j$) are not the same due to the different stop plans. Indeed, very different stop plans are derived for the two models: in particular, DRM shows a much larger number of stop changes (about 100) than NPRM.

Tables 5 and 6 show the results obtained for TCRM, when the Δ_{ij} ($i, j \in S, i < j$) are distributed on trains either uniformly or randomly, respectively. In each table, we report the results obtained by validating, through the validation model, the robust solutions computed with the two alternative termination conditions, i.e., having the optimality gap below 5% or reaching the time limit of two hours. Note that the computing times shown in Tables 5 and 6 are very short, since the train stop plans and timetable are fixed in the validation model. In both tables, the UC value is considerably reduced with respect to the nominal solution (UC is, respectively, 1.99 and 1.78 times smaller in the robust solution than in the nominal one). However, the reduction is not as large as it is for the UD value in NPRM: indeed, solving TCRM is usually harder than solving NPRM, as we require a certain protection level for each train, i.e., the passenger distribution on trains is more constrained in TCRM. We can also observe that requiring a uniform distribution or a random one does not give very different results. Finally, we note that by evaluating the TCRM-un solution obtained in two hours we obtain an average unavailable capacity value that is slightly larger than that of the solution obtained in 180 seconds. This happens because the protection buffers Δ_{ijk} are placed on different trains in the two robust solutions, and the fewer buffers of the solution computed in shorter time turn out to be more effective for the considered scenarios than those of the solution computed in two hours. On the contrary, in TCRM-rand, a smaller average unavailable capacity is found for the solution obtained in two hours of computation. Indeed, when using two hours of computation time, a smaller optimality gap (1.9%) was obtained in the case of randomly distributed demand with respect to the case of uniformly distributed demand, for which the gap was still 3.2% (see Table 3).

Finally, in Table 7, we show the results obtained for PDRM. We recall that this model can be seen as a compromise between TCRM and NPRM, since it requires a certain protection on every pair of stations but also controls the passenger distribution on every train through H_k ($k \in K$). PDRM significantly reduces the UC value with respect to the nominal case (UC is more than three times smaller for the robust solution). Thus, PDRM obtains a better reduction than TCRM, by allowing the model to distribute passengers in a less constrained way. Indeed, the UC value obtained by PDRM is about two times smaller than that of TCRM.

Scenarios	$\sum_{i,j\in S}\Delta_{ij}^{\omega}$	Nominal solution		Robust solution	
		UD	CPU time	UD	CPU time
1	1915	871	0.6	207	2.0
2	1936	849	0.4	189	0.6
3	1866	771	0.5	112	0.6
4	1895	781	0.5	186	0.6
5	1925	825	0.5	203	0.6
6	1873	794	0.7	137	0.7
7	1920	820	0.4	205	0.7
8	1865	784	0.4	156	0.6
9	1903	775	0.5	180	0.5
10	1894	777	0.5	161	0.6
11	1875	806	0.7	159	0.7
12	1921	824	0.9	140	0.6
13	1872	790	0.4	174	0.7
14	1866	779	0.5	130	0.5
15	1903	802	0.8	144	0.6
16	1885	787	0.5	198	0.7
17	1893	804	0.5	192	0.6
18	1842	741	0.5	112	0.5
19	1938	860	0.8	188	0.6
20	1823	721	0.5	113	0.5
Sum	37810	15961		3286	
Avg	1890.5	798.1		164.3	

Table 4: Comparison of the nominal and robust solutions for NPRM.

Scenarios	$\sum_{i,j \in S} \Delta_{ij}^{\omega}$	Nominal solution		Robust solution		Robust solution 2H	
		UC	CPU time	UC	CPU time	UC	CPU time
1	1915	708	0.5	349	0.6	363	0.7
2	1936	659	0.9	328	0.5	358	0.6
3	1866	610	0.5	292	0.5	293	0.6
4	1895	612	0.6	332	0.7	328	0.6
5	1925	636	0.6	306	0.5	350	0.6
6	1873	643	0.6	327	0.5	310	0.7
7	1920	678	0.8	328	0.6	342	0.8
8	1865	593	0.7	290	0.4	294	0.6
9	1903	597	0.8	301	0.4	307	0.6
10	1894	614	0.6	308	0.5	313	0.8
11	1875	633	0.6	305	0.5	322	0.8
12	1921	652	0.6	346	0.5	330	1.4
13	1872	641	1.0	309	0.8	324	0.6
14	1866	620	0.7	319	0.5	301	0.7
15	1903	640	0.5	319	0.7	322	0.5
16	1885	609	0.8	293	0.5	322	0.8
17	1893	599	0.6	306	0.5	322	0.7
18	1842	584	0.8	309	0.5	296	0.8
19	1938	630	0.6	301	0.5	320	0.5
20	1823	543	0.7	285	0.4	274	0.7
Sum	37810	12501		6253		6391	
Avg	1890.5	625.1		312.7		319.6	

Table 5: Comparison of the nominal and robust solutions for TCRM with demands uniformly distributed over trains.

Scenarios	$\sum_{i,j\in S} \Delta_{ij}^{\omega}$	Nominal solution		Robust solution		Robust solution 2H	
		UC	CPU time	UC	CPU time	UC	CPU time
1	1915	733	0.5	404	0.4	373	0.7
2	1936	687	0.7	383	0.6	423	0.7
3	1866	650	0.7	330	0.5	323	0.5
4	1895	643	0.5	378	0.4	285	0.6
5	1925	661	0.7	361	0.6	353	0.6
6	1873	664	0.7	353	0.7	347	0.6
7	1920	671	0.6	417	0.6	262	0.7
8	1865	615	0.7	380	0.8	275	0.7
9	1903	591	0.6	383	0.6	333	0.6
10	1894	625	0.7	374	0.5	340	0.6
11	1875	652	0.6	348	0.4	328	0.6
12	1921	677	0.6	330	1.1	311	0.7
13	1872	688	0.6	382	0.5	273	0.6
14	1866	652	0.7	378	0.5	283	0.8
15	1903	666	0.6	335	0.7	305	0.8
16	1885	617	0.6	356	0.5	338	0.7
17	1893	623	0.6	378	0.5	342	0.6
18	1842	618	0.6	349	0.5	274	0.7
19	1938	654	0.6	378	0.5	341	1.1
20	1823	576	0.8	324	0.7	297	1.2
Sum	37810	12963		7321		6406	
Avg	1890.5	648.2		366.1		320.3	

Table 6: Comparison of the nominal and robust solutions for TCRM with demands randomly distributed over trains.

Scenarios	$\sum_{i,j\in S}\Delta^{\omega}_{ij}$	Nominal solution		Robust solution	
		UC	CPU time	UC	CPU time
1	1915	636	0.9	207	0.7
2	1936	594	0.7	189	0.7
3	1866	548	0.7	126	0.6
4	1895	547	0.8	186	0.7
5	1925	548	0.8	203	0.8
6	1873	574	1.3	137	0.6
7	1920	594	0.8	205	0.8
8	1865	522	0.9	170	0.7
9	1903	499	0.8	181	0.6
10	1894	534	0.7	161	0.6
11	1875	554	0.8	159	0.7
12	1921	582	0.8	140	0.7
13	1872	586	0.8	174	0.7
14	1866	549	0.7	132	0.7
15	1903	566	0.7	144	0.6
16	1885	534	1.0	198	0.6
17	1893	513	0.8	192	0.7
18	1842	531	0.8	131	0.7
19	1938	559	0.7	188	0.7
20	1823	476	0.9	144	0.7
Sum	37810	11046		3367	
Avg	1890.5	552.3		168.4	

Table 7: Comparison of the nominal and robust solutions for PDRM.

We can conclude that the three proposed robust models deal effectively with uncertain passenger demand, being able to significantly reduce the unsatisfied demand or unavailable capacity with respect to the nominal solution, while requiring limited increase of the total travel time and number of stops.

5.2.1. Summary of comparison

We report in Table 8 the summary of the results obtained in Sections 5.1 and 5.2 for all the robust models. As explained in Section 4.5, although all the proposed robust models aim at achieving robustness against uncertain passenger demand, they use different ways for reaching it. To compare the performances of the models that share the same goal, in Table 8, we report, for each model, the average unsatisfied demand or unavailable capacity computed over the 20 considered scenarios, and the corresponding travel time and number of train stops: the first term measures the robustness of the solution, while the other two terms assess its efficiency.

Model	Avg. UD	Avg. UC	Travel time	# Stops
DRM	164.3		7662	145
NPRM	164.3		7548	141
TCRM-un		312.7	7662	145
TCRM-rand		366.1	7662	145
PDRM		168.4	7662	145

Table 8: Summary of comparison.

We can see that, on the considered instance, NPRM has a better performance than DRM, since it achieves the same unsatisfied demand with a better efficiency, shown by shorter travel time and smaller number of stops. However, as observed earlier, the solution obtained for NPRM is also feasible for DRM, and the former model should be preferred when it is useful to utilize a stop plan similar to one chosen by the practitioners. All the other models obtain solutions with the same travel time and number of train stops, but PDRM reaches the smallest unavailable capacity. This model turns out to be the most promising one when using buffer capacity for robustness. However, we observe that the level of detail used in TCRM allows inserting protection for every train, while this is not possible in the other models. Similarly, PDRM allows controlling the distribution of additional passengers over different trains, while this is not possible for DRM and NPRM. Therefore, each model has its advantages, but PDRM includes most of them, as also described in Section 4.5.

5.2.2. Comparison with different scenario sets for the protection level and the robustness evaluation

The protection levels Δ_{ij} (Δ_{ijk}, resp.), used in the previous experiments, have been determined based on 20 scenarios of uncertain demand, and the same set of scenarios has been used to evaluate the robustness quality of the obtained solutions. In this section, we show that by determining Δ_{ij} (Δ_{ijk}, resp.) based on the first ten scenarios, and evaluating the corresponding robust solutions on the second set of ten scenarios, we obtain very similar (and in some cases better) results. We first report, in Table 9, the results obtained by solving the robust models with Δ_{ij} (Δ_{ijk}, resp.) determined according to the first ten scenarios. The models are named with suffix '10' to show that Δ_{ij} (Δ_{ijk}, resp.) are based on the first ten scenarios. Then, we report one table for each model (Tables 10, 11, 12 and 13), showing the comparison between the nominal and new robust solutions evaluated on the second set of ten scenarios. In these tables, we also report, for ease of comparison, the results over the second set of ten scenarios, already reported in Tables 4, 5, 6 and 7.

Model	UD	UC	Gap%	CPU time	Travel time	# Stops
NPRM10	272		0.0	199	7506	141
TCRM-un10		667	4.8	7026	7662	145
TCRM-rand10		889	3.3	261	7662	145
PDRM10		272	0.0	115	7662	145

Table 9: Comparison of the performance of the robust models with protection levels based on the first ten scenarios.

Table 9 shows that by solving the models with protection levels determined according to the first ten scenarios we obtain objective function values that are very similar to those computed with protection levels based on 20 scenarios. The optimality gaps and computing times are also in line with those reported in Table 3, except for TCRM-un10 which requires longer computing time than TCRM-un.

Results reported in Tables 10, 11, 12 and 13 show that the robust solutions obtained by using only the first ten scenarios to determine the protection levels are equally or more robust than those obtained by using all scenarios. In particular, NPRM10 obtains the same average UD as NPRM: we observed that Δ_{ij} values, although similar, are not the same for NPRM and NPRM10, but the same robust solution is obtained in both cases, since the limitation of N^{change} restricts the set of feasible solutions. This robust solution has two different UD values (268 and 272, respectively), due to the different Δ_{ij} values in the two models. On the contrary, TCRM-un10, TCRM-rand10 and PDRM10 obtained smaller UC than the corresponding models solved with protection levels based on 20 scenarios. Indeed, we observed that, in some cases, the values of Δ_{ij} (Δ_{ijk}, resp.) can be larger if computed on the first ten scenarios than when considering all scenarios: in fact, we require that $\Delta_{ij} \geq \Delta_{ij}^{\omega}$ for 90% of the scenarios, and a smaller scenario set can even imply higher protection. Moreover, we note that the unsatisfied demand and unavailable capacity are significantly smaller than for the nominal solution. Therefore, we can conclude that the performance of the robust models has not worsened by setting the protection levels based on a set of scenarios and evaluating them under a different set.

Scenarios	$\sum_{i,j \in S} \Delta_{ij}^{\omega}$	Nominal solution		Robust NPRM		Robust NPRM10	
		UD	CPU time	UD	CPU time	UD	CPU time
11	1875	806	0.7	159	0.7	159	0.9
12	1921	824	0.9	140	0.6	140	0.7
13	1872	790	0.4	174	0.7	174	0.6
14	1866	779	0.5	130	0.5	130	0.5
15	1903	802	0.8	144	0.6	144	0.5
16	1885	787	0.5	198	0.7	198	0.6
17	1893	804	0.5	192	0.6	192	0.6
18	1842	741	0.5	112	0.5	112	0.5
19	1938	860	0.8	188	0.6	188	0.5
20	1823	721	0.5	113	0.5	113	0.5
Sum	18818	7914		1550		1550	
Avg	1881.8	791.4		155		155	

Table 10: Comparison of the nominal and robust solutions for NPRM10.

Scenarios	$\sum_{i,j\in S}\Delta_{ij}^{\omega}$	Nominal solution		Robust TCRM-un		Robust TCRM-un10	
		UC	CPU time	UC	CPU time	UC	CPU time
11	1875	633	0.6	305	0.5	233	0.6
12	1921	652	0.6	346	0.5	220	0.6
13	1872	641	1.0	309	0.8	243	0.6
14	1866	620	0.7	319	0.5	206	0.7
15	1903	640	0.5	319	0.7	236	0.6
16	1885	609	0.8	293	0.5	257	0.8
17	1893	599	0.6	306	0.5	241	0.7
18	1842	584	0.8	309	0.5	196	0.5
19	1938	630	0.6	301	0.5	247	0.7
20	1823	543	0.7	285	0.4	181	0.6
Sum	18818	6151		3092		2260	
Avg	1881.8	615.1		309.2		226	

Table 11: Comparison of the nominal and robust solutions for TCRM-un10.

Scenarios	$\sum_{i,j\in S}\Delta_{ij}^{\omega}$	Nominal solution		Robust TCRM-rand		Robust TCRM-rand10	
		UC	CPU time	UC	CPU time	UC	CPU time
11	1875	652	0.6	348	0.4	314	0.6
12	1921	677	0.6	330	1.1	302	0.7
13	1872	688	0.6	382	0.5	318	0.7
14	1866	652	0.7	378	0.5	326	0.6
15	1903	666	0.6	335	0.7	296	0.6
16	1885	617	0.6	356	0.5	316	0.6
17	1893	623	0.6	378	0.5	295	0.6
18	1842	618	0.6	349	0.5	281	0.6
19	1938	654	0.6	378	0.5	309	0.6
20	1823	576	0.8	324	0.7	266	0.6
Sum	18818	6423		3558		3023	
Avg	1881.8	642.3		355.8		302.3	

Table 12: Comparison of the nominal and robust solutions for TCRM-rand10.

5.3. Sensitivity Analysis

In this section, we perform an analysis of the impact of the variation of the parameters, considered in the proposed robust models, on the obtained robust solutions. In particular, we show the results obtained by testing the PDRM model, as it embeds effectively the features of NPRM and TCRM.

5.3.1. Trade-off between efficiency and robustness

We consider the variation of α, i.e., the parameter used in constraint (25) to limit the travel time increase in the robust solution. In particular, we want to determine how different values of α,

Scenarios	$\sum_{i,j \in S} \Delta_{ij}^{\omega}$	Nominal solution		Robust PDRM		Robust PDRM10	
		UC	CPU time	UC	CPU time	UC	CPU time
11	1875	554	0.8	159	0.7	159	0.7
12	1921	582	0.8	140	0.7	140	0.7
13	1872	586	0.8	174	0.7	174	0.7
14	1866	549	0.7	132	0.7	130	0.7
15	1903	566	0.7	144	0.6	144	0.8
16	1885	534	1.0	198	0.6	198	0.7
17	1893	513	0.8	192	0.7	192	0.8
18	1842	531	0.8	131	0.7	112	0.8
19	1938	559	0.7	188	0.7	188	0.9
20	1823	476	0.9	144	0.7	113	0.7
Sum	18818	5450		1602		1550	
Avg	1881.8	545		160.2		155	

Table 13: Comparison of the nominal and robust solutions for PDRM10.

leading to different travel times, affect the unavailable capacity of the robust solution. To this aim, we fix $\beta = \infty$, i.e., we allow any variation of the number of stops in the stop plan. Table 14 reports the results obtained by varying α from 1% up to the case of $\alpha = 20\%$. In particular, we display the average unavailable capacity (Avg. UC) over the 20 uncertain demand scenarios, computed, for each α value, by solving the validation model for PDRM, as in Section 5.2. In addition, we show the total travel time, the number of stops, the objective value UC of the robust model, the optimality gap and the computing time.

Value	Avg. UC	Travel time	# Stops	UC	Gap%	CPU time
$\alpha = 1\%$	164.3	7370	172	268	0.0	447
$\alpha = 5\%$	164.3	7662	178	268	0.0	131
$\alpha = 10\%$	164.3	8027	162	268	0.0	88
$\alpha = 20\%$	164.3	8368	174	268	0.0	85

Table 14: Trade-off between efficiency and robustness for PDRM.

We can see that the model is solved to optimality for all the considered values of α in rather short computing times, and the largest time is reached when α is 1%, which is the most restrictive condition. The travel time increases, as expected, when α becomes larger, since, as soon as constraint (25) is respected, the solver can stop the solution process. The robust objective value UC and the average unavailable capacity, computed over the 20 scenarios, are the same for all cases: therefore, for this instance, robustness cannot be increased by allowing longer travel times. This happens because trains are almost full to satisfy the nominal passenger demand, and the lowest possible unavailable capacity achievable with the available number of trains is 164.3. It is interesting to see that it is possible to achieve the same robustness quality even with only 1% increase of the total travel time: therefore, it is very important to impose constraint (25) in order to limit the total travel time of the robust solution. Finally, we can observe that the number of stops varies without following a specific relation with the value of α, because no limit is imposed on the number of additional stops in these experiments.

5.3.2. Impact of parameters α and β

Varying α without any constraint on the number of additional train stops has no impact on the robustness of the solution. We now consider the simultaneous variation of both α and β to evaluate how their values affect the robust solution. We report the obtained results in Table 15, where the columns have the same meaning as in Table 14. In the last row, we also consider the special case in which constraints (21) and (25) are neglected from the model.

Value	Avg. UC	Travel time	# Stops	UC	Gap%	CPU time
$\alpha = 1\%$, $\beta = 5\%$	166.1	7370	145	268	0.0	542
$\alpha = 5\%$, $\beta = 1\%$	178.9	7662	139	268	0.0	290
$\alpha = 5\%$, $\beta = 5\%$	168.4	7662	145	268	0.0	231
$\alpha = 10\%$, $\beta = 10\%$	164.3	8027	152	268	0.0	107
$\alpha = \infty$, $\beta = \infty$	164.3	8338	168	268	0.0	45

Table 15: Comparison of the performance of PDRM with different values of α and β.

We can see that different values of α and β, ruling constraints (21) and (25), have an impact on the level of robustness that can be reached: in particular, when β is 1%, the average unavailable capacity increases to 178.9, i.e., about 15 additional passengers cannot be transported with respect to the best case that has 164.3 as unavailable capacity value. When β is 5%, the average unavailable capacity decreases and becomes closer to the best case. We observe that neglecting constraints (21) and (25) by letting $\alpha = \beta = \infty$ does not further improve robustness, since the train capacity limits the amount of passengers that can be transported. Even though the optimal solution can be obtained, for all the considered parameter values, in rather short computing times, we can see that, when constraints (21) and (25) are not included in the model, the computing time is about one order of magnitude smaller than when $\alpha = 1\%$, $\beta = 5\%$. However, we notice that it is very relevant to impose constraints (21) and (25), in order to guarantee the efficiency of the robust solution by limiting its travel time and number of train stops.

5.3.3. Impact of parameter H_k

We report, in Table 16, the results obtained by varying parameter H_k that is used, in constraints (34), to limit the number of additional passengers that can be distributed on each train $k \in K$. We do not report results corresponding to values of H_k smaller than 50, since they led to infeasible solutions: indeed, constraints (32) require to transport the entire additional passenger demand and constraints (34) limit the number of additional passengers on each train. Therefore, parameter H_k must be chosen so as to satisfy both requirements. In Table 16, we report the results obtained for $H_k = 50$, i.e., the case considered in the previous experiments, and H_k set to 80 and 100. In particular, we show for comparison the results reported in Table 7 and add the unavailable capacity obtained with the larger values of H_k. In all three cases ($H_k = 50$, $H_k = 80$ and $H_k = 100$), the robust solutions obtained by solving PDRM have objective value UC equal to 268, although they are three different solutions.

As it can be seen, by increasing H_k we can slightly decrease the unavailable capacity. Indeed, more additional passengers can be transported on each train and, consequently, the unavailable capacity is reduced. However, we can also note that the results obtained with $H_k = 80$ and $H_k = 100$ are the same: therefore, further increasing the number of additional passengers on each

Scenarios	$\sum_{i,j\in S}\Delta_{ij}^{\omega}$	Nominal solution		$H_k = 50$		$H_k = 80$		$H_k = 100$	
		UC	CPU time	UC	CPU time	UC	CPU time	UC	CPU time
1	1915	636	0.8	207	0.7	207	0.7	207	0.7
2	1936	594	0.8	189	0.7	189	0.9	189	1.1
3	1866	548	0.7	126	0.6	112	1.0	112	0.7
4	1895	545	0.8	186	0.7	186	0.8	186	0.9
5	1925	548	0.8	203	0.8	203	0.7	203	0.7
6	1873	573	0.7	137	0.6	137	0.8	137	0.8
7	1920	594	0.8	205	0.8	205	0.8	205	0.7
8	1865	522	0.7	170	0.7	156	0.8	156	0.8
9	1903	499	0.7	181	0.6	180	0.7	180	0.7
10	1894	534	0.8	161	0.6	161	0.7	161	0.7
11	1875	552	0.7	159	0.7	159	0.8	159	0.9
12	1921	582	0.7	140	0.7	140	0.7	140	1.1
13	1872	585	0.7	174	0.7	174	0.8	174	0.8
14	1866	549	0.7	132	0.7	130	0.6	130	0.9
15	1903	566	0.7	144	0.6	144	0.8	144	0.8
16	1885	534	0.7	198	0.6	198	0.7	198	0.9
17	1893	513	0.9	192	0.7	192	0.8	192	0.8
18	1842	528	0.7	131	0.7	112	0.8	112	0.9
19	1938	559	0.9	188	0.7	188	0.7	188	0.9
20	1823	476	0.8	144	0.7	113	0.7	113	0.7
Sum	37810	11037		3367		3286		3286	
Avg	1890.5	551.85		168.4		164.3		164.3	

Table 16: Comparison of the nominal and robust solutions for PDRM with different H_k values.

train does not reduce the unavailable capacity, since the train capacity C_k in constraints (33) limits the global number of passengers that can be on each train. We observe that we have chosen H_k to be the same for all the trains $k \in K$, but it is also possible to select different H_k values for different trains: for example, higher values could be allowed for those trains for which a larger number of passengers is expected.

5.3.4. Impact of parameters α and N^{change}

PDRM, presented in Section 4.4, contains constraint (21) to limit the number of additional stops with respect to the nominal plan. In this section, we replace constraint (21) with constraint (26), used in NPRM. The latter constraint requires to limit to N^{change} the number of changes to the nominal stop plan. Therefore, this constraint is significantly more restrictive than (21). For this reason, in the original PDRM model, we have adopted constraint (21), in order to give more freedom to the solver for finding a robust solution. Table 17 reports the results obtained by solving PDRM with different values of α and N^{change}. The columns have the same meaning as in Tables 14 and 15.

We observe that, when α is 5% and N^{change} is at least 5, the average unavailable capacity is very similar to that obtained with constraint (21) (which is 168.4, as shown in Table 7), and the computing time is short. By keeping α to 5% and decreasing N^{change} to 3, we obtain a slight increase of the average unavailable capacity which reaches 170.5. However, when N^{change} is decreased to 1, the average unavailable capacity significantly worsens achieving 277.2, the computing time increases, and the optimal solution is not found: indeed, in this case, the problem becomes harder because we require to change only one stop with respect to the nominal stop plan. When α is 1% a similar performance is observed, even though the computing times are larger than for $\alpha = 5\%$, since the constraint limiting the total travel time increase is more restrictive. Therefore, for the considered

Value	Avg. UC	Travel time	# Stops	UC	Gap%	CPU time
$\alpha = 5\%$, $N^{change} = 10$	164.9	7662	145	268	0.0	73
$\alpha = 5\%$, $N^{change} = 5$	164.3	7662	143	268	0.0	91
$\alpha = 5\%$, $N^{change} = 3$	170.5	7662	141	268	0.0	234
$\alpha = 5\%$, $N^{change} = 1$	277.2	7662	139	366	4.9	878
$\alpha = 1\%$, $N^{change} = 10$	164.3	7370	147	268	0.0	400
$\alpha = 1\%$, $N^{change} = 5$	164.3	7370	143	268	0.0	558
$\alpha = 1\%$, $N^{change} = 3$	164.3	7370	141	268	0.0	330
$\alpha = 1\%$, $N^{change} = 1$	278.1	7370	139	366	0.0	1504

Table 17: Comparison of the performance with different values of α and N^{change} for PDRM.

instance, we can conclude that constraint (26) can be used in place of (21), as long as N^{change} is not too restrictive.

5.3.5. Impact of the protection level

We report, in Table 18, a comparison of the results obtained with different values of Δ_{ij} for PDRM. Up to now, Δ_{ij} (for every pair of stations $i, j \in S$, $i < j$) was chosen so that $\Delta_{ij} \geq \Delta_{ij}^{\omega}$ for 90% of the scenarios in the set Ω. Here, we consider $\Delta_{ij} \geq \Delta_{ij}^{\omega}$ for a number of scenarios between 10% and 100%, and show the corresponding average unavailable capacity evaluated on the 20 considered scenarios, the total travel time and number of stops of the robust solution, the objective value UC obtained by solving PDRM and the corresponding optimality gap and computing time.

Value	Avg. UC	Travel time	# Stops	UC	Gap%	CPU time
10%	164.3	7662	145	58	0.0	93
30%	164.3	7662	145	104	0.0	753
50%	164.3	7662	145	149	0.0	188
70%	164.3	7662	145	212	0.0	203
90%	168.4	7662	145	268	0.0	231
100%	179.5	7662	145	321	0.0	85

Table 18: Comparison of the performance with different values of Δ_{ij} for PDRM.

As it can be seen, the average unavailable capacity does not significantly change with different protection levels accounting for between 10% and 90% of the scenarios: even with rather small values, the PDRM solution reaches the best average unavailable capacity. The increase from 10% to 90% is not helpful to reduce the average unavailable capacity, which is slightly worse when Δ_{ij} accounts for 90% of the scenarios: indeed, even when Δ_{ij} accounts for only 10% of the scenarios, UC is larger than zero, i.e., not all the desired protection can be inserted between all pairs of stations, and, thus, requiring a higher protection does not imply that this protection is reached. When Δ_{ij} accounts for 100% of the scenarios, the average unavailable capacity is larger than in the other cases: with a larger protection we would expect the average unavailable capacity to be smaller, since a larger protection corresponds to a larger number of seats reserved to the additional passengers. However, when Δ_{ij} accounts for 100% of the scenarios, we require to satisfy all the scenarios, even those, with very large passenger demand, that occur very rarely. As a consequence, some pairs of stations will get a high protection, but other ones will not get enough protection, thus making the overall average unavailable capacity higher. Therefore, the value of the required protection level should be chosen so as to satisfy a high percentage of scenarios but should exclude the most extreme cases.

5.3.6. Constraint (26) *in TCRM and PDRM*

In this section, we compare the effect of using constraint (26) instead of (21) in TCRM-rand and PDRM. In particular, we always consider $\alpha = 5\%$, and then use $\beta = 5\%$ in (21), or $N^{change} \in \{1, 3, 5, 10\}$ in (26). The obtained results are reported in Table 19, where, for ease of comparison, we also include the results of PDRM shown in Table 17.

Model	Avg. UC	Travel time	# Stops	UC	Gap%	CPU time
PDRM with $\beta = 5\%$	168.4	7662	145	268	0.0	231
PDRM with $N^{change} = 10$	164.9	7662	145	268	0.0	73
PDRM with $N^{change} = 5$	164.3	7662	143	268	0.0	91
PDRM with $N^{change} = 3$	170.5	7662	141	268	0.0	234
PDRM with $N^{change} = 1$	277.2	7662	139	366	4.9	878
TCRM with $\beta = 5\%$	366.1	7662	145	1020	4.3	434
TCRM with $N^{change} = 10$	303.0	7662	148	1004	1.0	500
TCRM with $N^{change} = 5$	381.5	7662	143	1051	4.8	91
TCRM with $N^{change} = 3$	348.5	7662	141	1024	1.6	135
TCRM with $N^{change} = 1$	453.9	7662	139	1113	1.4	56

Table 19: Impact of constraint (26) in TCRM-rand and PDRM.

As it can be seen, the impact of using constraint (26), instead of (21), on the robustness quality is similar for both models, except for the case of $N^{change} = 5$ in TCRM, in which, due to the large optimality gap, the average unavailable capacity is larger than when using constraint (21). For the remaining cases, when N^{change} is at least 3, the average unavailable capacity is very similar to the one obtained in the case of $\beta = 5\%$. On the contrary, when the number of stops that can be changed is decreased to 1, the average unavailable capacity significantly increases in both models. Therefore, similar to what happens for PDRM, in TCRM constraint (26) can replace (21) without significantly worsening the unavailable capacity, as long as N^{change} is not too restrictive.

5.3.7. Larger demand scenarios between given pairs of stations and given trains

In this section, we evaluate the performance of TCRM when we increase the demand for some specific pairs of stations and trains. In particular, we consider higher demands of, respectively, 120, 50 and 100 passengers, between stations 1 and 18 on train 4, stations 1 and 7 on train 1, and stations 7 and 18 on train 13, and modify the first five scenarios, considered in the previous experiments, by keeping the same total demand but increasing the demand for these specific pairs of stations and trains. The robust solutions of TCRM-un and TCRM-rand obtained with 20 scenarios (reported in Table 3) are evaluated on these five scenarios, and the results are reported in Table 20, where we show the comparison of the unavailable capacity obtained for the nominal and TCRM solutions.

As it can be seen, both TCRM-un and TCRM-rand solutions are capable of handling these larger scenarios more effectively than the nominal solution: indeed, the unavailable capacity is almost halved. As expected, due to the larger demand required for some specific pairs of stations and trains, the unavailable capacity increases with respect to the case of the previous 20 scenarios. However, we can see that the average unavailable capacity is not much larger than before, where it

Scenarios	$\sum_{i,j \in S} \Delta_{ij}^{\omega}$	Nominal solution		TCRM-un		TCRM-rand	
		UC	CPU time	UC	CPU time	UC	CPU time
1	1915	758	0.6	407	0.6	437	0.6
2	1936	725	0.6	389	0.6	448	0.6
3	1866	672	0.5	352	0.5	387	0.6
4	1895	663	0.5	378	0.5	444	0.6
5	1925	690	0.6	397	0.5	440	0.5
Sum	9537	3508		1923		2156	
Avg	1907.4	701.6		384.6		431.2	

Table 20: Comparison of the solutions obtained by the nominal model and by TCRM on the new scenarios.

was, by computing the average over the first previous five scenarios, 321.4 for TCRM-un and 371.2 for TCRM-rand (see Tables 5 and 6).

5.3.8. Summary of the sensitivity analysis

We conclude this section by summarizing the results obtained with the sensitivity analysis. We have seen that, for the considered instance, parameter α, which limits the increase of travel time, does not have an impact on the level of robustness achieved, while β, which limits the number of additional train stops, affects it in a slightly more evident way. In addition, constraints (21) and (25) are very important, since, when we neglect them, the efficiency of the robust solutions significantly decreases. Setting the limit H_k on the number of additional passengers that can be transported on each train k is relevant for PDRM: if this parameter is set to a very small value, then no feasible solution can be obtained, because constraints (32) require to transport the entire additional demand. We have also evaluated the behavior of including constraint (26) in PDRM, used in NPRM, to limit the changes with respect to the nominal train stop plan: as we have seen, this constraint is effective and allows finding robust solutions that have shorter travel time and smaller number of train stops than when using constraint (21). Experiments conducted on the variation of the values of α and N^{change} have shown that when N^{change} is set to a very small value (e.g., 1), the average unavailable capacity increases significantly, but if the value is slightly larger it is beneficial to have constraint (26) in the model. We have also seen that the variation of the protection level does not have significant impact on the quality of robustness for the considered instance, except when it takes into account 100% of the scenarios, since, in the latter case, we require to satisfy all the scenarios, even extreme ones that occur very rarely.

We report, in Figure 4, the best robust train stop plans and timetable obtained by solving PDRM with $\alpha = 1\%$, $N^{change} = 3$, and compare it with the nominal one. In the figure, different colors correspond to different passenger flow densities on trains. In particular, a blue line corresponds to a large number of passengers transported along that section while a yellow line corresponds to a small number. As it can be seen, the structure of the diagrams is similar in both cases, even though the robust timetable shows a larger number of light blue lines. Moreover, the robust PDRM solution gives an average unavailable capacity of 164.3, total travel time 7370 and total number of stops 141. With respect to the nominal solution, the unavailable capacity has been reduced by more than 70%, with an increase of travel time of 1% and of the number of train stops of 1.4%. Therefore, a much more robust timetable has been obtained with slight worsening of efficiency.

The considered instance contains 36 trains, and, as shown in Table 3, the computing time required by PDRM is 231 seconds to compute the optimal robust solution, while NPRM needs 840 seconds, and TCRM cannot determine the optimal solution even in two hours of computation. In order to compute robust solutions for larger size instances, PDRM seems more appropriate, not only because it requires shorter computing times, but also because it has the advantage of finding a robust solution that distributes the additional passengers on all trains in a more balanced way than NPRM. By looking at the results of the sensitivity analysis, when the parameters α and N^{change} are set to stricter values, the computing time required by PDRM increases. Therefore, to solve PDRM on larger size instances, it would be better to allow larger increases of the travel time and more changes to the nominal stop plan or to use constraint (21) instead of (26) in PDRM. Clearly, depending on the size of the instance, metaheuristic algorithms could be developed to determine robust solutions, starting from the solution of a relaxation of PDRM, in which some of the constraints could be relaxed, and combined with constructive heuristic algorithms. The development of these algorithms is left for future work.

6. Conclusions and Future Research

We have studied the problem of determining train stop plans and timetables that are robust against uncertain passenger demand. Three Mixed Integer Linear Programming (MILP) models, Nominal-Plan based Robust Model (NPRM), Train-Capacity based Robust Model (TCRM) and Passenger-Distribution based Robust Model (PDRM) have been proposed, all using the Light Robustness method. In each model, a desired protection level is inserted to manage additional demand occurring in real-time, and the goal is to minimize the total protection that cannot be achieved. To guarantee that the stop plan and timetables are efficient, constraints on the maximum allowed worsening of the nominal travel time and number of stops are imposed. The three MILP models mainly differ in the way of inserting the required protection, either by considering additional demand or by reserving capacity. All models consider the passenger distribution on the trains and take into account train capacity. We tested the proposed models on real-world data of the Wuhan-Guangzhou high-speed railway line in China, and compared the obtained results with the nominal solution in terms of robustness quality under 20 demand scenarios. The results show that the unsatisfied demand or unavailable capacity in the robust solutions is between two and five times smaller than in the nominal case. In addition, it turned out that NPRM has better efficiency than a model proposed in Qi et al. (2018a), since timetables with the same robustness quality and shorter travel times are obtained by NPRM. TCRM, which requires protection for each pair of stations and for each train, requires longer computing times, but becomes useful when it is necessary to insert protection for specific trains. PDRM overcomes the drawbacks of NPRM and TCRM, notably reduces the unavailable capacity with respect to the nominal solution, and effectively combines the best features of the other models.

Future work will be dedicated to study the problem when uncertain passenger demand is split in given time intervals of the time horizon. This is a generalization of the proposed models, since the protection should be defined for every pair of stations and time period. We plan to extend (some of) the robust models to deal with this more specific protection requirement, and develop new solution methods to limit the computing times. In addition, the proposed models could be generalized by including the extensions described in Section 4.6. Finally, metaheuristic algorithms could be developed to determine robust solutions for large size instances.

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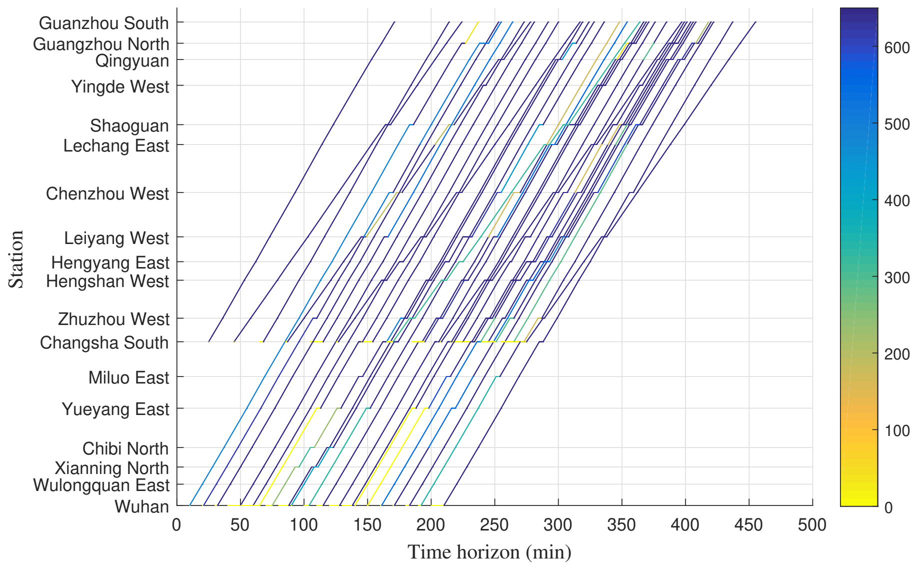
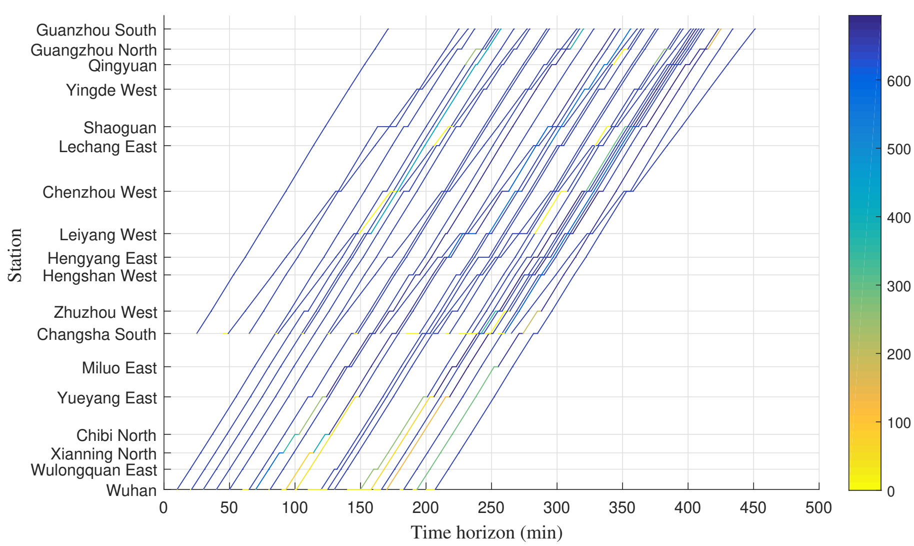


(a) Optimized nominal timetable.



(b) Optimized PDRM robust timetable.

Figure 4: Diagrams of nominal and robust timetables with colors showing passenger density.

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