Coordinating Research and Development Efforts for Quality Improvement along a Supply Chain



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# Coordinating research and development efforts for quality improvement along a supply chain*

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**Abstract**

The optimal design of two-part tariffs is investigated in a dynamic model where two firms belonging to the same supply chain invest in R&D (research and development) activities to increase the perceived quality of the final product. It is shown that the replication of the vertically integrated monopolist's performance can be attained using a two-part tariff in which the fee is a linear function of either the upstream R&D effort or product quality itself. The possibility of relying on R&D figures appearing in the upstream firm's balance sheet is desirable as quality enhancement might not be observable or verifiable.

**Keywords:** OR in research and development; product quality; vertical separation; vertical integration; outsourcing.

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# 1 Introduction

The purpose of this paper is to design optimal contracts allowing a vertical supply chain to exactly replicate the profit performance, R&D investments and product quality level, as perceived by consumers, of a vertically integrated monopolist in a dynamic model where quality improvement requires costly R&D efforts by the firms along the vertical channel or by different divisions of the same firm under vertical integration. Hence, the ensuing analysis is connected to the intense debate about supply chain coordination, which has taken into account several dimensions of firms' activities.[1] It is worth reconstructing the related literature, lying at the intersection of industrial economics, operations research and management, focussing on the aspects connected with the present paper.

To begin with, the downward distortion of product quality in monopoly markets is a long-standing issue in the theory of industrial organization. The incentive for a monopolist to undersupply quality in order to increase its own ability of extracting surplus from consumers has been highlighted by Spence (1975) and Mussa and Rosen (1978) and then further investigated by several other authors.[2] Quality supply has also repeatedly received attention in the fields of operations research, marketing and management (see Feichtinger *et al.*, 1994; and Jørgensen and Zaccour, 2004), where it has been often connected with the optimal coordination of supply chains. To the best of my knowledge, this strand of literature inserts product quality in static or dynamic models based on the representative consumer approach, so that it either shifts or inflates the market demand function, in such a way that the effects of quality improvements are analogous to those of increasing goodwill

[1] Exhaustive accounts of this literature can be found in Cachon (2003), Ingene and Parry (2004), Leng and Parlar (2005), Nagarajan and Sošić (2008) and Ingene *et al.* (2012).

[2] See Itoh (1983), Maskin and Riley (1984), Besanko, Donnenfeld and White (1987) and Champsaur and Rochet (1989), *inter alia*. For a survey, see Lambertini (2006).

through advertising.[3]

This particular aspect links the discussion about quality supply to a parallel debate concerning firms' make-or-buy decisions or, equivalently, the choice between carrying out production and R&D in house and outsourcing, with the related contractual problems which obviously accompany the latter, any time some relevant feature of the component being outsourced is subject to opportunistic behaviour (i.e., moral hazard) on the part of the OEM supplier. When this happens, with the supplier underinvesting along some key dimension, a hold-up problem obtains, to the disadvantage of the outsourcing firm facing the final customers. The hold-up phenomenon arising under opportunistic behaviour is a major issue in the theory of the firm ever since Williamson (1975, 1979) and has been extensively discussed in contract theory.[4]

The same issue is receiving a growing amount of attention in the fields of operations research and management, where product quality becomes a relevant additional element in the discussion about supply chain coordina-

[3] In this vein, see El Ouardighi and Pasin (2006), El Ouardighi and Kim (2010), Matsubayashi and Yamada (2008), Xie *et al.* (2011a,b), El Ouardighi and Kogan (2013) and Chen *et al.* (2015, 2017). Some of these contributions consider environmental quality and/or green R&D (Swami and Shah, 2013; Zhu and He, 2017; Yenipazarli, 2017). In others, quality is indeed treated as a hedonic feature of the product, as in Shi, Liu and Petruzzi (2013). Additionally, the design of the optimal range of product qualities has been investigated in connection with supply chain coordination (de Matta *et al.*, 2015) while in the initial literature in the field of industrial organization this aspect was investigated solely in relation to vertically integrated firms' ability of increasing profits through product proliferation (as in Mussa and Rosen, 1978; and Champsaur and Rochet, 1989, *inter alia*).

[4] See Grossman and Hart (1986), Hart and Moore (1988), Rogerson (1992), MacLeod and Malcomson (1993), Aghion *et al.* (1994) and Nöldeke and Schmidt (1995), *inter alia*. The most recent offshoot of this debate discussess the make-or-buy decision (or, vertical integration vs separation) in a global economy (see McLaren, 2000; Grossman and Helpman, 2002; Antras and Helpman, 2004, among others).

tion.[5] Therefore, quality is accounted for in connection with cost- or revenue-sharing contracts[6] as well as other types of contracts contemplating penalties, buybacks or lost sales clauses (as in, e.g., Leng and Parlar, 2010). Balachandran and Radhakrishnan (2005) tackle the moral hazard problem connected with unobservable quality using a model where the contract includes both a warranty and a penalty, the latter based in inspection and external failures, identifying the conditions whereby the contract ensures the attainment of the first best quality. Lee *et al.* (2013) use a model in which quality enters demand additively, to show that contracts based on revenue-sharing and buybacks fail to coordinate a supply chain in which uncertainty affects quality, while an alternative scheme contemplating compensations for defective items succeeds in coordinating the vertical relation. El Ouardighi and Kogan (2013) consider instead a multiplicative effect of quality on demand, to illustrate that even if a revenue-sharing contract improves the supplier's commitment to increase quality, such a contract does not entirely solve the coordination problem. These authors also outline a reward-based contract ensuring the replication of the vertically integrated firm's performance.

Here, I adopt a different approach in modelling the contract to be adopted to lead the vertical supply chain to deliver the optimal product quality. The analysis is confined to nondurables, as the model is constructed under the assumption that consumers buy the final product at any point in time. Hence,

[5] Throughout the paper, I confine the discussion to the literature where firms' investments have an impact on quality and not marginal cost. Of course, this side of a supply chain's techology has also been investigated. Among others, see Ge *et al.* (2014), where cartelization of R&D investments for process innovation may allow the supply chain to achieve efficiency.

[6] This class of contracts has been analysed in detail is several different setups. See Giannoccaro and Pontrandolfo (2004); Cachon and Lariviere (2005); Bhaskaran and Krishnan (2009); Leng and Parlar (2010); Chen *et al.* (2011), Chen *et al.* (2015), and Becker-Peth and Thonemann (2016), *inter alia*.

what follows can describe supply chains selling foodstuffs, in which outsourcing vs vertical integration may concern jars for marmalade or tomato sauce, with the quality and design of glass jars being a relevant element for the seller who wants to attract customers. Likewise, the ergonomic features of energy drink packaging matter and add up to the innovative efforts of the firm producing the drink itself. consumers' attention to these elements neatly emerges from websites (see, e.g., bpando.org). The ensuing model leaves instead out of the picture the analysis of the analogous problem related to durables, e.g., the supply of CPUs for PCs or laptops or winglets for a turbo engine to be mounted into a flagship sportscar.

I compare a vertically integrated monopolist with two divisions investing to increase product quality with the alternative industry structure in which the product quality level is the outcome of the efforts of two independent firms connected along a vertical supply chain. In the latter case, the contractual relation takes the form of a two-part tariff which may be designed in several alternative ways, thereby generating different outcomes, some of which are indeed achieving the desired outcome. The model is defined in continuous time, over an infinite horizon, under the assumption that the population of consumers is differentiated in terms of their marginal willingness to pay for quality. It is therefore a single agent dynamic programming model when a vertically integrated firm is considered, and a differential game with sequential moves at every instant if instead two independent firms are vertically related. From an analytical point of view, the procedure follows the same steps as in Lambertini (2014), where an analogous approach is used to design optimal contracts in a supply chain where firms have to build up goodwill over time.[7]

The main results can be spelled out as follows. After characterising the

[7] Lambertini (2014) nests into a large literature discussing the dynamics of brand equity and the use of two-part tariffs, from Jeuland and Shugan (1983) to Zaccour (2008).

efficient outcome engendered by the vertically integrated firm, the distortion induced by vertical separation is illustrated, to the effect that the sum of upstream and downstream R&D efforts do not match those taking place across divisions belonging to an integrated monopolist, and equilibrium quality consequently decreases. Then, it is shown that a two-part tariff consisting of an exogenously given fee combined with a wholesale price set at marginal cost creates a hold-up problem inducing the upstream firm not to invest at all in quality-increasing activities. As a consequence, the vertical channel falls short of the performance of the vertically integrated monopolist, which is instead attained modelling the fixed fee as an endogenous function of either (i) the R&D effort of the upstream firm, or (ii) the quality level itself. While being equally effective at first sight, these two alternative contractual designs may indeed be not entirely equivalent. This is because the quality level being developed along the supply chain may not be observable or verifiable along the chain itself (as well as by the final customer before purchasing),[8] and therefore the alternative contract based on the R&D effort - which can be verified from the balance sheet of the upstream firm, unless fraudulent behaviour is adopted by the latter - appears more reliable an instrument to cope with the issue represented by the vertical externality.

The remainder of the paper is structured as follows. The setup and the analysis of the vertically integrated monopolist are in Section 2. The case of vertical separation with double marginalization is dealt with in Section 3, while Section 4 contains the analysis of the alternative contractual designs based on three different definitions of the two-part tariff. Concluding remarks are in Section 5.

[8]If quality is not observable along the supply chain, this adds to the vertical externality associated with double marginalization in generating the hold up problem associated with the unpstream firm's R&D incentive.

# 2 Benchmark: the vertically integrated monopolist

The model is a variation on the setup introduced by Mussa and Rosen (1978), Gabszewicz and Thisse (1979) and Moorthy (1984). I assume the market is supplied by a single-product monopoly selling a nondurable good of quality $q(t) > 0$ at price $p(t) > 0$ over continuous time $t \in [0, \infty)$ . The assumption of an infinite horizon describes a situation in which the firm may not expect its product to remain on the market forever but is unable to forecast any specific finite time $T$ at which the demand for its product will become nil. To some extent, this idea is an implicit consequence of the absence of competitors on the same market. In the remainder, the same assumption applies to the vertical relation as well. In that case, the firms operating along the supply chain may indeed perceive their relationship as a long-term one, and cannot forecast its interruption.

The population of consumers is characterised by a level of marginal willingness to pay for quality $\theta \in [\Theta - 1, \Theta]$, where $\Theta > 1$, and is distributed with a uniform density $d = 1$ over such interval. Hence, the total mass of consumers amounts to 1. Net instantaneous consumer surplus is $u(t) = \theta q(t) - p(t) \geq 0$, so that parameter $\theta$ can be interpreted as a proxy of income or wealth. At any time $t \in [0, \infty)$, partial market coverage is assumed. At any instant, the marginal consumer is identified by the marginal willingness to pay $\widehat{\theta}(t)$ solving $\widehat{\theta}(t) q(t) - p(t) = 0$; hence, $\widehat{\theta}(t) = p(t)/q(t)$ and - assuming $p(t)/q(t) > \Theta - 1$ always - market demand at any time $t$ is $x(t) = \Theta - p(t)/q(t)$ .

Production takes place at marginal cost $c$, which can be normalised to zero without further loss of generality. The firm consists of two vertically related divisions, $U$ (for *upstream*) and $D$ (for *downstream*), each investing in R&D aimed at improving the quality level of the product supplied to consumers.

Define as $k_i(t)$ the instantaneous effort of division $i = D, U$. If R&D activity takes place at decreasing returns to scale, the total cost function borne by the firm is

$$C(t) = b\left[k_U^2(t) + k_D^2(t)\right] \tag{1}$$

where $b$ is a positive parameter. One can imagine the present setup as describing a situation in which each division cares for an input or component whose quality is crucial in determining the overall quality level of the final consumption good. I assume all controls (prices and R&D efforts) to be adjustable at all times. This amounts to saying that quality can be enhanced and prices modified at any instant during the life cycle of the product.[9] The state dynamics describing the evolution of the state variable $q(t)$ over time is

$$\frac{dq(t)}{dt} \equiv \dot{q} = z\left[k_U(t) + k_D(t)\right] - \delta q(t) \tag{2}$$

in which $z$ is a positive constant and $\delta > 0$ is the decay rate of quality. The presence of quality depreciation is a reasonable assumption even in absence of competitors in the same market, insofar as attention is confined to consumers' perception of quality, which (i) depends on brand equity and consumer's experience with other goods (not modelled in this partial equilibrium model) and (ii) may indeed differ from the actual quality level.[10]

The transfer price along the supply chain being nil (because the constant marginal cost has been normalised to zero), the vertically integrated

[9] This is reasonable for prices, as these are considerably more flexible than output levels, in particular when capacity constraints do not bite (as for soft drinks). As for R&D, the assumption describes a scenario in which firms choose an investment smoothing process instead of investing upfront a large amount of resources.

[10] In fact, recent contributions focus instead on the role of learning-by-doing in increasing the actual quality level (for an exhaustive account of the related literature, see Thompson, 2010), although some also include R&D (see Li and Ni, 2016; and Lambertini *et al.*, 2017).

monopolist's instantaneous profits are

$$\pi(t) = p(t)\left[\Theta - \frac{p(t)}{q(t)}\right] - b\left[k_U^2(t) + k_D^2(t)\right] \tag{3}$$

and the firm wants to maximise the discounted profit flow

$$\Pi(t) = \int_0^\infty \pi(t)\, e^{-\rho t} dt \tag{4}$$

w.r.t. controls $p(t)$, $k_U(t)$ and $k_D(t)$, under the constraints posed by the state equation (2) and the initial condition $q(0) = q_0 > 0$. Profits are discounted at the constant rate $\rho > 0$.

The Bellman equation is

$$\rho V_{VI}(q(t)) = \max_{p(t),k_U(t),k_D(t)} \left\{\pi(t) + V'_{VI}(q(t)) \cdot \frac{dq(t)}{dt}\right\} \tag{5}$$

where subscript $VI$ mnemonics for *vertical integration* and $V'_{VI}(q(t)) \equiv \partial V_{VI}(q(t))/\partial q(t)$. Concerning the guess about the form of the value function, it is worth noting that, by construction, the Bellman equation (5) is neither linear nor linear-quadratic, since the position of the marginal consumer, $\widehat{\theta}(t)$, is hyperbolic in the quality level. For the moment, I pose $V_{VI}(q(t)) = \gamma q^2(t) + \alpha q(t) + \beta$, so that $V'_{VI}(q(t)) = \alpha + 2\gamma q(t)$.

Taking the first order conditions (FOCs) on $\{p(t), k_u(t), k_d(t)\}$ and solving, one obtains the following triple of optimal feedback controls:

$$p^*(t) = \frac{\Theta q(t)}{2}\,;\; k_U^*(t) = \frac{z V'_{VI}(q(t))}{2b} = k_D^*(t) \tag{6}$$

Plugging (6) into (5) and simplifying, one obtains the following equation:

$$\begin{gathered} 4\gamma\left[b(2\delta + \rho) - 2\gamma z^2\right] q^2(t) + \\ \left[4\left(b(\delta + \rho) - 2\gamma z^2\right)\alpha - b\Theta^2\right] q(t) + 2\left(2b\beta\rho - \alpha^2 z^2\right) = 0 \end{gathered} \tag{7}$$

Now, since $\partial \dot{q}/\partial q < 0$, the state dynamics is stable. However, this is not sufficient to ensure stability as one has to check the slope of the feedback

R&D controls w.r.t. $q$. The coefficient of $q^2(t)$ in (7) is nil at $\gamma = 0$ and $\gamma = b(2\delta + \rho)/(2z^2) > 0$. Given that $\dot{q} \gtreqless 0$ for all $q(t) \lesseqgtr z[k_U(t) + k_D(t)]/\delta$, the solution engendered by $\gamma = b(2\delta + \rho)/(2z^2)$ is necessarily unstable, while that associated with $\gamma = 0$ is stable. The reason is the following: given the shape of (2), any $\gamma > 0$ makes the R&D controls positively sloped in $q$ and therefore unstable, which in turn amounts to saying that the stability of the state-control system requires the R&D controls to be either flat (as here) or decreasing in $q(t)$. The expressions of the coefficients solving

$$\begin{aligned} 4\left(b(\delta+\rho) - 2\gamma z^2\right)\alpha - b\Theta^2 = 0 \\ 2b\beta\rho - \alpha^2 z^2 = 0 \end{aligned} \tag{8}$$

are

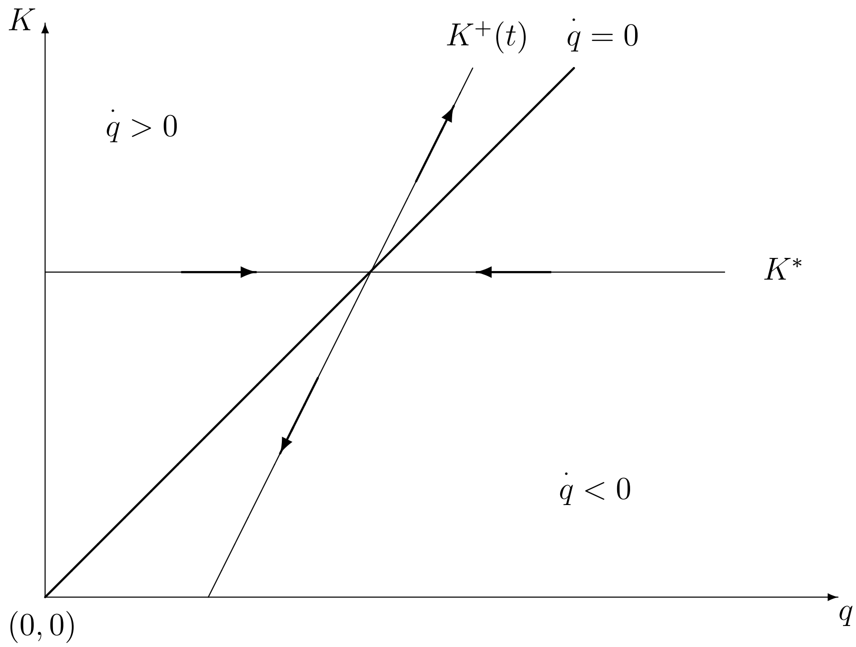
$$\beta = \frac{\alpha^2 z^2}{2b\rho}\,;\ \alpha = \frac{b\Theta^2}{4\left(b(\delta+\rho) - 2\gamma z^2\right)} \tag{9}$$

so that the R&D controls simplify to

$$k_U^+(t) = k_D^+(t) = \frac{4b\delta(2\delta+\rho)\,q(t) - z^2\Theta^2}{8bz\delta} \tag{10}$$

if $\gamma = b(2\delta + \rho)/(2z^2)$, and $k_U^* = k_D^* = k^* = z\Theta^2/[8b(\delta + \rho)]$ if $\gamma = 0$. Using $K^+(t) = k_U^+(t) + k_D^+(t)$, $K^* = 2k^*$ and $K^{ss} = \delta q/z$ to identify the steady state locus, one can draw the phase diagram illustrated in Figure 1, where the arrows describe the state dynamics and therefore also the stability or instability of feedback R&D controls. Obviously, the two aggregate R&D controls intersect the steady state locus in correspondence of the same quality level, $q^* = z^2\Theta^2/[4b\delta(\delta + \rho)]$, but the upward-sloping control appears to be unstable. The remaining equilibrium magnitudes are $p^* = z^2\Theta^3/[8b\delta(\delta + \rho)]$, $x^* = \Theta/2$ and $\pi^* = z^2\Theta^4(\delta + 2\rho)/\left[32b\delta(\delta + \rho)^2\right]$.

**Figure 1** The phase diagram under vertical integration




A few additional remarks about the solution outlined above are in order. In view of the aforementioned stability properties of the state dynamics (2) and the optimal pricing rule in (6), one may guess the linear value function $V_{VI}(q(t)) = \alpha q(t) + \beta$, so that, using (6), (5) reduces to

$$q(t)\left[b\left(\Theta^2 - 4\alpha(\delta+\rho)\right)\right] + 2\left(z^2\alpha^2 - 2b\beta\rho\right) = 0 \tag{11}$$

The reason for the emergence of a linear form is indeed that $p^*(t)$ is linear in $q(t)$, producing a state-linear model from an initial structure in which the state variable seemingly implies non-linearity.[11]

[11] This is not the only example of models whose initial form is not state-linear, and yet admit the adoption of a linear value function. See, for instance, the advertising game in Sethi (1983) and Sorger (1989), based on a modification of the Case (1979) model. See

# 3 Vertical separation: the effect of double marginalization

Now I illustrate the game in which $U$ and $D$ are independent firms playing noncooperatively, with the upstream firm endogenously setting a wholesale price $w(t)$ when selling each unit of its part or component to firm $D$, which then combines it with its own one and then sells the final good to consumers on the market. The two firms' instantaneous profit functions are (henceforth, the time argument is omitted for the sake of brevity):

$$\pi_U = wx - bk_U^2\, ; \, \pi_D = (p - w)\, x - bk_D^2 \tag{12}$$

Firm $U$ controls $w$ and $k_U$; firm $D$ controls $p$ and $k_D$. Their respective Bellman equations are:

$$\rho V_U(q) = \max_{w,k_U} \left\{ \pi_U + V_U'(q) \cdot \frac{dq}{dt} \right\} \tag{13}$$

$$\rho V_D(q) = \max_{p,k_D} \left\{ \pi_D + V_D'(q) \cdot \frac{dq}{dt} \right\} \tag{14}$$

Proceeding by backward induction, I take $w$ and $k_U$ as given and solve firm $D$'s optimum problem. The relevant FOCs on controls $p$ and $k_D$ yield:

$$p^{VS} = \frac{\Theta q + w}{2}\, ; \, k_D^{VS} = \frac{zV_D'(q)}{2b} \tag{15}$$

where superscript $VS$ stands for *vertical separation*. Controls (15) can be substituted into (13) together with $V_D(q) = \gamma q(t) + \varepsilon$ and $V_D'(q) = \gamma$, in such a way that (13) can be rewritten as follows:

$$\rho V_U(q) = \max_{w,k_U(t)} \left\{ \frac{(\Theta q - w)\, w}{2q} - bk_U^2 + \frac{V_U'(q)\left[z^2\gamma + 2b\,(zk_U - \delta q)\right]}{2b} \right\} \tag{16}$$

also Dockner *et al.* (2000, pp. 286-95). A different case is that in which the model is linear in the state and quadratic in controls, and looks so from the very outset. In this class of games, one of the linear feedback strategies degenerates to a constant: for instance, the Cournot-Nash output in the unregulated version of the differential Cournot game with polluting emissions in Benchekroun and Long (1998).

This generates the following FOCs:

$$\begin{aligned} \frac{\Theta}{2} - \frac{w}{q} &= 0 \\ zV_U'(q) - 2bk_U &= 0 \end{aligned} \tag{17}$$

which deliver $w^{VS} = \Theta q/2$ and $k_U^{VS} = zV_U'(q)/(2b)$. Then, posing $V_U(q) = \zeta q + \eta$, so that $V_U'(q) = \zeta$, the two Bellman equations simplify as follows:

$$\frac{bq\left[8(\delta+\rho)\zeta - \Theta^2\right] + 8b\eta\rho - 2z^2\zeta(2\gamma+\zeta)}{8b} = 0 \tag{18}$$

for firm $U$, and

$$\frac{bq\left[16(\delta+\rho)\gamma - \Theta^2\right] + 16b\varepsilon\rho - 4z^2\gamma(\gamma+2\zeta)}{16b} = 0 \tag{19}$$

for firm $D$. The unique solution of the system of four Riccati equations associated with (18-19) is

$$\gamma = \frac{\Theta^2}{16(\delta+\rho)}; \zeta = \frac{\Theta^2}{8(\delta+\rho)}; \ \varepsilon = \frac{5z^2\Theta^4}{1024b(\delta+\rho)\rho}; \ \eta = \frac{z^2\Theta^4}{128b(\delta+\rho)\rho} \tag{20}$$

and the equilibrium levels of R&D efforts and product quality are, respectively:

$$k_U^{VS} = \frac{z\Theta^2}{16b(\delta+\rho)}; k_D^{VS} = \frac{z\Theta^2}{32b(\delta+\rho)}; q^{VS} = \frac{3z^2\Theta^2}{32b\delta(\delta+\rho)} \tag{21}$$

with $k_U^{VS} + k_D^{VS} < 2k^*$ and consequently also $q^{VS} < q^*$. Additionally, output $x^{VS} = \Theta/4 = x^*/2$. As a result, equilibrium channel profits

$$\pi^{VS} = \frac{z^2\Theta^4(12\delta + 18\rho)}{1024b\delta(\delta+\rho)^2} \tag{22}$$

are lower than $\pi^*$. The analysis carried out in this section entails the following:

**Proposition 1** *The double marginalization associated with vertical separation brings about a reduction in R&D efforts, quality level and channel profits as compared to the vertically integrated solution.*

However, it is also worth noting that, although a hold-up effect is indeed operating because $k_i^{VS} < k^*$, $i = U, D$, it is nonetheless true that $k_U^{VS} = 2k_D^{VS}$, a property which is spelled out in

**Corollary 2** *Vertical separation and double marginalization lead the upstream firm to invest twice as much as the downstream firm.*

The reason driving this result lies in the fact that firm $U$ has an incentive to increase quality to keep output unaltered while at the same time driving upward the input price $w^{VS}$, both variables influencing positively its revenues. Put differently, firm $U$ may exploit double marginalisation to distort its own share of R&D less than $D$. Conversely, firm $D$ is forced to operate a higher distortion along the R&D dimension because its capability to generate profits is compromised by the increase in both prices, with the rise in market price squeezing output as compared to the vertically integrated equilibrium. As a result, the downward R&D distortion characterises the entire supply chain but its intensity is asymmetric across firms.

# 4 Two-part tariffs

A subset of the extant literature on supply chains where product quality is explicitly treated as a relevant feature of the channel's performance (see Economides, 1999; Bacchiega and Bonroy, 2015, *inter alia*) relies on the adoption of a Nash bargaining solution to design the allocation of profits along the channel itself, showing that this route fails to deliver the same total profits as the vertically integrated solution.

Here I rely on alternative definitions of a contract based on two-part tariffs to illustrate a twofold result:

- the traditional two-part tariff consisting of a fixed fee associated with a wholesale price does not allow the vertically separated firms to reproduce the performance of the vertically integrated monopolist. Instead,

- efficiency can be achieved by adopting, alternatively, a control-linear two-part tariff (where the control at stake is firm $U$'s R&D effort).

In both cases, the fee is accompanied by a wholesale price set at marginal production cost. As mentioned above, the second result is relevant in that the quality level may not be immediately observable or verifiable by the downstream firm, which would therefore be subject to the risk associated with opportunistic behaviour in the form of underinvestment on the part of the upstream firm. To complement the analysis, I also show that the replication of the vertically integrated outcome can indeed be attained by setting the fixed part of the tariff as a linear function of quality - in which case the aforementioned caveat should be kept in mind.

## 4.1 The exogenous two-part tariff

Here I consider the case in which the vertical relation between separated firms $U$ and $D$ takes the form of a 'classical' two-part tariff $T = wx + F$. The resulting instantaneous objective functions are therefore the following:

$$\pi_U = wx + F - bk_U^2\,;\ \pi_D = (p - w)\,x - F - bk_D^2 \tag{23}$$

where the fixed component $F$ of the TPT is an exogenous parameter, accompanied by a wholesale price equal to marginal production cost, $w = 0$.

The FOCs pertaining to firm $D$ yield the same controls as in (15). Now, posing $w = 0$, $V_D\,(q) = \gamma q + \varepsilon$ and $V_D'\,(q) = \gamma$ and proceeding as in the previous section, it is easily verified that, since $\pi_U = F - bk_U^2$, the optimal R&D effort by firm $U$ solving its first order condition is again $k_U^F = zV_U'\,(q)\,/\,(2b)$, superscript $F$ indicating the adoption of a TPT with an exogenous fee.

The partial derivative of the downstream firm's value function is again $V_U'(q) = \zeta$. However, firm $U$'s Bellman equations simplifies as follows:

$$\frac{z^2\zeta\left[\Theta^2 + 2\zeta(\delta+\rho)\right] - 8b(\delta+\rho)\left[\eta\rho + \zeta q(\delta+\rho) - F\right]}{8b(\delta+\rho)} = 0 \qquad (24)$$

whereby one of the two Riccati equation generated by (24) is

$$8b\zeta q(\delta+\rho)^2 = 0 \qquad (25)$$

which implies $\zeta = 0$, so that $V_U'(q) = 0$ and therefore also $k_U^F = 0$. This shows that the exogeneity of the fixed fee appearing in the tariff altogether eliminates any R&D incentive upstream. It is also worth stressing that, typically, $F$ should be posed equal to

$$p^* x^* = \frac{z^2\Theta^4}{16b\delta(\delta+\rho)} \qquad (26)$$

in order for the upstream firm to appropriate the revenues generated by sales, but this of course wouldn't do the job of restoring R&D incentives upstream either.

Accordingly, one may claim:

**Proposition 3** *The adoption of a classical TPT of the form $T = wx + F$ altogether eliminates the upstream firm's incentive to invest in product quality improvement.*

That is, here the classical hold-up problem emerges upstream in its entirety, being clearly generated by the presence of a fixed fee transferring upwards the whole of firm $D$'s revenues. Firm $D$'s investment being $k_D^F = k^*$, the resulting steady state quality level is $q^F = z^2\Theta^2/\left[8b\delta(\delta+\rho)\right] = q^*/2$.

## 4.2 The control-linear two-part tariff

The definition of the two-part tariff is the same as in the previous case. Therefore, the instantaneous profit functions are as in (23). Here, however, I will pose $F = \phi + \psi k_U$. That is, the contract governing the supply chain must describe the TPT as a combination of marginal cost wholesale pricing plus a transfer linear in the upstream technological effort. It is worth stressing that this construction clearly differs from cost- or revenue-sharing contracts appearing in the extent literature reviewed in the introduction. It is instead based on the idea that the TPT should be designed in terms of the upstream R&D investment, which must be accounted for in the balance sheet of the upstream supplier.

As a result of this definition of the TPT, all of the relevant variables and profits will be identified by a superscript $k_U$ revealing that the TPT specified in the contract is a function of the upstream firm's R&D control. Setting $w = 0$, the optimal controls of firm $D$ are $p_{VS} = \Theta q/2$ and $k_D^{k_U} = zV_D'(q)/(2b)$. Specifying the upstream firm's value function as $V_D(q) = \gamma q + \varepsilon$ and solving the resulting system w.r.t. $\gamma$ and $\varepsilon$, we obtain:

$$\gamma = \frac{\Theta^2}{4(\delta+\rho)}\,;\ \varepsilon = \frac{z^2\Theta^4 + 16b(\delta+\rho)\left[zk_U\Theta^2 - 4F(\delta+\rho)\right]}{64b\rho(\delta+\rho)^2} \tag{27}$$

Now define the fee as $F = \phi + \psi k_U$. *Prima facie*, this might recall cost- or revenue-sharing contracts adopted to coordinate a supply chain. Indeed, a closer inspection reveals the different nature of the present type of contract. The fee $F = \phi + \psi k_U$ intervenes in a situation in which, by the very nature of the good being produced, firm $U$ has a responsibility in determining its quality from the very outset and the contract is designed so as to restore the upstream firm's proper incentive (as embodied by the FOC on its R&D control). It does so by writing a contract which is linear in *effort* and does not imply transferring part of the upstream *costs* to the downstream firm. By sharing costs or revenues when the original structure of the vertical relation does not imply it (for instance, when a portion of the cost materially generated upstream is accounted for by the seller), firms produce a degree of cooperation based on transferring part of the burden (or advantage) along the supply chain itself. It is also worth stressing that the contract relying on $F = \phi + \psi k_U$ does not contemplate buybacks or lost sales clauses.

Proceeding backward to the Bellman equation of the upstream firm, the FOC on $k_U$ delivers $k_U^{k_U} = (zV_U'(q) + \psi)/(2b)$. Conjecturing $V_U(q) = \zeta q + \eta$, the resulting system of Riccati equations is solved by $\zeta = 0$ and $\eta = (4b\phi + \psi^2)/(4b\rho)$, and the state equation simplify as follows:

$$\dot{q} = \frac{z\left[z\Theta^2 + 4(\delta+\rho)\psi\right]}{8b(\delta+\rho)} - \delta q \tag{28}$$

whereby the equilibrium quality level is

$$q_U^{k_U} = \frac{z\left[z\Theta^2 + 4(\delta+\rho)\psi\right]}{8b\delta(\delta+\rho)} \tag{29}$$

It is then immediate to check that $k_U^{k_U} = \psi/(2b)$ and $k_D^{k_U} = k^*$. Hence, we have that $k_U^{k_U} = k^*$ and $\pi_U^{k_U} + \pi_D^{k_U} = \pi^*$ at $\psi = z\Theta^2/4(\delta+\rho)$. Firms' profits in steady state are:

$$\pi_U^{k_U} = \frac{z^2\Theta^4}{64b(\delta+\rho)^2} + \phi\,;\, \pi_D^{k_U} = \frac{z^2\Theta^4(\delta+4\rho)}{64b\delta(\delta+\rho)^2} - \phi \tag{30}$$

with

$$\pi_U^{k_U} \geq 0\,\forall\,\phi \geq -\frac{z^2\Theta^4}{64b(\delta+\rho)^2};\, \pi_D^{k_U} \geq 0\,\forall\,\phi \leq \frac{z^2\Theta^4(\delta+4\rho)}{64b\delta(\delta+\rho)^2} \tag{31}$$

The exact distribution of profits along the supply chain depends on the size and sign of $\phi$, which in turn will depend on the relative bargaining power of the two firms. Note that the conditions for the non-negativity of profits in (31) allow $\phi$ to take negative values.

The foregoing analysis boils down to the following:

**Proposition 4** *A two-part tariff $TPT = wx + F$, with $F = \phi + \psi k_U$,*

$$\phi \in \left[-\frac{z^2\Theta^4}{64b(\delta+\rho)^2}, \frac{z^2\Theta^4(\delta+4\rho)}{64b\delta(\delta+\rho)^2}\right]$$

*and $\psi = z\Theta^2/4(\delta+\rho)$ allows the vertically separated industry to reproduce the same performance attained by the vertically integrated monopolist.*

### 4.3 The state-linear two-part tariff

A natural way out of the problem outlined above consists in defining the fee $F$ as a linear function of the quality level, i.e., $F = \phi + \psi q$, coupled with $w = 0$.[12] Of course, this solution can be pursued as long as the quality level of the component or intermediate good supplied by $U$ to $D$ is observable by $D$. If not (or, if it is verifiable after a significant lag), then such a contract will not be, in general, a solution to the aforementioned hold-up problem. State and control variables, as well as output and profits will carry superscript $q$ to recall that the TPT is a function of the quality level.

For the moment, I keep $F$ as exogenous and just set $w = 0$. The maximum problem of firm $D$ is solved by (15), with $w = 0$. Then, posing $V_D(q) = \gamma q(t) + \varepsilon$ and $V_D'(q) = \gamma$ and taking $k_U$ as given, the Bellman equation of firm $D$ is solved by the pair $(\gamma, \varepsilon)$ solving the following system of Riccati equations:

$$\begin{aligned} \Theta^2 - 4\gamma(\delta + \rho) = 0 \\ z^2\gamma^2 - 4b(F - \gamma z k_U + \varepsilon\rho) = 0 \end{aligned} \tag{32}$$

System (32) delivers

$$\varepsilon = \frac{\gamma(4\gamma z^2 + 4bk_U) - 4bF}{4b\rho}\,;\ \gamma = \frac{\Theta^2}{4(\delta + \rho)} \tag{33}$$

The downstream firm's profit simplifies as follows:

$$\pi_D^q = \frac{\Theta^2 q}{4} - \frac{z^2\Theta^4}{64b(\delta + \rho)^2} - F \tag{34}$$

and it is nil in correspondence of

$$F = \frac{\Theta^2 q}{4} - \frac{z^2\Theta^4}{64b(\delta + \rho)^2} \tag{35}$$

[12] This is the standard approach to obtain (degenerate) Markovian equilibria in Stackelberg differential games where the leader's policy is taken to be a linear function of the relevant state variable (see Dockner *et al.* 2000, pp. 134-41).

The expressions appearing in (33) and (35) can be substituted into the Bellman equation of the upstream firm, which generates a FOC w.r.t. $k_U$ delivering the by now familiar result $k_U^q = zV_U'(q)/(2b)$. Assuming again $V_U(q) = \zeta q + \eta$, the Bellman equation of firm $U$ produces the following system:

$$4\zeta(\delta+\rho) - \Theta^2 = 0 \quad (36)$$

$$64b\eta\rho(\delta+\rho)^2 - z^2\left[16\zeta^2\left(\delta^2+\rho^2\right) + 8\zeta\left(\Theta^2(\delta+\rho) + 4\zeta\rho\right) - \Theta^4\right] = 0 \quad (37)$$

whose unique solution is identified by the pair

$$\zeta = \frac{\Theta^2}{4(\delta+\rho)}\,;\; \eta = \frac{z^2\Theta^4}{32b\rho(\delta+\rho)^2} \quad (38)$$

At this point it is quickly checked that $q = q^*$, $k_U^q = k_D^q = k^*$, $x^q = x^*$ and $\pi_U^q + \pi_D^q = \pi^*$. Accordingly, I may formulate

**Proposition 5** *If the fee appearing in the TPT is (i) linear in the quality level and (ii) extracts the full surplus from the pockets of the downstream firm, the equilibrium attained under vertical separation replicates the performance of the vertically integrated monopolist.*

Although apparently this type of contract produces the same equilibrium as the one based on a TPT linear in the upstream firm's control, the approach illustrated in this section is somewhat problematic as it leaves room to a moral hazard problem. If any given quality increase along the supply chain is verifiable (and therefore contractible), then the TPT incorporating (35) represents a feasible efficient solution to the hold-up problem. If not, (35) is a gamble the downstream firm should not be willing to accept as it exposes the same firm to an obvious opportunistic behaviour on the part of the upstream supplier.

# 5 Concluding remarks

I have investigated the efficient design of the contract based on a two-part tariff that should be adopted to lead a supply chain along which quality-improving investments take place to entirely replicate the performance of a vertically integrated firm. In particular, the foregoing analysis has shown that there exist two alternative specification of the TPT achieving this outcome: one contemplates a fee defined as a linear function of the upstream R&D endeavour, the other specifies the fee as a linear function of product quality. The latter might not be a feasible solution if quality improvements along the vertical relation are not immediately observable/verifiable, and therefore not contractible, while the adoption of the former hinges upon reliable financial reports on the part of the upstream OEM firm.

Several extensions of the above analysis can be envisaged. First of all, the setup can be extended to allow for oligopolistic competition to take place either downstream or upstream, or in both stages. Secondly, the presence of some other type of investment, e.g., in cost-reducing innovation, could also be accounted for, as in Lambertini and Orsini (2000; 2015). Thirdly, here I have confined my attention to nondurables; using the same approach to analysing contractual design based on TPT's for durables looks like a natural *addendum*. These tasks are left for future research.

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