



ALMA MATER STUDIORUM  
UNIVERSITÀ DI BOLOGNA

ARCHIVIO ISTITUZIONALE  
DELLA RICERCA

## Alma Mater Studiorum Università di Bologna Archivio istituzionale della ricerca

Deformation and stress in hydrothermal regions: The case of a disk-shaped inclusion in a half-space

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

*Published Version:*

Mantiloni, L., Nespoli, M., Belardinelli, M.E., Bonafede, M. (2020). Deformation and stress in hydrothermal regions: The case of a disk-shaped inclusion in a half-space. JOURNAL OF VOLCANOLOGY AND GEOTHERMAL RESEARCH, 403, 1-12 [10.1016/j.jvolgeores.2020.107011].

*Availability:*

This version is available at: <https://hdl.handle.net/11585/769000> since: 2020-10-22

*Published:*

DOI: <http://doi.org/10.1016/j.jvolgeores.2020.107011>

*Terms of use:*

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (<https://cris.unibo.it/>).  
When citing, please refer to the published version.

(Article begins on next page)

1 Deformation and stress in hydrothermal regions: the case of a disk-shaped  
2 inclusion in a half-space

3 Lorenzo Mantiloni<sup>a,b</sup>, Massimo Nespoli<sup>a</sup>, Maria Elina Belardinelli<sup>a</sup>, Maurizio Bonafede<sup>a</sup>

4 <sup>a</sup>*Department of Physics and Astronomy, Alma Mater Studiorum, University of Bologna, Bologna, Italy*

5 <sup>b</sup>*GFZ German Research Centre for Geosciences, Potsdam, Germany*

---

---

6 **Abstract**

7 Hydrothermal regions are affected by a wide variety of phenomena, including ground inflation and deflation  
8 episodes. Among them, calderas offer the opportunity to study the complex interactions between magmatic  
9 processes at depth and permeable rocks saturated with fluids in the upper sedimentary layers. One of  
10 such regions is the Campi Flegrei caldera in southern Italy, where several source models have been applied  
11 over the years to reproduce the ground displacement and seismicity observed during the most recent phase  
12 of major unrest (1982-1984). The present work aims at introducing a new source model consisting of a  
13 thermo-poro-elastic inclusion embedded in a homogeneous poroelastic half-space. The inclusion is meant to  
14 represent a permeable rock layer stressed and strained by hot and pressurized volatiles released upward by  
15 an underlying magmatic reservoir and is modeled as a thin horizontal disk inside which a sudden change of  
16 temperature and pore pressure occurs. We provide semi-analytical solutions for the displacement and stress  
17 fields both within and outside the source and check them by comparison with those obtained through a  
18 fully numerical approach. Results provided by our model are compared with two other deformation source  
19 models often used to describe volcanic environments in terms of pressurized cavities describing a spherical  
20 magma chamber (Mogi source) or a sill-like magma intrusion (Fialko source). For the Campi Flegrei 1982-84  
21 unrest, our model provides a better reproduction of ground deformation data and manages to explain the  
22 widespread presence of compressive focal mechanisms, since the stress field promoted both inside and outside  
23 the thermo-poro-elastic inclusion is very different from pressurized cavities.

24 **Keywords:** Campi Flegrei, Thermo-poro-elasticity, Focal mechanisms, Deformation sources, Volcanism.

57  
58 **1. Introduction**  
59  
60

61 Hydrothermal regions are found in many areas of the Earth, and are in some cases associated with  
62 calderas. They are affected by complex interactions in which convection of water and other fluids of magmatic  
63 origin within the Earth's crust transfer heat and mass towards the surface. This leads to a variety of  
64 observable phenomena, including ground deformation, gravity changes, hot springs, fumaroles and seismicity  
65 (see e.g. the Yellowstone caldera, USA, Tizzani et al., 2015; the Rabaul caldera, Papua New Guinea,  
66 Robertson and Kilburn, 2016; the Masaya complex, Nicaragua, Williams-Jones et al., 2003; the Long Valley  
67 caldera, USA, Hill, 2006; Prejean et al., 2002; Sorey et al., 1991; the Hengill volcanic system, Iceland, Feigl  
68 et al., 2000). According to physical models, these effects are generally connected with hydrothermal processes  
69 (Rinaldi et al., 2010, Todesco et al., 2014), involving temperature and pore-pressure changes of fluids flowing  
70 through permeable rocks, but also with the inflation or deflation of the parent magma chamber related to  
71 the mass input/output, to internal differentiation processes or to the emplacement of a new magmatic body  
72 (Macedonio et al., 2014; Di Vito et al., 2016). In particular Lima et al. (2009) consider ground deformation  
73 episodes as due to the cooling and crystallization of a magma volume at shallow depth, accompanied by  
74 release of magmatic fluids which are occasionally expelled from a deep, pressurized, region into the shallow  
75 hydrothermal system. In the Lima et al. (2009) conceptual model, subsidence could result from a volume  
76 decrease due to both crystallization and a decrease in the flux of magmatic fluids entering the system, or  
77 a rapid permeability increase (and pore pressure decrease) that occurs when the fluid pressure exceeds the  
78 local strength of the crust, leading to failures in the elastic matrix of the porous media. As the discrimination  
79 between these processes is not trivial, the modelling of these phenomena is most important to improve the  
80 comprehension of volcanic hazard.

81 Ground deformation in volcanic areas is usually modeled in terms of the surface effects of a deformation  
82 source at depth, typically consisting of a pressurized cavity representing a magma chamber (e.g. Mogi, 1958,  
83 Yang et al., 1988) or a horizontal circular crack, suited to model sill-like magma intrusions (e.g. Fialko et al.,  
84 2001). Such models assume the source to be embedded in a homogeneous, elastic half-space and neglect  
85 the presence of fluids within the rocks. In the present paper we consider the mechanical effects induced  
86 by temperature and pore-pressure changes within a thermo-poro-elastic inclusion surrounded by an elastic  
87 medium. Conceptually similar thermo-poro-elastic models were employed to study the effects of pressure

113  
114 53 and temperature gradients around wellbores, accounting for deformation sources with cylindrical geometries  
115  
116 54 located within unbounded media (e.g. Myklestad, 1942; Perkins et al., 1984 and Perkins et al., 1985). To  
117  
118 55 model subduction above gas or oil reservoirs, Geertsma et al. (1973) considered the effect of a drop in pore  
119  
120 56 pressure within a finite cylindrical volume in an elastic half-space, retrieving analytical solutions for surface  
121  
122 57 displacement components. Myklestad (1942) developed analytical solutions for stress components close to a  
123  
124 58 semi-infinite circular cylinder inside which a uniform increase of temperature occurs.

125  
126 59 In the present work we introduce a deformation source consisting of a disk-shaped horizontal Thermo-  
127  
128 60 Poro-Elastic (TPE) inclusion embedded in a poro-elastic half space in free drainage conditions. As in  
129  
130 61 Belardinelli et al. (2019) the TPE inclusion is meant as a region of permeable rock being affected by a  
131  
132 62 sudden increase in temperature and pore pressure, embedded in a surrounding medium in isothermal drained  
133  
134 63 conditions. It is worth to notice that purely magmatic models hardly explain long-lasting subsidence (Calò  
135  
136 64 and Tramelli, 2018 and Troise et al., 2018) and are not suitable for the shallow source regions where the  
137  
138 65 presence of large magma bodies can be ruled out. Moreover, differently from a pressurized cavity, the TPE  
139  
140 66 model provides a strong deviatoric stress field even within the source. Belardinelli et al. (2019) consider a  
141  
142 67 spherical shell-shaped TPE inclusion surrounding a fluid filled magma chamber and embedded within an  
143  
144 68 unbounded poro-elastic medium; in the present work we (i) include the free surface boundary condition and  
145  
146 69 (ii) consider a disk-shaped TPE inclusion. Including the free surface is fundamental in order to compare  
147  
148 70 model predictions with observed fault mechanisms above the magma reservoir and with surface displacement.  
149  
150 71 With respect to a spherical shell surrounding the magmatic intrusion, a disk-shaped region is better suited  
151  
152 72 to describe a horizontal permeable rock layer stressed and strained by hot and pressurized volatiles. For  
153  
154 73 example, at Campi Flegrei at about 2 km depth, there is evidence of a seismic layer separating a deeper  
155  
156 74 magmatic body from the shallower aquifer (Figure 8 in Calò and Tramelli, 2018), the most permeable part  
157  
158 75 of which may allow the magmatic fluids to flow upward.

156  
157 76 In the next sections we present the semi-analytical formulation of the model. As the present model is  
158  
159 77 inspired by observations made in the Campi Flegrei caldera in southern Italy (fig. 1), in the last section we  
160  
161 78 provide an application focused on one of its unrest episodes. During the period 1982-84 the recorded uplift  
162  
163 79 at Campi Flegrei was nearly axi-symmetric and centered in the town of Pozzuoli (Bonafede and Ferrari,  
164  
165 80 2009) where it reached its maximum with rate values up to 1 m/yr. One of the most relevant aspects of  
166  
167 81 the 1982-84 unrest was the important increase in seismic activity, while the previous episodes of uplift were  
168

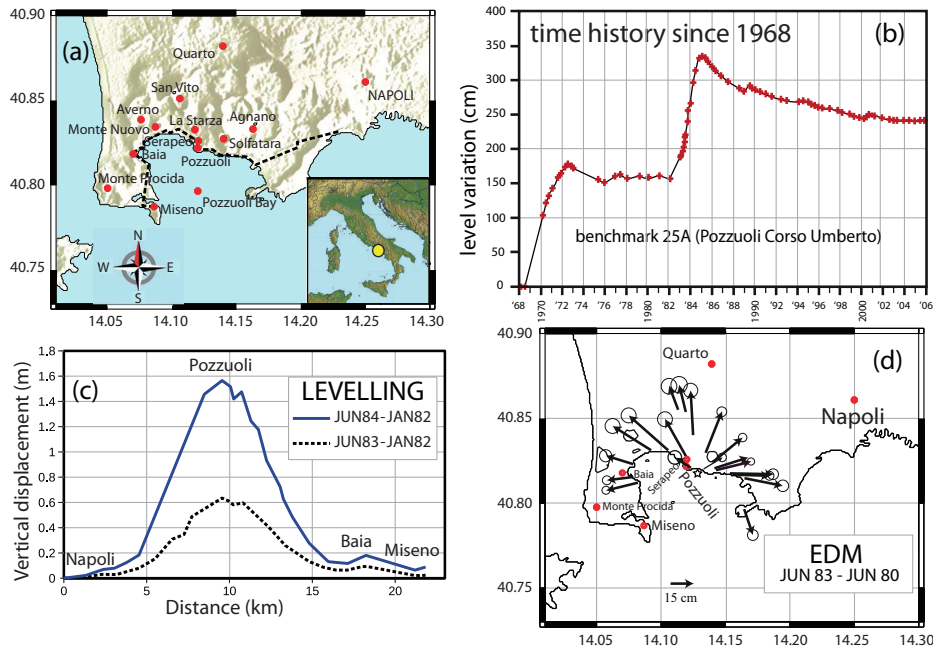


Figure 1: Map and deformation data of the studied area. a) Map of the Campi Flegrei region. b) evolution of uplift at benchmark 25A (Pozzuoli Corso Umberto) since 1968 to 2006. c) pattern of uplift measured on the baseline between Napoli and Miseno (drawn in a) as a dashed black line) in June 1983 (black dotted line) and in June 1984 (blue) with respect to January 1982; the maximum uplift was close to the center of Pozzuoli. d) displacement vectors estimated from EDM (Electromagnetic Distance Measurement) from Jun 80 to Jun 83 referred to the point shown as a star (Amoruso et al., 2014). White circles represent errors.

82 accompanied by weak to moderate seismicity (D’Auria et al., 2014). The contribution of both magmatic  
 83 intrusions and hydrothermal dynamics to surface ground deformation was envisaged for this episode (e.g.  
 84 Belardinelli et al., 2019). Our results will be compared with some of the principal source models used for  
 85 the 1982-84 unrest, in particular attention is paid to inversion of surface deformation data and the expected  
 86 distributions of focal mechanisms versus related evidences.

87 It is worth to notice that, despite having been inspired by the features of one particular case of study,  
 88 the simple geometry and characteristics of our model make it applicable to the study of other hydrothermal  
 89 regions around the world.

## 90 2. Methods

91 Following Eshelby (1957) we retrieve the displacement and stress fields associated to the TPE inclusion.  
 92 The procedure has already been outlined in details by Belardinelli et al. (2019). The strain field  $e_{ij}$  of a  
 93 thermo-poro-elastic medium (McTigue, 1986) undergoing changes of stress  $\tau_{ij}$ , temperature  $\Delta T$  and pore  
 94 pressure  $\Delta p$  is

$$e_{ij} = \frac{1}{2\mu} \left( \tau_{ij} - \frac{\nu}{1+\nu} \tau_{kk} \delta_{ij} \right) + \frac{1}{3H} \Delta p \delta_{ij} + \frac{1}{3} \alpha \Delta T \delta_{ij} \quad (1)$$

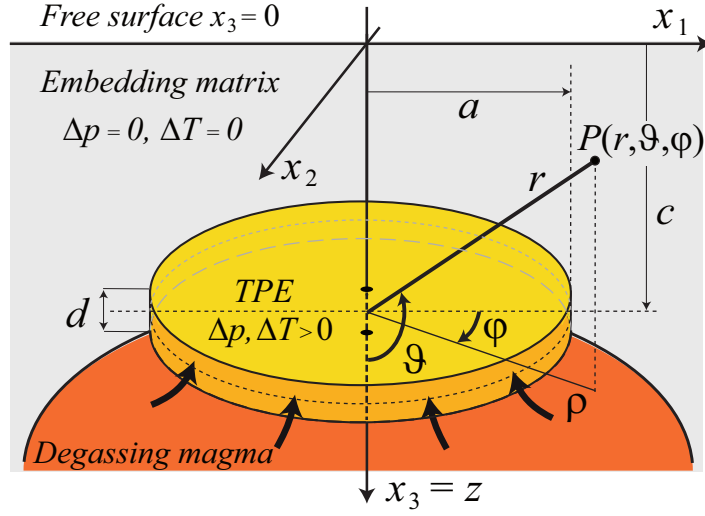


Figure 2: Schematic picture of the disk-shaped thermo-poro-elastic inclusion. The inclusion (yellow region) has a radius  $a$  and thickness  $d$ ; it is located at depth  $c$  and embedded in a poro-elastic half-space (grey region). The inclusion undergoes a sudden change in temperature  $\Delta T$  and pore pressure  $\Delta p$  caused by degassing of a underlying magma body (orange region). The median plane of the disk is drawn with a dotted line. The spherical and cylindrical coordinates  $(r, \theta, \varphi)$  and  $(\rho, \varphi, z)$ , respectively, are expressed in a reference frame with origin in  $x_1 = 0, x_2 = 0, x_3 = c$ .

95 while the inverse relation is

$$\tau_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - K \left( \frac{1}{H}\Delta p\delta_{ij} + \alpha\Delta T\delta_{ij} \right) \quad (2)$$

96 where  $H$  is the Biot's constant,  $\alpha$  the coefficient of thermal expansion,  $\mu$  the rigidity,  $\nu$  the drained isothermal

97 Poisson's ratio and  $K = \frac{2\mu(1+\nu)}{3(1-2\nu)} = \lambda + \frac{2}{3}\mu$  the drained isothermal bulk modulus of the poroelastic medium.

98 Following eq. (1), the stress-free strain  $e_{ij}^*$  that the inclusion would undergo in absence of the hosting medium

99 (Belardinelli et al., 2019) can be expressed as:

$$e_{ij}^* = e_0\delta_{ij} \quad \text{where} \quad e_0 = \frac{1}{3H}\Delta p + \frac{1}{3}\alpha\Delta T \quad (3)$$

100 Surface tractions  $T_k = -3Ke_0n_k$  must be applied in isothermal and drained conditions to restore the original

101 volume and shape of the inclusion. Outside the inclusion the tractions vanish, so that a traction discontinuity

102  $[T_k]_-^+ = 3Ke_0n_k$  appears on the TPE inclusion boundary  $S$ . When removing the traction discontinuity across

103  $S$ , the following displacement is produced (see e.g. Aki Richards, p. 58)

$$u_i(\mathbf{x}) = \oint_S G_{ik}(\mathbf{x}, \mathbf{x}') [T_k]_-^+ dS' = 3Ke_0 \oint_S G_{ik}(\mathbf{x}, \mathbf{x}') n_k(\mathbf{x}') dS' \quad (4)$$

104 where  $G_{ik}$  is the elastic Green's tensor for a half-space with drained, isothermal elastic parameters, whose

105 components are given by Mindlin (1936). The Green's function  $G_{ik}(\mathbf{x}, \mathbf{x}')$  yields the displacement in the

281  
282 106  $i$  –  $th$  direction at point  $\mathbf{x}$  due to a unitary point force acting in the  $k$  –  $th$  direction at  $\mathbf{x}'$ . By applying  
283  
284 107 Gauss' theorem we obtain  
285  
286

$$287 \quad u_i(\mathbf{x}) = 3Ke_0 \int_{V_S} \frac{\partial G_{ik}}{\partial x'_k}(\mathbf{x}, \mathbf{x}') dv(\mathbf{x}') \quad (5)$$

290 where  $V_S$  is the volume of the TPE inclusion. The displacement caused by the TPE source everywhere in  
291  
292 109 the half-space is provided by equation (5). Instead the stress field  $\tau_{ij}$  caused by the TPE source is provided  
293  
294 110 by eq. (2) and should be defined separately within the inclusion, where  $\tau_{ij} = \tau_{ij}^{in}$ , and outside it, where  
295  
296 111  $\Delta p = 0$ ,  $\Delta T = 0$  and  $\tau_{ij} = \tau_{ij}^{out}$ , so that

$$300 \quad \tau_{ij}^{in} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - 3Ke_0 \delta_{ij} \quad (6a)$$

$$302 \quad \tau_{ij}^{out} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} \quad (6b)$$

303  
304  
305 112 with  $e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ . Since  $G_{ik}(\mathbf{x}, \mathbf{x}')$  is singular, when  $\mathbf{x} \rightarrow \mathbf{x}'$  particular care must be taken when  
306  
307 113 computing  $u_i$ ,  $e_{ij}$  and  $\tau_{ij}$  within the inclusion.  
308  
309

### 310 2.1. Retrieval of the displacement field: singular and non-singular terms

311  
312 115 The three components of the displacement field  $u_i$  are found by first evaluating the sum of Green's tensor  
313  
314 116 partial derivatives in eq. (5), employing cartesian coordinates. Their expressions can be written as  
315  
316

$$317 \quad u_1 = 3KCe_0 \int_{-a}^a dx'_1 \int_{-f(x'_1)}^{f(x'_1)} dx'_2 \int_{c-\frac{d}{2}}^{c+\frac{d}{2}} dx'_3 (x_1 - x'_1) \left\{ \frac{1}{R_1^3} + \frac{(3-4\nu)}{R_2^3} - \frac{6x_3(x_3 + x'_3)}{R_2^5} \right\}$$

$$319 \quad u_2 = 3KCe_0 \int_{-a}^a dx'_1 \int_{-f(x'_1)}^{f(x'_1)} dx'_2 \int_{c-\frac{d}{2}}^{c+\frac{d}{2}} dx'_3 (x_2 - x'_2) \left\{ \frac{1}{R_1^3} + \frac{(3-4\nu)}{R_2^3} - \frac{6x_3(x_3 + x'_3)}{R_2^5} \right\} \quad (7)$$

$$321 \quad u_3 = 3KCe_0 \int_{-a}^a dx'_1 \int_{-f(x'_1)}^{f(x'_1)} dx'_2 \int_{c-\frac{d}{2}}^{c+\frac{d}{2}} dx'_3 \left\{ \frac{(x_3 - x'_3)}{R_1^3} - \frac{(3-4\nu)(x_3 + x'_3)}{R_2^3} - \frac{6x_3(x_3 + x'_3)^2}{R_2^5} + \frac{2x_3}{R_2^3} \right\}$$

328  
329 117 where  
330  
331  
332  
333  
334  
335  
336

$$R_1 = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + (x_3 - x'_3)^2} \quad (8a)$$

$$R_2 = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + (x_3 + x'_3)^2} \quad (8b)$$

$$f(p) = \sqrt{a^2 - p^2}, \quad C = \frac{1 - 2\nu}{8\pi\mu(1 - \nu)} \quad (8c)$$

and the intervals of integration are given by the geometry of the TPE inclusion (Figure 2). The integrand functions in eqs. (7) can be divided into two parts: the terms depending on  $\frac{1}{R_1^3}$  which diverge within the volume of the inclusion ( $V_s$ ) and those depending on powers of  $\frac{1}{R_2}$  which are bounded within  $V_s$ . For this reason, the terms depending on  $\frac{1}{R_1^3}$  are referred to as the *singular* terms (apex *s*), while those depending on powers of  $\frac{1}{R_2}$  are referred to as the *non-singular* terms (apex *ns*).

Accordingly, even the displacement field  $\mathbf{u}$  is found by summing up two contributions, as follows:

$$\mathbf{u} = \mathbf{u}^s + \mathbf{u}^{ns} \quad (9)$$

The singular contribution to displacement,  $\mathbf{u}^s$ , can be written as the gradient of a scalar potential  $\Phi$  (Belardinelli et al., 2019) so that:

$$\mathbf{u}^s = -\frac{e_1}{4\pi} \nabla \Phi \quad \text{with} \quad \Phi(\mathbf{x}) = \int_{V_s} \frac{1}{R_1} dv(\mathbf{x}') \quad (10)$$

where

$$e_1 = e_0 \frac{1 + \nu}{1 - \nu} \quad (11)$$

The potential in eq. (10) is formally equivalent to the Coulomb electrostatic potential due to a cylindrical volume  $V_S$  of charge density  $4\pi\epsilon_0$  (see Jackson, 1999), and therefore the integral can be computed employing an expansion in Legendre polynomials  $P_l(x)$  if we make the assumption that the thickness  $d$  of the cylinder is much smaller than its radius  $a$  ( $\frac{d}{a} \ll 1$ )

$$\begin{aligned} \Phi(r, \vartheta) &= 2\pi ad \left[ 1 - |\cos \vartheta| \frac{r}{a} + \sum_{m=1}^{\infty} c_{2m} P_{2m}(\cos \vartheta) \frac{1}{2m-1} \left(\frac{r}{a}\right)^{2m} \right] \quad \text{if } r < a \\ \Phi(r, \vartheta) &= 2\pi ad \sum_{m=0}^{\infty} c_{2m} P_{2m}(\cos \vartheta) \frac{1}{2m+2} \left(\frac{a}{r}\right)^{2m+1} \quad \text{if } r > a \end{aligned} \quad (12)$$

where  $c_{2m} = (-1)^m 4^{-m} (2m)! (m!)^{-2}$  and  $(r, \vartheta, \varphi)$  are the spherical coordinates of a point ( $\vartheta$  is the colatitude measured from the  $z$  axis) in a reference frame with origin in the disk center (see Figure 2). When  $r \approx a$ , the

393 convergence of the above series is extremely slow, so that analytical continuation may be employed. On the  
394  
395  
396 134 other side, the integrals of the *non-singular* terms in (7) are dealt with by performing analytical integrations  
397  
398 135 and simplifying them into single integrals over one coordinate  $dx'_i$ , which are computed numerically, yielding  
399  
400 136 the *non-singular* contribution  $\mathbf{u}^{ns}$  to  $\mathbf{u}$  (see supplementary material).

### 401 402 403 137 2.2. Retrieval of the stress field within and outside the inclusion

404  
405 138 The strain tensor  $e_{ij} = e_{ij}^s + e_{ij}^{ns}$ , can be also separated into a singular part,  $e_{ij}^s$ , and a non-singular  
406  
407 139 one,  $e_{ij}^{ns}$  related to derivatives of  $\mathbf{u}^s$  and  $\mathbf{u}^{ns}$ , respectively. The singular components  $e_{ij}^s$  can be obtained  
408  
409 140 analytically from spatial derivatives of the scalar potential (eq. 12) as follows

$$410 \quad e_{rr}^s = u_{r,r}^s, \quad e_{\vartheta\vartheta}^s = r^{-1}(u_{\vartheta,\vartheta}^s + u_r^s), \quad e_{\varphi\varphi}^s = (r \sin \vartheta)^{-1}(u_{\varphi,\varphi}^s + u_r^s \sin \vartheta + u_{\vartheta}^s \cos \vartheta), \quad (13)$$

411  
412  
413 141 where the spatial derivative of a scalar field  $\Psi$  with respect to the variable  $x$  is indicated as  $\Psi_{,x}$ ,

$$414 \quad e_{r\vartheta}^s = u_{\vartheta,r}^s, \quad e_{r\varphi}^s = 0, \quad e_{\vartheta\varphi}^s = 0. \quad (14)$$

415  
416  
417  
418 142 The second members of the last equation are obtained considering that  $u_{\vartheta,r}^s = r^{-1}(u_{r,\vartheta}^s - u_{\vartheta}^s)$ , being from  
419  
420 143 (10)  $u_r^s = \Phi_{,r}$  and  $u_{\vartheta}^s = r^{-1}\Phi_{,\vartheta}$  and  $u_{\varphi}^s = (r \sin \vartheta)^{-1}\Phi_{,\varphi} = 0$ , while  $u_r^s$  and  $u_{\vartheta}^s$  do not depend on  $\varphi$ . In  
421  
422 144 analogy with Belardinelli et al. (2019), it may be shown that the singular dilation outside the inclusion is  
423  
424 145  $e_{kk}^s = 0$ , while inside it we have  $e_{kk}^s = e_1$ . The non-singular components are retrieved by analytical spatial  
425  
426 146 derivatives of  $\mathbf{u}^{ns}$ , evaluating the corresponding volume integrals in a semianalytical way as made for  $\mathbf{u}^{ns}$   
427  
428 147 itself (see supplementary material). Then the final expressions for  $\tau_{ij}^{out}$  and  $\tau_{ij}^{in}$  are

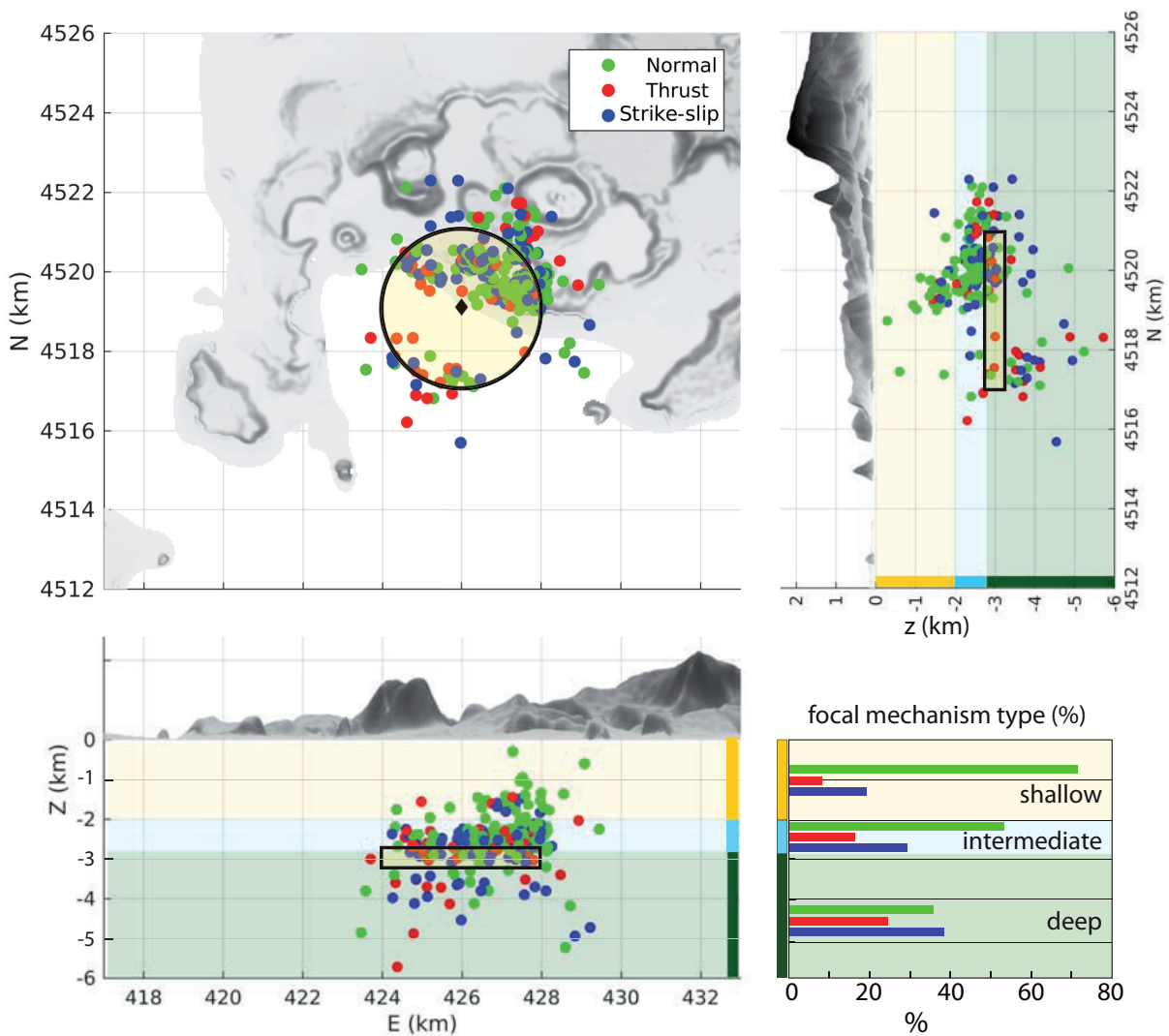
$$429 \quad \tau_{ij}^{out} = \lambda e_{kk}^{ns} \delta_{ij} + 2\mu (e_{ij}^s + e_{ij}^{ns}) \quad (15a)$$

$$430 \quad \tau_{ij}^{in} = \lambda (e_1 + e_{kk}^{ns}) \delta_{ij} + 2\mu (e_{ij}^s + e_{ij}^{ns}) - 3K e_0 \delta_{ij} \quad (15b)$$

431  
432  
433  
434  
435  
436  
437 148 In order to test the robustness of our results, and to check the correctness of the numerical integration used  
438  
439 149 in the present work, we compare our semi-analytical solutions to the one obtained through a completely  
440  
441 150 numerical method, which employs a surface distribution of orthogonal forces on the surface of the TPE  
442  
443 151 disk to account for the traction discontinuity on it. In fact, as the Green's function  $G_{km}(\mathbf{x}, \mathbf{x}')$  yields the  
444  
445 152 displacement in the  $k$ -th direction at point  $\mathbf{x}$  due to a point force in the  $m$ -th direction at  $\mathbf{x}'$ , the surface  
446  
447 153 integral in eq. (4) can be seen as the displacement field given by point forces distributed over the surface

449  
 450  $S$  of the inclusion and perpendicular to it. The difference between the results of the semi-analytical and  
 451  
 452 numerical methods for a shallow TPE inclusion with  $c/a < 3$  (when non-singular contributions are relevant)  
 453  
 454 for both surface displacement and stress in the plane  $x_3 = c$  (the median plane of the TPE disk, Figure S1)  
 455  
 456 are negligible, provided that, in the numerical model, the force distribution over the TPE source boundary  
 457  
 458 is dense enough.

461 **3. THE APPLICATION TO THE 1982-84 CAMPI FLEGREI UNREST**



496 Figure 3: Map and N-S (view from east) and E-W (view from south) vertical sections of the Campi Flegrei Caldera. The  
 497 topography is vertically exaggerated. Dots represent earthquake locations (D'Auria et al., 2014) occurred during the 1982-84  
 498 unrest episode. Normal, thrust and strike-slip mechanisms are associated respectively to green, red and blue colours. The black  
 499 circle and its projection on the vertical sections represent a tentative location of the TPE inclusion, whose center is shown with  
 500 a black diamond. Histograms show the percentage of focal mechanism type over the total number of earthquakes located in  
 501 the relative depth range. The three depth ranges define the shallow (0-2 km, yellow background), intermediate (2-2.8 km, light  
 502 blue background) and deep (2.8-6 km, dark green background) zone, respectively.

505  
506 160 Campi Flegrei is a nested caldera (Figure 1) located west of the city of Naples, with external and internal  
507  
508 161 diameters of about 14 km and 12 km, respectively. Volcanic activity has occurred there since 47,000 years  
509  
510 162 ago (De Vivo, 2006), seeing two major eruptive episodes approximately 39,000 and 14,900 years BP, the  
511  
512 163 last magmatic eruption being that of Monte Nuovo in 1538 AD (Di Vito et al., 2016). In historical times  
513  
514 164 the whole caldera has experienced several cycles of subsidence and uplift (e.g. Di Vito et al., 1999; Di Vito  
515  
516 165 et al., 2016). Two significant phases of uplift recorded by leveling data started in the second half of the 20th  
517  
518 166 century, reaching their peaks in two major unrest episodes in 1969-1972 and 1982-1984 (Figure 1). At the  
519  
520 167 end of 1984 the uplift trend stopped, starting a subsidence phase with a much slower rate which lasted until  
521  
522 168 2005, when a new period of inflation took over at a slower rate. Both the subsidence and the recent uplift  
523  
524 169 phases were characterized by minor peaks of uplift superimposed on the global trend, which have always  
525  
526 170 been followed by a fast recovery of their whole deformation (Gaeta et al., 2003).

526 171 The shape of ground deformation (Figure 1) remained practically unaltered during both up and down  
527  
528 172 movements, maintaining the same features of the 1982-84 episode (Troise et al., 2018). Phases of unrest at  
529  
530 173 Campi Flegrei have been monitored through several techniques over the time, including GPS and InSAR  
531  
532 174 data (Trasatti et al., 2015), seismic (D'Auria et al., 2014) and geochemical data (e.g. Chiodini et al., 2015),  
533  
534 175 gravimetry surveys (Berrino, 1994) and deep drillings (De Natale et al., 2016). Moreover, thanks to the  
535  
536 176 seismic tomography the annular shaped buried rim of the caldera was detected from 800-2000 m to 1800-  
537  
538 177 4000 m of depth beneath which a depressed limestone basement is present at less than 4000 m depth (Zollo  
539  
540 178 et al., 2003, Judenherc and Zollo, 2004). The TPE inclusion is expected within the buried rim of the inner  
541  
542 179 caldera, then in a depth range of 2-4 km as suggested by the tomographic study of Calò and Tramelli (2018).  
543  
544 180 Actually most of geothermal processes (gas emission and boiling pools) are located within few kilometers  
545  
546 181 from the center of the caldera (e.g. Solfatarata crater in Figure 1; Chiodini et al., 2015) below which we assume  
547  
548 182 that the TPE source is located (Figure 3).

548 183 It is worth to notice that even if the caldera is located in the tectonic environment of the Campania  
549  
550 184 margin, which is characterized by extensional structures and normal fault activity (Lima et al., 2009), the  
551  
552 185 focal mechanisms distribution retrieved from the 1982-84 seismic data series, below the caldera (D'Auria  
553  
554 186 et al., 2014), is very heterogeneous (Figure 3), suggesting a dominant role of local deformation mechanisms  
555  
556 187 related to the volcanic environment. Moreover, the distribution of focal mechanisms is not uniform along  
557  
558 188 depth, as confirmed by the percentage of focal mechanism type computed over the total number of earth-

561  
562 189 quakes occurred in the shallow (0-2 km), intermediate (2-2.8 km) and deep (2.8-6 km) zones, respectively  
563  
564 190 (Figure 3). Below the caldera there is a progressive increase of strike-slip mechanisms over depth (from 20  
565  
566 191 to 39%). The same is true for thrust mechanisms whose percentage changes from about 8 to 25%, while,  
567  
568 192 in contrast, there is a strong decrease in normal mechanisms percentage that reduces from 72 to 36%. The  
569  
570 193 cut-off of the seismicity can be identified at about 4 km depth, even if the hypocenter depth was generally  
571  
572 194 above 3 km (D’Auria et al., 2014).

573 195 Different deformation sources have been considered over the years to interpret the cause of the 1982-84  
574  
575 196 unrest. Berrino et al. (1984) found that the observed bell-shaped pattern of ground uplift can be nicely fitted  
576  
577 197 by a Mogi source located at about 3 km depth beneath the center of the caldera. Battaglia et al. (2006)  
578  
579 198 inverted deformation and gravity data determining pressurized penny-shaped horizontal cracks located in  
580  
581 199 the depth range 2.5 and 3.5 km, probably filled with aqueous fluids, as the probable sources of inflation at  
582  
583 200 Campi Flegrei. Other authors (Amoruso et al., 2008), considering the same source model within a layered  
584  
585 201 embedding medium, support the presence of magma in its interior. More recently, based on considerations  
586  
587 202 about the ratios of the three moment tensor eigenvalues retrieved from the data, Trasatti et al. (2011)  
588  
589 203 concluded that a mixed mode dislocation with both shear and tensile components, through which a magma  
590  
591 204 volume might have intruded, is the most suitable deformation source for the event, ruling out the applicability  
592  
593 205 of a pressurized ellipsoid.

593 206 Shallow magmatic intrusions (3-4 km depth) have been advocated as the origin of both the 1982-84 and  
594  
595 207 the 2011-13 unrest episodes (Dvorak and Berrino, 1991; Macedonio et al., 2014). Purely magmatic models,  
596  
597 208 however, fail in explaining the observed long lasting subsidence after the 1982-84 peak (Troise et al., 2018).  
598  
599 209 Moreover, seismic tomography surveys (Judenherc and Zollo, 2004) found no evidence of shallow magma  
600  
601 210 batches in the 3-4 km depth range, while they have highlighted a large sill at about 8 km depth which may  
602  
603 211 feed the entire Neapolitan volcanic area (Zollo et al., 2008). Even the temperature profiles inferred from  
604  
605 212 deep drilling projects (Carlino et al., 2012) are generally incompatible with the presence of magma at shallow  
606  
607 213 depths (Trasatti et al., 2011).

### 608 609 214 *3.1. Choice of parameters*

610  
611 215 Firstly, we have to define an adequate set of parameters both for the dimensions of the inclusion and  
612  
613 216 the properties of the medium. However we normalize all the TPE inclusion results to  $|u_z|^{max}$ , the maximum  
614  
615  
616

617  
618 217 uplift at the free surface, which realizes on the symmetry axis of the system, and we show patterns using  
619  
620 218 spatial coordinates normalized to the radius of the TPE disk. Accordingly the choice of parameters slightly  
621  
622 219 affects the results shown. The radius of the TPE inclusion and its depth are preliminarily chosen as  $a = 2000$   
623  
624 220 m and  $c = 3000$  m as suggested by Battaglia et al. (2006), Amoruso et al. (2008) and D’Auria et al. (2014),  
625  
626 221 employing pressurized horizontal cavities. These parameters are also suggested by the seismicity distribution  
627  
628 222 and the location (between 2 and 4 km) of a shallow  $V_P/V_S$ -anomaly possibly related to an overpressurized  
629  
630 223 fluid volume (Chiarabba and Moretti, 2006, Zollo et al., 2008; Calò and Tramelli, 2018). The disk height is  
631  
632 224 chosen so that the ratio  $\frac{d}{a} \ll 1$  is suitable to allow the potential expansion in equation (12). For the chosen  
633  
634 225 parameters  $|u_z|^{max}$  is in the order of tens of centimeters.

635 226 According to Belardinelli et al. (2019), the elastic parameters in isothermal and drained conditions of  
636  
637 227 the poro-elastic matrix are  $\lambda = 4$  GPa,  $\mu = 6$  GPa ( $\nu = 0.2$ ). The thermal expansion coefficient of the TPE  
638  
639 228 source is  $\alpha = 3 \cdot 10^{-5} \text{K}^{-1}$ , while  $H = 10$  GPa (see eq. 3). These values are pertinent to highly porous  
640  
641 229 sedimentary rocks (Rice and Cleary, 1976), such as those constituting much of the upper stratigraphy of the  
642  
643 230 Campi Flegrei caldera (Lima et al., 2009).

644 231 Finally, the changes in temperature and pore pressure within the inclusion are assumed respectively in  
645  
646 232 the order of  $\Delta T = 100$  K,  $\Delta p = 10$  MPa. The assumption of a 100 K temperature jump is a reasonable  
647  
648 233 order of magnitude if we consider the injection of overheated and overpressurized volatiles from a deep  
649  
650 234 reservoir into a shallower system as sketched in Figure 2. Shallow water reservoirs in the Campi Flegrei  
651  
652 235 area are associated with temperatures between  $150^\circ \text{C}$  and  $250^\circ \text{C}$  (Carlino et al., 2012), while the critical  
653  
654 236 temperature of water is  $373.9^\circ \text{C}$ . An order of magnitude of tens MPa for  $\Delta p$  is well within the difference  
655  
656 237 between the lithostatic and hydrostatic pore pressure at 3 km depth.

#### 657 238 4. RESULTS

660 239 Given the axial symmetry of the TPE inclusion with respect to the vertical axis  $z$ , we provide results  
661  
662 240 using the cylindrical reference frame  $(\rho, \varphi, z = x_3)$  shown in Figure 2.

663 241 At the free surface, the resulting displacement components are illustrated in Figure 4a (solid lines) as  
664  
665 242 functions of  $\rho/a$ , where  $\rho$  is the horizontal distance from the  $z$  axis (see Figure 2). Figure 5a and b show the  
666  
667 243 components of the stress tensor over the median plane of the TPE inclusion ( $x_3 = 3$  km) and slightly above  
668  
669 244 it ( $x_3 = 2.5$  km), respectively. In Figure 5a inside the TPE inclusion ( $\rho < a$ ), the diagonal stress components

673  
674  
675  
676  
677  
678  
679  
680  
681  
682  
683  
684  
685  
686  
687  
688  
689  
690  
691  
692  
693  
694  
695  
696  
697  
698  
699  
700  
701  
702  
703  
704  
705  
706  
707  
708  
709  
710  
711  
712  
713  
714  
715  
716  
717  
718  
719  
720  
721  
722  
723  
724  
725  
726  
727  
728

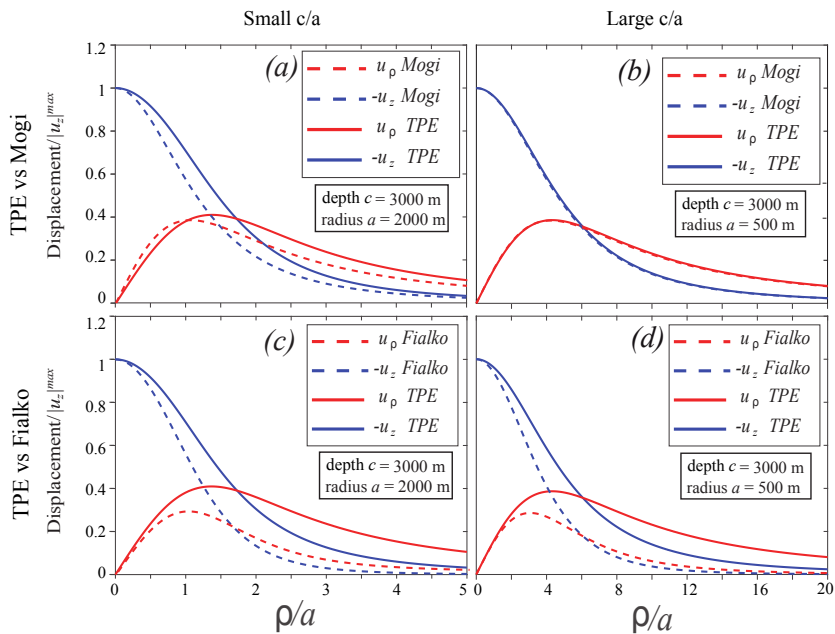


Figure 4: Displacement at the free surface. Comparison between the TPE inclusion and Mogi source (a, b) and TPE inclusion and Fialko source (c, d) of displacement ( $u_\rho$ , red lines) and vertical uplift ( $-u_z$ , blue lines) at free surface. Displacement components are normalized to the maximum value of the vertical uplift for each model ( $|u_z|^{max}$ ). The horizontal distance  $\rho$  is normalized to the TPE inclusion radius  $a$ . All the source centers are placed in  $(0, 0, c)$  with  $c = 3000$  m. In panels (a) and (c) we assume a large  $c/a$  ratio for the TPE inclusion ( $a=500$  m,  $d = 40$  m), in panel (b, d) we assume a small  $c/a$  ratio for the TPE inclusion ( $a=2000$  m,  $d = 200$  m as used in the present work). The volume of the Mogi source is always assumed as equal to the one of the TPE inclusion, so its radius is 843 m in panel (a) and 196 m in panel (b), while Fialko sources have the same radius as the TPE source. Note the different scales in abscissa.

245 are almost constant for  $\rho < 0.8a$  and  $\tau_{zz}^{in} \gg \tau_{\rho\rho}^{in} > \tau_{\varphi\varphi}^{in}$  while, outside it ( $\rho > a$ ),  $\tau_{zz}^{out} > \tau_{\varphi\varphi}^{out} > \tau_{\rho\rho}^{out}$ . Outside  
246 the inclusion the stress components rapidly decay with  $\rho$  in agreement with the observed cut-off of seismicity  
247 getting outside the TPE inclusion boundaries (black circle in Figure 3). All shear components vanish over  
248 the median plane. Above the TPE inclusion, the stress strongly decreases and, at a depth of 2.5 km, it is  
249 already reduced by two orders of magnitude (Figure 5b) even if the decay with  $\rho$  is less pronounced than in  
250 Figure 5a. It is worth to notice that, for  $\rho < a$ , inside the TPE inclusion (Figure 5a),  $\tau_{zz}$  is the maximum  
251 normal stress, while above it (Figure 5b), it is the least one. Furthermore, a significant shear component  
252  $\tau_{\rho z}$  appears above the inclusion while other shear components  $\tau_{\rho\varphi}$  and  $\tau_{z\varphi}$  vanish as a consequence of axial  
253 symmetry.

254 Myklestad (1942) addressed the problem of a semi-infinite circular cylinder in an infinite solid inside  
255 which a uniform increase in temperature occurs, retrieving analytical solutions for normal and shear stresses  
256 both within and outside the source. Notably, both the models predict the same compressive stress regime  
257 within the sources, with both  $\tau_{\phi\phi}$  and  $\tau_{zz}$  changing sign from inside to outside the cylinder (compare fig.  
258 5 a with Myklestad, 1942, fig. 2, bottom right). Some differences arise in the normal stress components

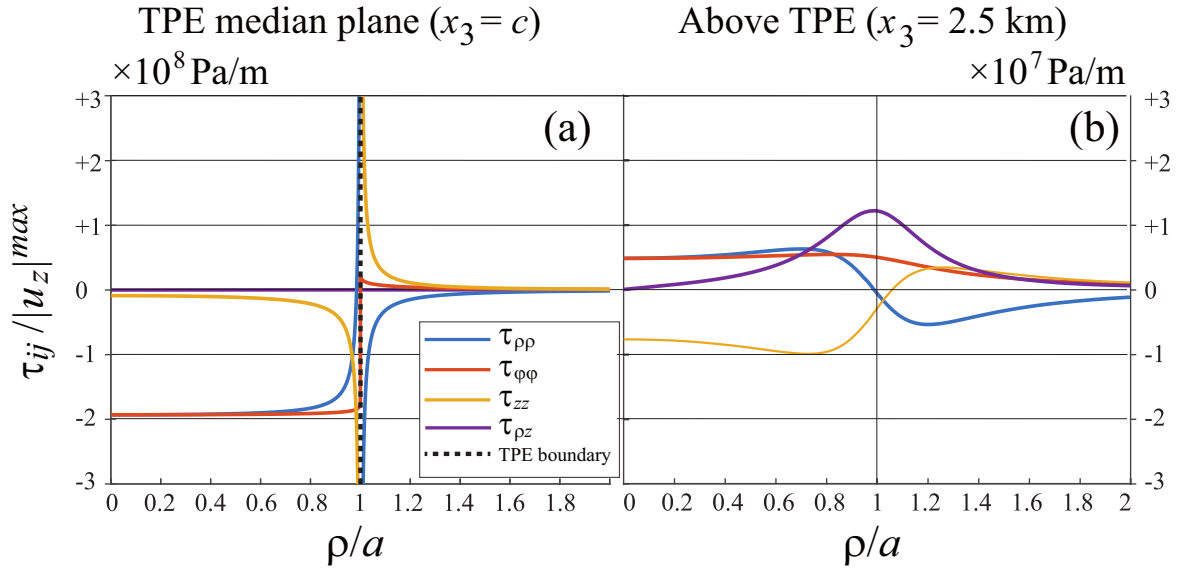


Figure 5: Stress components generated by the TPE disk. a) On the median plane ( $z = c = 3$  km) of the TPE inclusion and b) above it ( $z = 2.5$  km), stress components  $\tau_{ij}$  as functions of horizontal distance from the center  $\rho/a$ .  $|u_z|^{max}$  is the maximum value of vertical uplift. The black dashed line in panel (a) represents the TPE disk boundary  $\rho = a$ . The TPE disk radius is  $a = 2$  km.

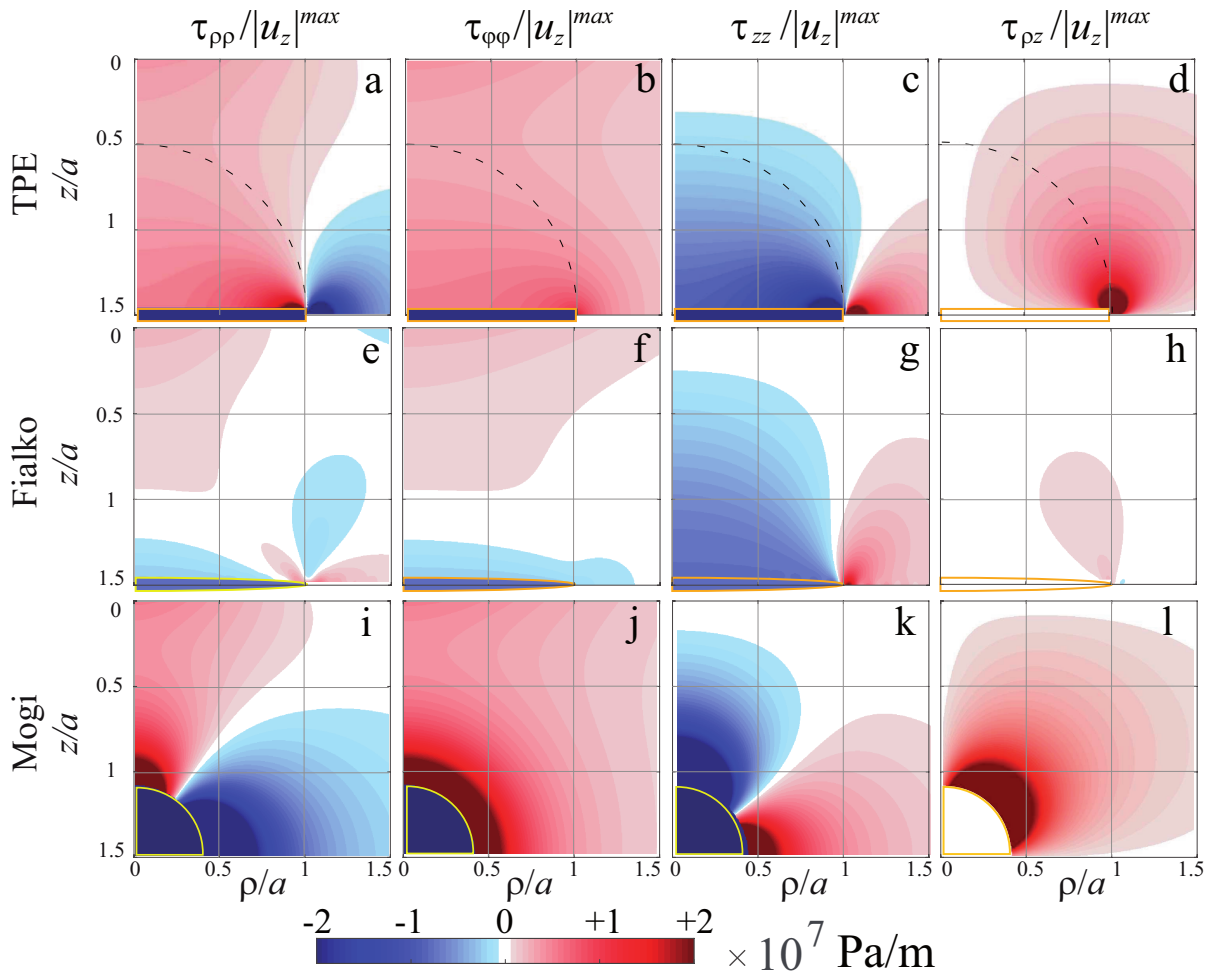
259 calculated on a plane perpendicular to the axis of the cylinder and just below its base (Myklestad, 1942,  
 260 fig. 2, bottom left) with respect to the ones we retrieved above the TPE disk (fig. 5 b), likely due to the  
 261 different geometry of the sources and the free surface condition affecting the results of fig. 5 b.

262 In Figure 4 the TPE disk displacement is compared with results for a point-source approximation of a  
 263 spherical pressurized source (Mogi, 1958, Figure 4a and b, dashed lines) and a penny-shaped crack (Fialko  
 264 et al., 2001, Figure 4c and d, dashed lines); in the following these sources are simply referred as Mogi and  
 265 Fialko, respectively. We recall that outside the spherical TPE shell inclusion considered in Belardinelli et al.  
 266 (2019) for assumed values of  $e_1$ , external radius  $a_2$  and internal radius  $a_1 < a_2$ , (please note the different  
 267 notation with respect to that paper), results are the same of a Mogi source with the same center, radius  
 268  $a = a_2$  and overpressure  $\Delta P = \frac{4}{3}\mu e_1 \frac{a_2^3 - a_1^3}{a_2^3}$ . Accordingly outside the source,  $r > a_2$ , the Mogi source results  
 269 are coincident with the ones for the TPE shell inclusion considered in Belardinelli et al. (2019).

270 In order to compare results for both displacement and stress, we assume the same source depth ( $c =$   
 271 3000 m) while the same volume as in the TPE inclusion is assumed for the Mogi source and the same radius  
 272 ( $a = 2000$  m) for the Fialko source. Results are normalized to  $|u_z|^{max}$ , the maximum uplift predicted by  
 273 each model at the surface of the half-space. In this way we can compare the results of the three kinds of  
 274 sources as if each of them would produce the same (1 m) maximum uplift at free surface, regardless of the

785 particular choice made for the parameters which affect the displacement linearly.  
 786  
 787

788 In Figure 4b and d the displacement is evaluated assuming for the TPE inclusion a smaller radius  $a$  than  
 789 stated in section 3.1, in order to evaluate the effect of a TPE disk with greater  $c/a$  ratio. In the case of the  
 790 stated in section 3.1, in order to evaluate the effect of a TPE disk with greater  $c/a$  ratio. In the case of the  
 791 larger  $c/a$  ratio, both the radial and the vertical displacement components produced at the free surface by the  
 792 Mogi source and TPE disk are indistinguishable (Figure 4b). As the Mogi source already managed to fit in  
 793 good approximation the geodetic data at Campi Flegrei (Dvorak and Berrino, 1991), the similarity between  
 794 these results means that the model we consider cannot be ruled out in the first place in the interpretation  
 795 of the causes of the uplift. However we shall see that the stress field induced by the TPE disk and the Mogi  
 796 source are significantly different, in particular within the sources.  
 797  
 798  
 799  
 800  
 801  
 802



832 Figure 6: Depth maps of cylindrical stress components. They are plotted over the  $\rho - z$  section between the free surface ( $\frac{z}{a} = 0$ )  
 833 and the depth of the sources ( $\frac{z}{a} = 1.5$ ). a)-d): cylinder-shaped TPE source; e)-h): Fialko source; i)-l): Mogi source. Stress  
 834 values of each model are divided by  $|u_z|^{max}$ , the maximum uplift at the Earth surface predicted by the same model. Horizontal  
 835 and vertical axes are normalized to the radius  $a$  of the TPE inclusion. The singular components of the the TPE disk stresses  
 836 (obtained from equation 12 are not convergent along the circle  $r = a$  (black dashed line in panels a)-d) where the solution  
 837 should be compared by analytical continuation.  
 838  
 839  
 840

841  
842 284 As for the Fialko model (Figure 4*c* and *d*), the displacement components show similar trends, but the  
843  
844 285 maximum horizontal displacement in the case of the TPE source occurs farther from the origin than in  
845  
846 286 the case of the Fialko source, regardless of the  $c/a$  ratio. Furthermore, the amplitudes of displacement  
847  
848 287 components computed by TPE inclusion decrease more slowly away from the source than for Fialko. This  
849  
850 288 means that the TPE model may describe situations where the horizontal deformation is not negligible even  
851  
852 289 at considerable distances from the center of the area of maximum uplift, without requiring a greater depth.

853 290 Depth maps of the stress components for all the models considered are reported in Figure 6. For the  
854  
855 291 Mogi model, the strain (supplementary material) and stress components were retrieved from the expression  
856  
857 292 for displacement reported by Bonafede and Ferrari (2009) and the constitutive relation (2) with  $\Delta T = 0$  and  
858  
859 293  $\Delta p = 0$ . The stress components of the Fialko model were instead obtained through numerical integration  
860  
861 294 of the analytical expressions published in Fialko et al. (2001): this has been achieved through a modified  
862  
863 295 version of the USGS dMODELS tool (Battaglia, 2017). The stress field of the TPE source (Figure 6*a*, *b*, *c*  
864  
865 296 and *d*) differs considerably from the Mogi source (Figure 6*i*, *j*, *k* and *l*) and even more from Fialko (Figure  
866  
867 297 6*e*, *f*, *g* and *h*). Similarities may be noted between the  $\tau_{zz}$  components for the TPE inclusion and Fialko,  
868  
869 298 while only TPE and Mogi sources display a significant  $\tau_{\rho z}$  component. It is important to note finally that an  
870  
871 299 extremely high deviatoric stress is present within the TPE source (as shown in Figure 5), while it vanishes  
872  
873 300 within both the Mogi and Fialko sources where an isotropic pressure applies.

873 301 The differences between the stress components related to distinct models give rise respectively to a different  
874  
875 302 distribution of expected fault mechanisms on the basis of the Frohlich triangle (Frohlich, 2001). According to  
876  
877 303 this method, the favoured fault mechanisms in each point of the medium is computed by evaluating principal  
878  
879 304 stresses and related axes orientations.

880 305 Plots of the expected fault mechanisms and the maximum shear stress on the same vertical section as in  
881  
882 306 Figure 6 are reported for each model in Figure 7. The TPE source is associated with normal faults over an  
883  
884 307 area spanning from the free surface to the upper base of the disk (Figure 2). The lateral extension of this  
885  
886 308 domain reduces progressively with depth, laterally bounded by a region where thrust faults are expected.  
887  
888 309 This pattern is similar to that related to the Mogi source (Figure 7*c*); in particular, both give rise to thrust  
889  
890 310 faults on their median plane, but it is markedly different in the case of the the Fialko source (Figure 7*b*).  
891  
892 311 It is important to note that inside the TPE source, thrust mechanisms are predicted with extremely high  
893  
894 312 deviatoric stress, while the other sources (Mogi and Fialko) are pressurized cavities with internal vanishing  
895  
896

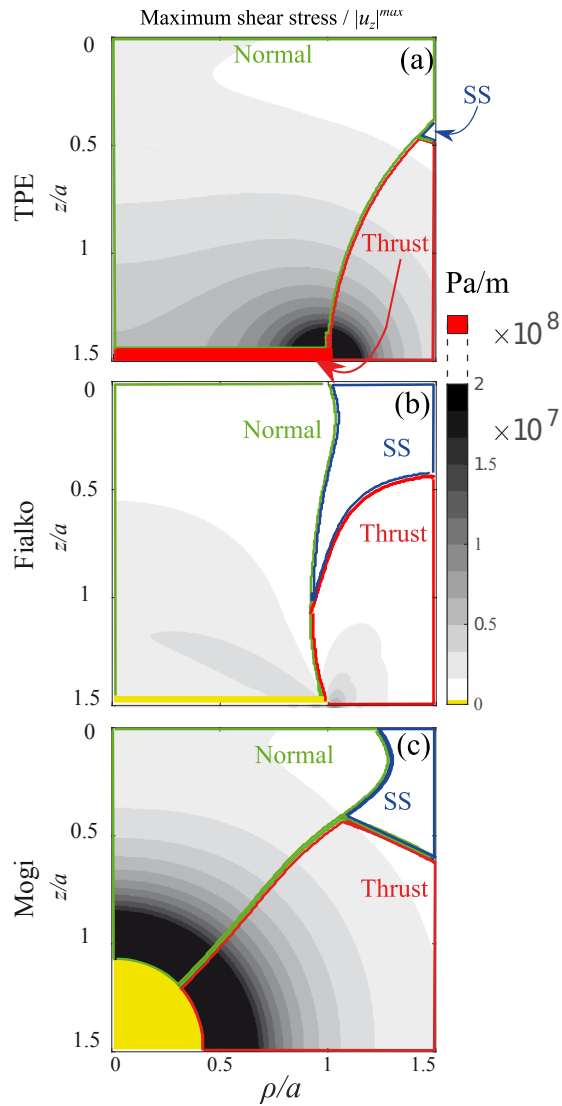


Figure 7: Vertical sections of maximum shear stress. The maximum shear stress (gray coloured palette) is plotted over the  $\rho - z$  section between the free surface ( $\frac{z}{a} = 0$ ) and the depth of the sources ( $\frac{z}{a} = 1.5$ ). (a) TPE inclusion, (b) Fialko, (c) Mogi source. Contour includes areas in which each source promotes Normal, Thrust and Strike-Slip (SS) mechanisms. Horizontal and vertical axes are normalized to the radius  $a$  of the TPE inclusion. In panels (b) and (c) the internal domain of the sources, where shear stress vanishes, is represented in yellow.

313 deviatoric stress components.

314 In order to test the reasonability of parameters of the different models when applied to the Campi  
 315 Flegrei unrest it is necessary to reproduce the actual deformation field observed during an unrest phase.  
 316 We considered the data recorded through the EDM technique (changes of distance between benchmarks) and  
 317 the vertical displacement recorded by leveling during the period June 1980 - June 1983 (Figure 1). The  
 318 maximum uplift was 1.80 m in November 1984 (w.r.t. January 1982), about three times the uplift at the  
 319 end of the considered observation period (Figure 1b). In order to accurately infer model parameter values  
 320 from inversion of surface data, the hypothesis of a homogeneous medium, common to three different models

Table 1: Results of the inversions and misfits associated to the three models considered. Parameters estimated by inversion of surface data are in bold. TPE-Disk refers to the TPE disk models with fixed aspect ratios  $\frac{d}{a} = 0.3$ . In the case of the Mogi model the parameter estimated by inversion is  $Q = \Delta P \cdot a^3 \frac{1-\nu}{\mu}$ , while in the case of the TPE shell  $Q = \frac{4}{3} e_0 (1 + \nu) (a_2^3 - a_1^3)$ , representing the scaling factor for displacement at the surface. We assume  $a = a_2 = 0.843$  km as in figures 6-7 and  $\frac{a_2 - a_1}{a_2} = 0.3$ , being  $a_1$  the internal radius of the TPE shell. Values of  $\Delta P$  for the Mogi model and  $e_0$  for the TPE-shell are retrieved from the  $Q$  value estimated by inversion. Values of the  $\Delta p$  are retrieved from  $e_0$  estimated through inversion assuming  $\Delta T = 100$  K. The misfit in the last column refers to the sum of the absolute difference between predicted and observed EDM and leveling.

Model	$c$ (km)	$Q$ ( $\text{m}^3$ )	$\Delta P$ (MPa)	$e_0$	$a$ (km)	$\Delta p$ (MPa)	Total misfit (m)
Mogi	<b>2.7</b>	<b><math>5.121 \cdot 10^6</math></b>	64.1	–	–	–	3.868
Fialko	<b>2.9</b>	–	<b>3</b>	–	<b>2.5</b>	–	4.678
TPE-DISK	<b>1.9</b>	–	–	<b><math>1.7 \cdot 10^{-3}</math></b>	<b>1.9</b>	21	2.904
TPE-SHELL	<b>2.7</b>	<b><math>5.121 \cdot 10^6</math></b>	–	$8.1 \cdot 10^{-3}$	–	214	3.868

here considered, is inadequate (Trasatti et al., 2011). We are aware of this, but at least for the purpose of model comparison, the inversion of surface data is suitable.

For each model, a direct search in the parameter space was performed using a Monte Carlo sampling. Then the posterior probability density distribution (PPD) of each parameter was estimated by Bayesian inference (*e.g.* Sambridge, 1999). In Table 1 best fit values of parameters allowed free to vary during the inversion are indicated with bold numbers. Other values reported in Table 1 refer to parameters depending on free parameters and the fixed ones. Results for the Mogi source allow us to estimate the parameters of a TPE-shell model (Belardinelli et al., 2019) with the same center, an external radius  $a_2$  and an internal radius  $a_1$  assigned by fixing the ratio  $\frac{a_2 - a_1}{a_2} = 0.3$ . For the TPE-disk we fixed the geometrical ratio  $\frac{d}{a} < 1$  at different values finding that smaller values require shallower and wider disks to reproduce data and the minimum misfit is realized by fixing  $\frac{d}{a} = 0.3$ . From Table 1 we can see that the TPE-disk provides the minimum misfit among the three considered models. An Akaike test (*e.g.* Hurvich and Tsai, 1989) shows that the misfit improvement justifies the increase in the number of parameters.

In Figure 8 we can note that employing best fit values of parameters, the TPE-disk reproduces well both kinds of data, while the Mogi model describes worse leveling data and the Fialko model underestimates EDM data. It is worth to mention that, according to Dieterich and Decker (1975), horizontal data have greater resolving power among different deformation source models.

## 5. DISCUSSION

We consider a disk-shaped thermo-poro-elastic inclusion embedded in a poro-elastic semi-infinite medium bounded by a free surface (Figure 2) in order to model a sudden input of hot and pressurized fluids from

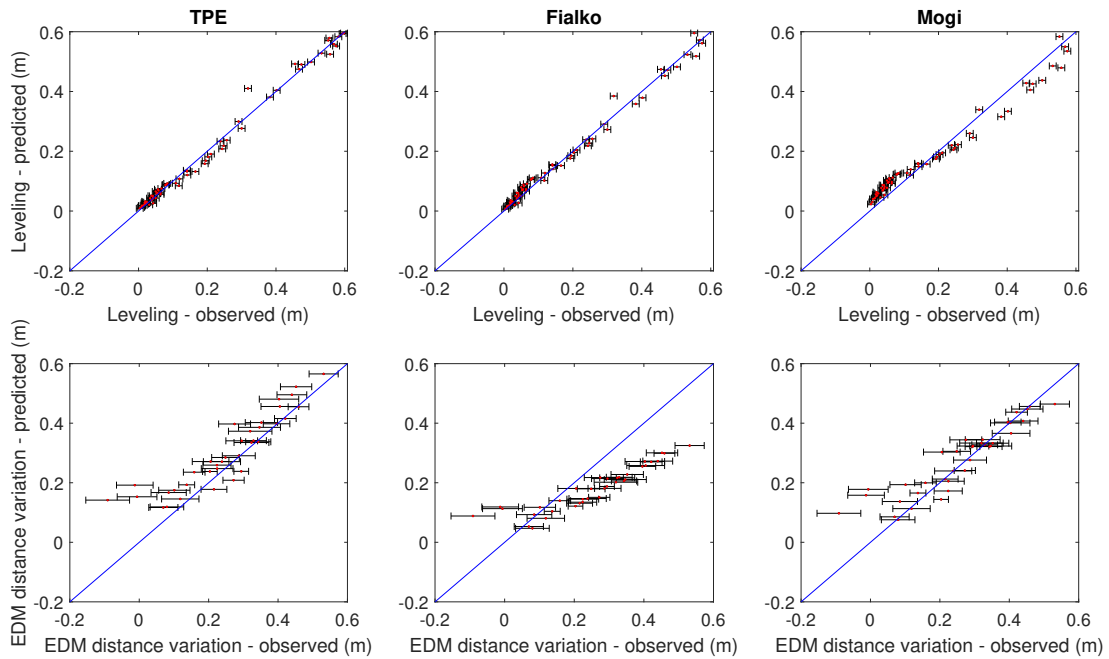


Figure 8: Results of the inversion of leveling (upper row) and EDM (lower row) data of displacement for the period June 1980 and in June 1983 at Campi Flegrei using three different source models (indicated).

an underlying magma body into a permeable region as envisaged by many authors for the Campi Flegrei caldera (e.g. Chiodini et al., 2015; Trasatti et al., 2019, Calò and Tramelli, 2018 ). Our semi-analytical computations are tested with a fully numerical approach (Figure S1).

The present model is intended to describe surface ground deformation and stress field at depth in hydrothermal regions, and we focus on the 1982-84 unrest episode at Campi Flegrei caldera. The adopted elastic parameters for the external medium and the inclusion represent highly-porous sedimentary rocks which constitute the upper layers of Campi Flegrei stratigraphy.

We compare our results to those of two axially-symmetric source models that have been employed in similar situations: Mogi and Fialko sources. The displacements on the free surface (Figure 4) are in good agreement with those of a Mogi source, in the case of a large  $c/a$  ratio, while there are some differences with the Fialko source for both small and large  $c/a$  ratio; in that, in our case, the amplitudes of the displacement components decrease more slowly with distance from the source.

All considered sources promote normal fault mechanisms above them (Figure 7) in agreement with data at Campi Flegrei (Figure 3), and thrust mechanisms laterally. A strong deviatoric stress is retrieved within the TPE inclusion (e.g. Figure 5a), unlike Mogi and Fialko sources. The large deviatoric stress inside the

1065  
1066 356 TPE disk is able to promote thrust faults and exceeds by one order of magnitude the values at the same  
1067  
1068 357 depth outside the source, explaining the increasing percentage of thrust fault mechanisms at increasing depth  
1069  
1070 358 (Figure 3).

1071  
1072 359 Results of inversion (Table 1) show that in order to obtain 1/3 of the maximum uplift observed at  
1073  
1074 360 the surface during the 1982-1984 unrest at Campi Flegrei, the Mogi and Fialko sources require magma  
1075  
1076 361 overpressures of  $\Delta P = 64.1$  and 3 MPa, respectively, for a reasonable value of the radius of the Mogi source,  
1077  
1078 362  $a = 843$  m, while the TPE-disk requires a pore pressure change of  $\Delta p = 21$  MPa, for a temperature change  
1079  
1080 363  $\Delta T = 100$  K (we recall that according to equation 3, for the same uplift, the requested  $\Delta p$  decreases with  
1081  
1082 364 increasing  $\Delta T$ ). Following Trasatti et al. (2011), we can assume that to realize the 1.8 m of maximum uplift  
1083  
1084 365 observed in November 1983, these pressure estimates must be scaled by a factor of 3, leading to unrealistically  
1085  
1086 366 high magma overpressure values for the Mogi source ( $\Delta P \approx 190$  MPa,  $Q \approx 1.5 \cdot 10^7$  m<sup>3</sup>) with respect to  
1087  
1088 367 lithostatic values at less than 3 km depth. These parameters are comparable with previous estimates (e.g.  
1089  
1090 368 Berrino et al., 1984,  $Q = 1.3 \cdot 10^7$  m<sup>3</sup>,  $c = 2.8 \pm 0.2$  km and Bonafede and Ferrari, 2009,  $Q = 1.6 \cdot 10^7$   
1091  
1092 369 m<sup>3</sup>,  $c = 3$  km). A previous inversion for the Fialko source (Amoruso et al., 2008), despite considering a  
1093  
1094 370 different rigidity modulus with respect to the present work, confirms that this kind of source leads to much  
1095  
1096 371 lower overpressure estimation than that of the Mogi one ( $\Delta P = 7$  MPa,  $c = 3$  km,  $a = 2.7$  km). The same  
1097  
1098 372 scaling (factor of 3) of the estimates in Table 1 leads, however, to unrealistically high pore pressure changes  
1099  
1100 373  $\Delta p$  also in the case of both the TPE-disk and the TPE-shell. Therefore, we can exclude that the big uplift  
1101  
1102 374 observed during that episode of unrest was totally due to the hydrothermal processes modeled by the TPE  
1103  
1104 375 source. Instead the present model could be suitable to represent subsequent smaller episodes of uplift ( $\sim$   
1105  
1106 376 cm) at Campi Flegrei (1989, 1994, 2000 and 2006), that were most likely related to shallow hydrothermal  
1107  
1108 377 processes (D’Auria et al., 2011). Actually, since 1989 volcanotectonic hypocenters have been confined almost  
1109  
1110 378 exclusively between 1 and 3 km depth, within the area of most important geothermal output (D’Auria et al.,  
1111  
1112 379 2011).

1110 380 The 1982-84 unrest could be likely ascribed to the combined effects of both the emplacement of a magma  
1111  
1112 381 body at shallow depths and hydrothermal processes. According to Trasatti et al. (2011), the magmatic  
1113  
1114 382 intrusion can be modeled as due to a dike emplacement in a compressive stress regime region below the  
1115  
1116 383 center of the caldera, consisting of a tensile dislocation with a reverse-slip component. As the TPE source  
1117  
1118 384 provides strong compressive stress regime inside, it can give support to the model of Trasatti et al. (2011)

1121  
1122 385 suggesting that during dike emplacement, the latter may have met the TPE source. Furthermore thrust  
1123  
1124 386 faulting mechanisms are reported by Ekstrom (1994) and Nettles and Ekstrom (1998) in different volcanic  
1125  
1126 387 regions.

1127  
1128 388 The Fialko model requires smaller  $\Delta P$  than the Mogi source (Table 1). However, the presence of a  
1129  
1130 389 large magmatic reservoir at 2.9 km depth (Table 1) seems incompatible with the brittle rheology and with  
1131  
1132 390 temperatures met during deep drilling in nearby wells (400° C at 3 km depth, e.g. Carlino et al., 2012 ).With  
1133  
1134 391 respect to both Fialko and Mogi model , the main advantage of the TPE inclusion is the retrieval of a stress  
1135  
1136 392 field at different depths with strong differences between the interior and the exterior of the source, which  
1137  
1138 393 could account for the high heterogeneity of closely located seismic mechanisms observed at Campi Flegrei  
1139  
1140 394 during the 1982-84 episode. Moreover the TPE source: (i) differently from the Fialko model, can easily  
1141  
1142 395 explain the increase of the percentage of thrust mechanisms over depth (Figure 3); (ii) compared to the  
1143  
1144 396 Fialko model for the same maximum uplift at the surface the TPE disk generates much larger shear stresses  
1145  
1146 397 (Figure 7). The reason is that the crack represented by the Fialko model is very efficient in producing high  
1147  
1148 398 displacement with low overpressure and then low stresses.

1149  
1150 399 All models fail to produce strike slip faulting apart from shallow far field regions, where in any case the  
1151  
1152 400 induced shear stress is small (Figure 7). Instead at Campi Flegrei strike-slip faulting is frequent in near field  
1153  
1154 401 (Figure 3). However, even a small additional component of regional stress may easily exchange the order of  
1155  
1156 402  $\tau_{zz}$  and  $\tau_{\rho\rho}$  (Figure 5b), so that strike-slip faulting can be promoted in external regions close to TPE disk.

1157  
1158 403 Both poro-elastic and thermo-elastic effects are considered in our model. Temperature changes are more  
1159  
1160 404 effective than pore-pressure changes in inducing strain due to the relative magnitudes of  $\alpha\Delta T$  and  $\frac{\Delta p}{H}$  in eq.  
1161  
1162 405 (3) for reasonable values of sudden increases of  $\Delta T$  and  $\Delta p$ . However it may be argued that, as demonstrated  
1163  
1164 406 by previous studies on ground deformation in hydrothermal regions (e.g Hutnak et al., 2009; Fournier and  
1165  
1166 407 Chardot, 2012), surface uplift due to the fluid migration from a deep input of hot and pressurized fluids is  
1167  
1168 408 predominantly driven by the poro-elastic contribution for short timescales (as depending on the hydraulic  
1169  
1170 409 diffusivity and the depth of the basis of the reservoir). In the present work we assume changes in  $\Delta p$  and  
1171  
1172 410  $\Delta T$  to occur suddenly and uniformly over a specific volume at basis of the reservoir, that is the TPE, so  
1173  
1174 411 that the model does not account for fluid migration and it is suited to estimate the contribution to the  
1175  
1176 412 uplift increase observed in an hydrothermal region during a given time interval. In order to reproduce the  
1177  
1178 413 temporal dependence of an unrest process, after the sudden  $\Delta p$  and  $\Delta T$  establishment within the TPE

1177  
1178 414 region, it might be necessary to model the progressive migration of the initial changes in temperature and  
1179  
1180 415 pore pressure that could affect a wider region, starting from the inclusion considered here. For the afore-  
1181  
1182 416 mentioned reasons, we expect that during unrest episodes also the subsidence following the peak of uplift  
1183  
1184 417 may be mainly related to the decrease of  $\Delta p$  due to the fluid discharge from the TPE inclusion toward the  
1185  
1186 418 hydrostatic aquifers above, while  $\Delta T$  may be considered unchanged during this stage. The assessment of  
1187  
1188 419 this hint is left for future developments of the present study.

## 1190 420 6. CONCLUSIVE REMARKS

1191  
1192 421 The main result of the present work is that unlike Mogi and Fialko sources, the TPE source here proposed  
1193  
1194 422 allows for a large deviatoric stress promoting thrust fault mechanisms inside. Accordingly, the heterogeneity  
1195  
1196 423 of focal mechanisms observed at Campi Flegrei as in other volcanic provinces supports the existence of a TPE  
1197  
1198 424 source. Moreover, inverted-displacement results indicate that a TPE source can better model the surface  
1199  
1200 425 deformation than other sources. As suggested by the case of the 1982-1984 unrest episode at Campi Flegrei a  
1201  
1202 426 TPE source can be considered as part of a complex system of deformation sources where both hydrothermal  
1203  
1204 427 and magmatic processes contribute to the observed displacement field.

1205 428 Another major advantage of the TPE disk model over the Mogi one is that a large pore pressure change  
1206  
1207 429  $\Delta p$  may be easily and quickly accomplished through vertical motion of the magmatic volatiles exolved at  
1208  
1209 430 lithostatic pressure by an underlying magma reservoir. Instead the pressure  $P$  of a dense and highly viscous  
1210  
1211 431 magma presumably decreases faster while uprising according to a "magmastatic" gradient (at least). Thus  
1212  
1213 432 within the same depth range, large  $\Delta p$  values are transferred much more easily and faster than similar  $\Delta P$   
1214  
1215 433 values.

1216 434 Further developments of this model could take into account the heterogeneity of the poro-elastic half-  
1217  
1218 435 space, attempting at simulating the observed stratigraphy at Campi Flegrei or in other volcanic areas.  
1219  
1220 436 We conclude remarking that such analytical or semi-analytical models as those we consider here are of  
1221  
1222 437 fundamental importance when it comes: *i*) to calibrate and assess the validity of more complex numerical  
1223  
1224 438 models; *ii*) to study sensitivities without having to re-grid, as may be necessary in numerical models; *iii*) to  
1225  
1226 439 quantify driving parameters using fast models in inversion / data assimilation ; *iv*) to study forecasts and  
1227  
1228 440 their range of uncertainties much easier than in numerical models because of the calculation speed.

1233  
1234 441 **7. Acknowledgements**  
1235

1236  
1237 442 Two anonymous reviewers are gratefully acknowledged for constructive comments and fruitful suggestions  
1238  
1239 443 that helped in improving the revised version of the manuscript. We thank Elisa Trasatti for useful suggestions  
1240  
1241 444 and Luca D’Auria for the catalog of focal mechanisms related to the 1982-1984 unrest episode at Campi  
1242  
1243 445 Flegrei.  
1244

1245 446 **References**  
1246

- 1247 447 Amoruso, A., Crescentini, L., and Berrino, G. (2008). Simultaneous inversion of deformation and gravity  
1248  
1249 448 changes in a horizontally layered half-space: evidences for magma intrusion during the 1982–1984 unrest  
1250  
1251 449 at campi flegrei caldera (italy). *Earth and Planetary Science Letters*, 272(1-2):181–188.  
1252  
1253 450 Amoruso, A., Crescentini, L., and Sabbetta, I. (2014). Paired deformation sources of the campi flegrei caldera  
1254  
1255 451 (italy) required by recent (1980-2010) deformation history. *Journal of Geophysical Research: Solid Earth*,  
1256  
1257 452 119(2):858–879.  
1258  
1259 453 Battaglia, M. (2017). dmodels: A software package for modeling volcanic deformation. In *EGU General*  
1260  
1261 454 *Assembly Conference Abstracts*, volume 19, page 4203.  
1262  
1263 455 Battaglia, M., Troise, C., Obrizzo, F., Pingue, F., and Natale, G. D. (2006). Evidence for fluid migration as  
1264  
1265 456 the source of deformation at campi flegrei caldera (italy). *Geophysical Research Letters*, 33(1).  
1266  
1267  
1268 457 Belardinelli, M., Bonafede, M., and Nespoli, M. (2019). Stress heterogeneities and failure mechanisms  
1269  
1270 458 induced by temperature and pore-pressure increase in volcanic regions. *Earth and Planetary Science*  
1271  
1272 459 *Letters*, 525:115765.  
1273  
1274 460 Berrino, G. (1994). Gravity changes induced by height-mass variations at the campi flegrei caldera. *Journal*  
1275  
1276 461 *of Volcanology and Geothermal Research*, 61(3):293 – 309. International Conference on Active Volcanoes  
1277  
1278 462 and Risk Mitigation.  
1279  
1280 463 Berrino, G., Corrado, G., Luongo, G., and Toro, B. (1984). Ground deformation and gravity changes  
1281  
1282 464 accompanying the 1982 pozzuoli uplift. *Bulletin volcanologique*, 47(2):187–200.  
1283  
1284 465 Bonafede, M. and Ferrari, C. (2009). Analytical models of deformation and residual gravity changes due to  
1285  
1286 466 a mogi source in a viscoelastic medium. *Tectonophysics*, 471(1-2):4–13.  
1287  
1288

- 1289  
1290 467 Calò, M. and Tramelli, A. (2018). Anatomy of the campi flegrei caldera using enhanced seismic tomography  
1291  
1292 468 models. *Scientific Reports*, 8(16254).  
1293  
1294 469 Carlino, S., Somma, R., Troise, C., and Natale, G. D. (2012). The geothermal exploration of campanian vol-  
1295 canoes: Historical review and future development. *Renewable and Sustainable Energy Reviews*, 16(1):1004  
1296 470 – 1030.  
1297  
1298 471  
1299  
1300 472 Chiarabba, C. and Moretti, M. (2006). An insight into the unrest phenomena at the campi flegrei caldera  
1301 473 from vp and vp/vs tomography. *Terra Nova*, 18(6):373–379.  
1302  
1303  
1304 474 Chiodini, G., Vandemeulebrouck, J., Caliro, S., D’Auria, L., Martino, P. D., Mangiacapra, A., and Petrillo,  
1305 Z. (2015). Evidence of thermal-driven processes triggering the 2005–2014 unrest at campi flegrei caldera.  
1306 475  
1307 476 *Earth and Planetary Science Letters*, 414:58–67.  
1308  
1309  
1310 477 D’Auria, L., Giudicepietro, F., Aquino, I., Borriello, G., Del Gaudio, C., Lo Bascio, D., Martini, M.,  
1311 478 Ricciardi, G. P., Ricciolino, P., and Ricco, C. (2011). Repeated fluid-transfer episodes as a mechanism for  
1312 479 the recent dynamics of campi flegrei caldera (19892010). *Journal of Geophysical Research: Solid Earth*,  
1313 480 116(B4).  
1314  
1315  
1316 481 D’Auria, L., Massa, B., Cristiano, E., Gaudio, C. D., Giudicepietro, F., Ricciardi, G., and Ricco, C. (2014).  
1317 482 Retrieving the stress field within the campi flegrei caldera (southern italy) through an integrated geodetical  
1318 483 and seismological approach. *Pure and Applied Geophysics*, 172(11):3247–3263.  
1319  
1320  
1321 484 De Natale, G., Troise, C., Mark, D., Mormone, A., Piochi, M., Vito, M. A. D., Isaia, R., Carlino, S.,  
1322 485 Barra, D., and Somma, R. (2016). The campi flegrei deep drilling project (cfddp): New insight on  
1323 486 caldera structure, evolution and hazard implications for the naples area (southern italy). *Geochemistry,*  
1324 487 *Geophysics, Geosystems*, 17(12):4836–4847.  
1325  
1326  
1327 488 De Vivo, B. (2006). *Volcanism in the Campania Plain: Vesuvius, Campi Flegrei and Ignimbrites*, volume 9.  
1328 489 Elsevier.  
1329  
1330  
1331 490 Di Vito, M. A., Acocella, V., Aiello, G., Barra, D., Battaglia, M., Carandente, A., Gaudio, C. D., Vita,  
1332 491 S. D., Ricciardi, G. P., Ricco, C., et al. (2016). Magma transfer at campi flegrei caldera (italy) before the  
1333 492 1538 ad eruption. *Scientific reports*, 6:32245.  
1334  
1335  
1336  
1337  
1338  
1339  
1340  
1341  
1342  
1343  
1344

- 1345  
1346 493 Di Vito, M. A., Isaia, R., Orsi, G., Southon, J. D., Vita, S. D., d'Antonio, M., Pappalardo, L., and Piochi,  
1347  
1348 494 M. (1999). Volcanism and deformation since 12,000 years at the campi flegrei caldera (italy). *Journal of*  
1349  
1350 495 *Volcanology and Geothermal Research*, 91(2-4):221–246.  
1351  
1352 496 Dieterich, J. H. and Decker, R. W. (1975). Finite element modeling of surface deformation associated with  
1353  
1354 497 volcanism. *Journal of Geophysical Research (1896-1977)*, 80(29):4094–4102.  
1355  
1356 498 Dvorak, J. J. and Berrino, G. (1991). Recent ground movement and seismic activity in campi flegrei, southern  
1357  
1358 499 italy: Episodic growth of a resurgent dome. *Journal of Geophysical Research: Solid Earth*, 96(B2):2309–  
1359  
1360 500 2323.  
1361  
1362 501 Ekstrom, G. (1994). Anomalous earthquakes on volcano ring-fault structures. *Earth and Planetary Science*  
1363  
1364 502 *Letters*, 128(3):707 – 712.  
1365  
1366 503 Eshelby, J. D. (1957). The determination of the elastic field of an ellipsoidal inclusion, and related problems.  
1367  
1368 504 *Proc. R. Soc. Lond. A*, 241(1226):376–396.  
1369  
1370 505 Feigl, K. L., Gasperi, J., Sigmundsson, F., and Rigo, A. (2000). Crustal deformation near hengill volcano,  
1371  
1372 506 iceland 19931998: Coupling between magmatic activity and faulting inferred from elastic modeling of  
1373  
1374 507 satellite radar interferograms. *Journal of Geophysical Research: Solid Earth*, 105(B11):25655–25670.  
1375  
1376 508 Fialko, Y., Khazan, Y., and Simons, M. (2001). Deformation due to a pressurized horizontal circular crack in  
1377  
1378 509 an elastic half-space, with applications to volcano geodesy. *Geophysical Journal International*, 146(1):181–  
1379  
1380 510 190.  
1381  
1382 511 Fournier, N. and Chardot, L. (2012). Understanding volcano hydrothermal unrest from geodetic observa-  
1383  
1384 512 tions: Insights from numerical modeling and application to white island volcano, new zealand. *Journal of*  
1385  
1386 513 *Geophysical Research: Solid Earth*, 117(B11).  
1387  
1388 514 Frohlich, C. (2001). Display and quantitative assessment of distributions of earthquake focal mechanisms.  
1389  
1390 515 *Geophysical Journal International*, 144(2):300–308.  
1391  
1392 516 Gaeta, F. S., Peluso, F., Arienzo, I., Castagnolo, D., Natale, G. D., Milano, G., Albanese, C., and Mita,  
1393  
1394 517 D. G. (2003). A physical appraisal of a new aspect of bradyseism: The miniuplifts. *Journal of Geophysical*  
1395  
1396 518 *Research: Solid Earth*, 108(B8).  
1397  
1398  
1399  
1400

- 1401  
1402 519 Geertsma, J. et al. (1973). Land subsidence above compacting oil and gas reservoirs. *Journal of Petroleum*  
1403  
1404 520 *technology*, 25(06):734–744.  
1405  
1406 521 Hill, D. P. (2006). Unrest in long valley caldera, california, 1978–2004. *Geological Society, London, Special*  
1407  
1408 522 *Publications*, 269(1):1–24.  
1409  
1410  
1411 523 Hurvich, C. M. and Tsai, C.-L. (1989). Regression and time series model selection in small samples.  
1412  
1413 524 *Biometrika*, 76(2):297–307.  
1414  
1415 525 Hutnak, M., Hurwitz, S., Ingebritsen, S., and Hsieh, P. (2009). Numerical models of caldera deformation:  
1416  
1417 526 Effects of multiphase and multicomponent hydrothermal fluid flow. *Journal of Geophysical Research: Solid*  
1418  
1419 527 *Earth*, 114(B4).  
1420  
1421 528 Jackson, J. D. (1999). Classical electrodynamics.  
1422  
1423  
1424 529 Judenherc, S. and Zollo, A. (2004). The bay of naples (southern italy): Constraints on the volcanic structures  
1425  
1426 530 inferred from a dense seismic survey. *Journal of Geophysical Research: Solid Earth*, 109(B10).  
1427  
1428 531 Lima, A., Vivo, B. D., Spera, F. J., Bodnar, R. J., Milia, A., Nunziata, C., Belkin, H. E., and Cannatelli, C.  
1429  
1430 532 (2009). Thermodynamic model for uplift and deflation episodes (bradyseism) associated with magmatic–  
1431  
1432 533 hydrothermal activity at the campi flegrei (italy). *Earth-Science Reviews*, 97(1-4):44–58.  
1433  
1434 534 Macedonio, G., Giudicepietro, F., D’auria, L., and Martini, M. (2014). Sill intrusion as a source mechanism  
1435  
1436 535 of unrest at volcanic calderas. *Journal of Geophysical Research: Solid Earth*, 119(5):3986–4000.  
1437  
1438 536 McTigue, D. F. (1986). Thermoelastic response of fluid-saturated porous rock. *Journal of Geophysical*  
1439  
1440 537 *Research: Solid Earth*, 91(B9):9533–9542.  
1441  
1442  
1443 538 Mindlin, R. D. (1936). Force at a point in the interior of a semi-infinite solid. *Physics*, 7(5):195–202.  
1444  
1445 539 Mogi, K. (1958). Relations between the eruptions of various volcanoes and the deformations of the ground  
1446  
1447 540 surfaces around them. *Earthq Res Inst*, 36:99–134.  
1448  
1449 541 Myklestad, N. O. (1942). *Two problems of thermal stress in the infinite solid*, volume 9. Cornell University.  
1450  
1451  
1452 542 Nettles, M. and Ekstrom, G. (1998). Faulting mechanism of anomalous earthquakes near brdarbunga volcano,  
1453  
1454 543 iceland. *Journal of Geophysical Research: Solid Earth*, 103(B8):17973–17983.  
1455  
1456

- 1457  
1458 544 Perkins, T., Gonzalez, J., et al. (1984). Changes in earth stresses around a wellbore caused by radially  
1459  
1460 545 symmetrical pressure and temperature gradients. *Society of Petroleum Engineers Journal*, 24(02):129–  
1461  
1462 546 140.
- 1463  
1464 547 Perkins, T., Gonzalez, J., et al. (1985). The effect of thermoelastic stresses on injection well fracturing.  
1465  
1466 548 *Society of Petroleum Engineers Journal*, 25(01):78–88.
- 1467  
1468  
1469 549 Prejean, S., Ellsworth, W., Zoback, M., and Waldhauser, F. (2002). Fault structure and kinematics of  
1470  
1471 550 the long valley caldera region, california, revealed by high-accuracy earthquake hypocenters and focal  
1472  
1473 551 mechanism stress inversions. *Journal of Geophysical Research: Solid Earth*, 107(B12):ESE 9–1–ESE 9–19.
- 1474  
1475 552 Rice, J. R. and Cleary, M. P. (1976). Some basic stress diffusion solutions for fluid-saturated elastic porous  
1476  
1477 553 media with compressible constituents. *Reviews of Geophysics*, 14(2):227–241.
- 1478  
1479 554 Rinaldi, A., Todesco, M., and Bonafede, M. (2010). Hydrothermal instability and ground displacement at  
1480  
1481 555 the campi flegrei caldera. *Physics of the Earth and Planetary Interiors*, 178(3):155 – 161.
- 1482  
1483 556 Robertson, R. M. and Kilburn, C. R. (2016). Deformation regime and long-term precursors to eruption at  
1484  
1485 557 large calderas: Rabaul, papua new guinea. *Earth and Planetary Science Letters*, 438:86–94.
- 1486  
1487  
1488 558 Sambridge, M. (1999). Geophysical inversion with a neighbourhood algorithmII. Appraising the ensemble.  
1489  
1490 559 *Geophysical Journal International*, 138(3):727–746.
- 1491  
1492 560 Sorey, M. L., Suemnicht, G. A., Sturchio, N. C., and Nordquist, G. A. (1991). New evidence on the  
1493  
1494 561 hydrothermal system in long valley caldera, california, from wells, fluid sampling, electrical geophysics,  
1495  
1496 562 and age determinations of hot-spring deposits. *Journal of Volcanology and Geothermal Research*, 48(3–  
1497  
1498 563 4):229–263.
- 1499  
1500 564 Tizzani, P., Battaglia, M., Castaldo, R., Pepe, A., Zeni, G., and Lanari, R. (2015). Magma and fluid  
1501  
1502 565 migration at yellowstone caldera in the last three decades inferred from insar, leveling, and gravity mea-  
1503  
1504 566 surements. *Journal of Geophysical Research: Solid Earth*, 120(4):2627–2647.
- 1505  
1506 567 Todesco, M., Costa, A., Comastri, A., Colleoni, F., Spada, G., and Quarenì, F. (2014). Vertical ground  
1507  
1508 568 displacement at campi flegrei (italy) in the fifth century: Rapid subsidence driven by pore pressure drop.  
1509  
1510 569 *Geophysical Research Letters*, 41(5):1471–1478.
- 1511  
1512

- 1513  
1514 570 Trasatti, E., Acocella, V., Di Vito, M. A., Del Gaudio, C., Weber, G., Aquino, I., Caliro, S., Chiodini, G.,  
1515  
1516 571 de Vita, S., Ricco, C., and Caricchi, L. (2019). Magma Degassing as a Source of Long-Term Seismicity at  
1517  
1518 572 Volcanoes: The Ischia Island (Italy) Case. *Geophysical Research Letters*.  
1519  
1520 573 Trasatti, E., Bonafede, M., Ferrari, C., Giunchi, C., and Berrino, G. (2011). On deformation sources in  
1521  
1522 574 volcanic areas: modeling the campi flegrei (italy) 1982–84 unrest. *Earth and Planetary Science Letters*,  
1523  
1524 575 306(3-4):175–185.  
1525  
1526 576 Trasatti, E., Polcari, M., Bonafede, M., and Stramondo, S. (2015). Geodetic constraints to the source mech-  
1527  
1528 577 anism of the 2011–2013 unrest at campi flegrei (italy) caldera. *Geophysical Research Letters*, 42(10):3847–  
1529  
1530 578 3854.  
1531  
1532 579 Troise, C., Natale, G. D., Schiavone, R., Somma, R., and Moretti, R. (2018). The campi flegrei caldera unrest:  
1533  
1534 580 Discriminating magma intrusions from hydrothermal effects and implications for possible evolution. *Earth-*  
1535  
1536 581 *Science Reviews*.  
1537  
1538 582 Williams-Jones, G., Rymer, H., and Rothery, D. A. (2003). Gravity changes and passive so<sub>2</sub> degassing at the  
1539  
1540 583 masaya caldera complex, nicaragua. *Journal of Volcanology and Geothermal Research*, 123(1-2):137–160.  
1541  
1542 584 Yang, X.-M., Davis, P. M., and Dieterich, J. H. (1988). Deformation from inflation of a dipping finite prolate  
1543  
1544 585 spheroid in an elastic half-space as a model for volcanic stressing. *Journal of Geophysical Research: Solid*  
1545  
1546 586 *Earth*, 93(B5):4249–4257.  
1547  
1548 587 Zollo, A., Judenherc, S., Auger, E., D’Auria, L., Virieux, J., Capuano, P., Chiarabba, C., de Franco, R.,  
1549  
1550 588 Makris, J., Michelini, A., and Musacchio, G. (2003). Evidence for the buried rim of campi flegrei caldera  
1551  
1552 589 from 3-d active seismic imaging. *Geophysical Research Letters*, 30(19).  
1553  
1554 590 Zollo, A., Maercklin, N., Vassallo, M., Iacono, D. D., Virieux, J., and Gasparini, P. (2008). Seismic reflections  
1555  
1556 591 reveal a massive melt layer feeding campi flegrei caldera. *Geophysical Research Letters*, 35(12).  
1557  
1558  
1559  
1560  
1561  
1562  
1563  
1564  
1565  
1566  
1567  
1568