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Chapter 1

Energy-Efficient Train Control

A practical application

Valentina Cacchiani, Antonio di Carmine, Giacomo Lanza, Michele Monaci,
Federico Naldini, Luca Prezioso, Rosalba Suffritti, and Daniele Vigo

Abstract This work presents a practical application of the *Energy Efficient Train Control* (EETC) problem, arising from a collaboration between the *OR* group of the University of Bologna and ALSTOM Ferroviaria SpA, within the framework of project *Swift*, funded by the Emilia-Romagna regional authority. Such problem requires to determine a speed profile for a given train, running on a given line, such that it minimizes the traction energy consumption. For the solution of this problem we introduce three methods: a constructive heuristic; a multi-start randomized constructive heuristic; and a Genetic Algorithm. Numerical experiments are executed on real-life instances. The results show that high quality solutions are produced and the computing time is suitable for real-time applications.

Key words: heuristic, railway optimization, energy, train control

1.1 Introduction

One of the major costs for railway companies is given by energy consumption. For this reason, the development into a more mature and competitive market makes an efficient energy management imperative for reducing the operating costs. Ecological awareness is also a major driver for energy efficiency in railway systems in an effort towards reducing air pollutants, e.g., carbon dioxide, whose emissions are one of the causes of global warming (Luijt et al., 2017).

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The construction of energy-efficient driving profiles has attracted large attention from researchers in the recent years, being an effective way of saving energy. The resulting problem is known as *Energy-Efficient Train Control* (EETC), and is sometimes also referred to as *eco-driving* or *train trajectory planning problem*. This problem aims at finding the most energy-efficient driving profile for a given train traveling on a certain line, while satisfying a number of operational constraints to ensure a safe and punctual journey. Recent surveys on EETC have been proposed by Yang et al. (2016) and Scheepmaker et al. (2017). A complete review on the Optimal Train Control Theory has been given in Albrecht et al. (2016a,b).

Analyses based on the *Pontryagin Maximum Principle* (PMP) have shown that an optimal driving strategy consists of a sequence of four *driving regimes*, namely, maximum acceleration/traction (MT), cruising/speed-holding (SH), coasting (CO) and maximum braking (MB), see, e.g., Howlett et al. (1994) and Albrecht et al. (2016a,b). An example speed profile consisting of MT-SH-CO-MB is shown in Figure 1.1.

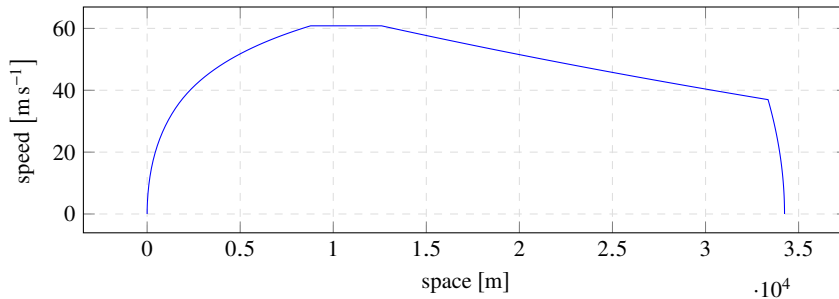


Fig. 1.1 A profile consisting of MT, SH, CO and MB. The train runs on a track approximately 35 kilometers long, and starts and ends at zero speed.

Based on this result, most of the algorithms proposed in the literature (Albrecht et al. (2016b), Yang et al. (2016) and Scheepmaker et al. (2017)) define the driving profile as a sequence of these four driving regimes, identifying suitable switching points between consecutive driving regimes.

In this paper we focus on the EETC in its basic version, arising when the driving profile of a single train has to be determined. We do not consider railway traffic management, as we assume that the schedule of the train has already been optimized by a (possibly online) scheduling algorithm (see, e.g., Bettinelli et al. (2017) and Fischetti and Monaci (2017)). In this context, safety requirements impose that the schedule of the train is given on input and cannot be changed. In addition, we do not consider possible interactions between trains in terms of power exchange, as it could happen in complex networks equipped with appropriate infrastructures and energy storage systems.

This paper is organized as follows. In Section 1.2 we define the problem, and in Section 1.3 the solution approaches are outlined. The results of the computational

testing on real-world instances are given in Section 1.4. Finally, Section 1.5 draws some conclusions.

1.2 Problem Definition

We are given a rail track having length D , and a train running on the track. The train has a fixed schedule, imposing its travel time be equal to exactly T time units, and is characterized by some known rolling-stock properties (e.g., its weight). The track geometry, e.g., the position-varying slope and radius of curvature, is also known. Finally, there are give speed limits that the train must respect, and that vary along the track. The problem requires to determine a driving profile for the train such that (i) the travel time of the train is exactly T ; (ii) speed limits are respected; and (iii) the total amount of energy required for running the train is minimized.

We represent the track as a segment $[0, D]$ and assume that the train travels on the track from time instant 0 to time instant T .

For the sake of simplicity, we do not consider the case of *steep* uphill/downhill tracks, i.e., we assume that the train, regardless of its speed, is always capable of negotiating uphill/downhills or speed-holding. Moreover, we assume a continuous control and neglect any form of energy recovery, as it happens with the so-called *Regenerative Brake*. For details on steep uphill/downhills and on Regenerative Brake, the reader is referred to Howlett et al. (1994); Albrecht et al. (2016a,b).

In our setting, the motion of a train is approximated using the following point-mass model

$$\frac{dv}{dt} = u(t) - R(v(t)) + G(x(t)) \quad (1.1)$$

$$\frac{dx}{dt} = v(t), \quad (1.2)$$

where time $t \in [0, T]$, is the independent variable, while speed $v(t)$, position $x(t)$ are coordinates of the dynamical system and $u(t)$ is the controlled acceleration.

Term $R(v)$ represents the so-called *Basic Resistance*, taking into account the speed-dependent resistive phenomena, and given by the well-known *Davis Equation*

$$R(v) = r_0 + r_1 v(t) + r_2 v(t)^2, \quad (1.3)$$

where r_0 , r_1 and r_2 are positive empirical constants depending on the rolling-stock (see, J.W. Davis Jr (1926)).

Term $G(x)$ is the *Line Resistance*, which measures the position-dependent forces acting on the train along its route, such as the gravity force (depending on the position-varying track slope) and the curve resistance (depending on the position-varying radius of curvature). The analytic expression of $G(x)$ has been provided by our industrial partner as a piece-wise function through the expression

$$G(x) := -g \sin \left[\arctan \left(\frac{0.8}{\rho(x)} + \tan \theta(x) \right) \right], \quad (1.4)$$

$\theta(x)$ and $\rho(x)$ being the (position-dependent) track grade and radius of curvature respectively, and g the gravity constant (see, Fayet, 2008; Saprionova et al., 2017).

The controlled acceleration $u(t)$ is bounded by two functions, i.e.

$$-u_L(v) \leq u(t) \leq u_U(v). \quad (1.5)$$

where bounds $u_L(v) \geq 0$ and $u_U(v) \geq 0$ are the *maximum deceleration* and the *maximum acceleration*, respectively, that the train engine can handle. In the following, we will always assume that u_L and u_U are decreasing non-linear functions of the speed.

As mentioned above, speed limits impose an upper bound on the maximum speed

$$0 \leq v(t) \leq \bar{V}(x(t)) \quad (1.6)$$

where the upper bound value $\bar{V}(x(t))$ depends on the geometry of the track. In addition, the following boundary conditions

$$x(0) = 0, \quad x(T) = D, \quad v(0) = v_{init}, \quad v(T) = v_{final}, \quad (1.7)$$

are imposed to fix the position and speed of the train at time instants $t = 0$ and $t = T$, with v_{init} and v_{final} given on input.

Finally, the objective is to minimize the total energy spent by the train, given by

$$E = \int_0^T \frac{u(t) + |u(t)|}{2} v(t) dt \quad (1.8)$$

Equation (1.8) considers that no energy is recovered when $u(t) < 0$, in fact, the term $(u + |u|)/2$ is equal to zero in that case.

Observe that the Equations (1.1)-(1.8) constitute a well-known Optimal Control Formulation for the EETC, see Albrecht et al. (2016a,b) and Scheepmaker et al. (2017).

1.2.1 Overview of the Solution Approach

In most practical cases, track geometry and speed-limits are described as piece-wise constant functions of the space. Therefore, it is possible to partition the track $[0, D]$ into a sequence of n consecutive sections $S_k = [x_o^{(k)}, x_f^{(k)}]$, with $x_o^{(0)} = 0$, $x_f^{(n)} = D$ and $x_f^{(k)} = x_o^{(k+1)}$, ($k = 0, 1, \dots, n-1$), that have constant slope, radius of curvature and speed limit (see Haahr et al. (2017); Wang et al. (2012, 2013)). We will refer to those sections as *segments*. This results in having a fixed *Line Resistance* along with a fixed speed limit. In particular $\forall k = 1 \dots n$:

$$G(x) = g^{(k)} \quad \bar{V}(x) = \bar{v}^{(k)}. \quad (1.9)$$

As mentioned in the introduction, similar to what is done in Li and Lo (2014), our approach is based on the Optimal Train Control theory and assumes that a concatenation of at most four given driving regimes is used to define the speed profile in each section. The driving regimes are characterized by the following conditions:

$$\text{Maximum Traction, MT:} \quad u_1 = u_U(v) \quad (1.10)$$

$$\text{Speed Holding (or Cruising), SH:} \quad u_2 = R(v) - g^{(k)} \quad (1.11)$$

$$\text{Coasting, CO:} \quad u_3 = 0 \quad (1.12)$$

$$\text{Maximum Braking, MB:} \quad u_4 = -u_L(v), \quad (1.13)$$

where u_1 , u_2 , u_3 and u_4 are the accelerations in every regime. Within a segment, each regime can be executed at most once, and regimes must appear in fixed order, namely, MT, SH, CO, and MB. Furthermore, some regimes may not be used in a segment.

As a consequence, Equations (1.1) and (1.2) can be simplified, because the term u corresponds to one of the u_j ($j = 1, \dots, 4$) shown in Equations (1.10)-(1.13) and $G(x) = g^{(k)}$ in each segment k .

We observe that, within our settings, energy is only consumed during the MT and SH driving regimes. Let $E_{MT}^{(k)}$ and $E_{SH}^{(k)}$, respectively, be the energy consumed in the MT and SH regimes for segment k ($k = 1 \dots n$). Then Equation (1.8) reduces, for MT, to

$$E_{MT}^{(k)} = \int_{t_1^{(k)}}^{t_2^{(k)}} u_1(v(t)) v(t) dt, \quad (1.14)$$

where $t_1^{(k)}$ and $t_2^{(k)}$ are the start and end time for MT in segment k . Similarly, for regime SH, we have that (1.8) simplifies to

$$E_{SH}^{(k)} = u_2(v_2^{(k)}) v_2^{(k)} (t_3^{(k)} - t_2^{(k)}), \quad (1.15)$$

where $v_2^{(k)}$ is the constant speed value, while $t_2^{(k)}$ and $t_3^{(k)}$ are respectively the start and end times for SH regime.

We note that, under appropriate time discretion, Equations (1.1), (1.2) and (1.14), can be pre-computed, for each segment and driving regime, by using numerical methods or approximated closed-form expressions under some assumptions on u as in Ye and Liu (2017).

1.3 Heuristic Solution Approaches

In this section we propose heuristic approaches for the problem at study. We first introduce a constructive heuristic (CH); then, we present a multi-start randomized constructive heuristic (RCH) and a Genetic Algorithm (GA).

1.3.1 Constructive Heuristic

The proposed constructive heuristic is an iterative procedure that starts from an infeasible solution and, at each iteration, tries to reduce infeasibility, until a feasible solution is produced.

At the beginning, the algorithm computes the so-called *allout* speed profile, i.e. the solution obtained by running the train at its maximum allowed speed. When computing this speed profile, train motion laws and speed limits are taken into account.

Observe that the running time τ_{AO} associated with this solution provides a lower bound on the running time for the train in any feasible solution. If this running time is not equal to the required time T , the current solution is infeasible. To recover feasibility we consider the actual maximum speed $\tilde{V}^{(k)}$ reached in every segment k , and slow the train down by artificially reducing the speed limits. This speed reduction is applied in one segment at a time.

For each segment k , we reduce its maximum speed by a given amount s (which is a parameter of the algorithm), and re-compute the associated speed profile and running time τ . If $\tau > T$, then the last update of the maximum speed is canceled. If, instead $\tau < T$, the next segment is considered. This procedure continues until a feasible solution is obtained. The pseduo-code of the algorithm is shown in Algorithm 1.

```

1 Compute the allout speed profile for the given route. Store travel time in  $\tau = \tau_{AO}$ .
  Store in  $\tilde{V}^{(k)}$  the maximum speeds that are actually reached by the allout speed
  profile in each segment  $k$ ;
2 Set  $w_k := \tilde{V}^{(k)}$ ,  $\forall k = 1 \dots n$ ;
3 while  $\tau < T$  do
4   for  $k = 1 \dots n$  do
5      $w_k := w_k - s$ ;
6     Re-compute speed profile using the current speed limits in each segment (i.e.
7      $w_1, \dots, w_n$ ). Store travel time in  $\tau$ ;
8     if  $\tau > T$  then
9       Undo the change, namely  $w_k := w_k + s$ 
10    end
11  end

```

Algorithm 1: CH

1.3.2 Multi-start Randomized Constructive Heuristic

The constructive heuristic of the previous section is used in a multi-start fashion to determine a pool of feasible solutions. This allows to possibly find better solutions and to determine an initial population of the genetic algorithm (see Section 1.3.3).

The multi-start randomized constructive heuristic performs a fixed number (say, NI) of iterations, changing a set of parameters in a random way. In particular, at each iteration, γ_1 segments are selected (where γ_1 is a parameter of the algorithm), and the maximum speed w_k of the selected segments is reduced by a random value. Then, the constructive heuristic of Section 1.3.1 is applied. It is worth to be noted that the sub-procedure which computes the allout speed profile needs to be executed only once. The best solution found among all iterations is then returned. The pseudo-code is reported in Algorithm 2.

```

1 Compute the allout speed profile for the given route. Store travel time in  $\tau = \tau_{AO}$ .
  Store in  $\tilde{V}^{(k)}$  the maximum speeds that are actually reached by the allout speed
  profile in each segment  $k$ ;
2 Set  $w_k := \tilde{V}^{(k)}$ ,  $\forall k = 1 \dots n$ ;
3 while  $\tau < T$  do
4   for  $c = 1 \dots \gamma_1$  do
5     Randomly select a segment  $\tilde{k}$  according to a uniform random distribution;
6     Reduce  $w_{\tilde{k}}$  by a uniform random number within  $(0, \gamma_2 w_{\tilde{k}}]$  with  $0 \leq \gamma_2 \leq 1$ ;
7   end
8   Re-compute speed profile using  $w_k$  as the maximum speed in each segment
     ( $k = 1 \dots n$ ). And let  $\tau$  be the resulting travel time;
9 end
10 Apply lines 3-11 of Algorithm 1;

```

Algorithm 2: RCH

1.3.3 Genetic Algorithm (GA)

In this section, we present a Genetic Algorithm, inspired by the work of Li and Lo (2014).

In our algorithm, each chromosome is associated with a solution and is represented by a vector containing the values of the initial speed for each driving regime.

The population contains M individuals. During each iteration, by means of crossover and mutation operators, the current population might grow larger than M . The selection operator restores the number of individuals to M by eliminating the lowest ranking ones from the current population. A fixed number h of *Elite* individuals is kept intact over the iterations.

The evaluation of each chromosome c of a current population is carried out by means of the following fitness function

$$f(c) = k_1 E(c) + k_2 H(c), \quad (1.16)$$

where k_1 and k_2 are tuning parameters, $E(c)$ represents the traction energy that is spent driving the train according to solution c , and $H(c)$ measures the diversity of chromosome c with respect to the rest of the current population (thus making the fitness function biased towards favoring diversity, instead of considering energy efficiency only). $H(c)$ is computed as a mean difference between c and the other individuals.

The main components of the GA are reported in the following list, according to the order in which chromosomes are processed in each iteration:

1. **Mutation Operator:** this operator is applied, with probability P_m , to each non-elite chromosome, producing new individuals. It consists of applying random variations of speed to a set of randomly selected segments, according to a uniform random distribution. The maximum variation amount is given as a percentage m (parameter of the algorithm) of the maximum speed in a each segment.
2. **Crossover Operator:** it implements a single-point crossover to produce two children chromosomes from a couple of parents, by recombining their genome after splitting on a randomly selected segment boundary. It has a probability P_c of affecting each chromosome, excluding Elites and those chromosomes produced by mutation at the current iteration;
3. **Repair Operator:** it restores feasibility of solutions after mutation and crossover, if needed. It implements a procedure similar to the constructive algorithm introduced in Section 1.3.1
4. **Selection Operator:** it ranks each chromosome c of the current population by means of the fitness function. Then, it deletes the lowest ranking individuals exceeding the maximum population size M , while preserving a group of E Elites intact.

1.4 Computational Experiments

The algorithms described in the previous section were implemented in C and executed on a 2.9-Ghz Intel Core i7-7500U with 16 GB of RAM. Our benchmark is composed by real-world instances provided by ALSTOM. All instances are associated with two railway lines, denoted as ROUTE 1 and ROUTE 2. These lines have lengths equal to 250 and 270 kilometers, respectively, and are composed by 42 and 67 segments, respectively. For each line, we considered five different train models, denoted with letters A to E in the following, and having different characteristics. This produced a benchmark of 10 different instances. To evaluate the quality of the obtained solutions, we used the following measure of efficiency, commonly used by ALSTOM's practitioners:

$$e = 100 \cdot \frac{E_A - E}{E_A}, \quad (1.17)$$

where E_A represents the energy consumption of the aforementioned allout profile, while term E corresponds to the energy consumption associated with the given solution.

The RCH algorithm (see Algorithm 2), which also initializes the GA, was iterated 32 times in each experiment in a multi-start fashion. Parameter s was set to 1 (m/s), $\gamma_1 = 20$ and $\gamma_2 = 40\%$. The GA was configured according to the following parameter values: $P_c = 0.97$, $P_m = 0.95$, $M = 9$, $h = 5$ and $m = 5\%$. A maximum number of 300 generations was imposed. Moreover the GA was set to terminate after 11 non-improving consecutive generations.

Table 1.1 shows the results associated with ROUTE 1. For each train model we report the fixed travel time T , the level of efficiency computed according to (1.17), and the associated CPU time (in seconds).

Train	T (s)	Efficiency			CPU time (seconds)		
		CH	RCH	GA	CH	RCH	GA
A	4560	3.2	5.4	7.2	0.2	1.5	4.4
B	4560	8.2	10.4	11.6	0.1	1.2	2.5
C	6525	6.6	11.5	25.3	0.3	4.9	10.6
D	10920	10.9	12.5	12.7	0.1	0.6	2.2
E	7079	18.8	20.2	21.6	0.2	0.7	2.1

Table 1.1 Results on ROUTE 1 (250 km, 42 segments).

The results show that, when the first line is considered, the constructive heuristic is able to reduce the energy consumption, with respect to the allout profile, by around 10% on average. The associated computing times are rather small, being always below 2 seconds. By executing the randomized multistart constructive heuristic and the genetic algorithm we obtain further savings, about 12% and 15%, respectively. While the increase in CPU time for the former is rather limited, the latter may require a significantly larger computing time, which may prevent its use in a real-time system.

Train	T (s)	Efficiency			CPU time (seconds)		
		CH	RCH	GA	CH	RCH	GA
A	7285	6.6	13.7	23.7	0.5	9.6	15.8
B	7142	8.7	14.8	29.0	0.6	11.4	21.7
C	7933	9.9	13.4	27.2	0.8	15.9	29.4
D	11729	12.7	13.2	13.9	0.2	0.9	2.4
E	7595	18.2	18.9	22.2	0.3	1.7	5.2

Table 1.2 Results on ROUTE 2 (270 km, 67 segments).

The results for the second line, reported in Table 1.2, confirm those obtained for the first line, although the computing times are larger than before. However, we observe that energy reduction in Table 1.2 is also higher than in Table 1.1.

1.5 Conclusions and future research

In this paper we studied the Energy-Efficient Train Control (EETC) problem. This problem was solved using three approaches: a constructive heuristic, a multi-start randomized constructive heuristic, and a genetic algorithm. Computational results on real-life instances show that, in most cases, the computing times of the algorithms are short enough to allow their use in a real-time application.

Future research directions will be to consider real-time traffic management aspects, related to the possibility of slightly changing the schedule of the train, as well as the adoption of a system which allows energy recovery through regenerative brake.

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