Logics for Normative Supervenience



*Published Version:*
Rotolo, A. (2017). Logics for Normative Supervenience. Dordrecht : Springer [10.1007/978-3-319-61046-7_1].

*Availability:*
This version is available at: https://hdl.handle.net/11585/626919 since: 2018-02-27

*Published:*
DOI: http://doi.org/10.1007/978-3-319-61046-7_1

# Logics for Normative Supervenience

Antonino Rotolo

*One virtue is that the plurality of the consequence relation comes at little or no cost. Another is that pluralism offers a more charitable interpretation of many important (but difficult) debates in philosophical logic than is otherwise available; [. . . ] pluralism does more justice to the mix of insight and perplexity found in many of the debates in logic in the last century. (Beall and Restall, 2000, 31)*

**Abstract** This essay addresses the problem of logically modelling the concept of normative supervenience. We will argue that alternatives of classical logic can grasp specific aspects of this concept. We will examine two cases: (a) the idea of institutional supervenience corresponding to the counts-as relation, (b) modal logics for jumping or generating the normative dimension of supervenient properties.

## 1 Introduction – Assumption I: A Plurality of Logics for Normative Supervenience

This essay addresses the problem of modelling forms of normative supervenience (hereafter, NS) from the logical point of view. We will offer in Section 2 some clarifications on what we mean by *normative* supervenience[1]. For the moment, let us assume that NS is a special case of the very general idea of supervenience.

With this said, a natural way of logically modelling NS would argue that this relation corresponds to a form of entailment. However, convincing counter-arguments in the literature have been proposed showing that the supervenience relation is in general *different* from entailment (cf. McLaughlin, 1995). Indeed, despite the fact that supervenience apparently shares with entailment formal properties, such as re-

CIRSFID, University of Bologna, Italy
e-mail: antonino.rotolo@unibo.it

[1] More intuitions are of course offered in the other essays in this volume.

flexivity[2], transitivity, and non-symmetry, if we take the sets of properties $A$ and $B$ we have that (McLaughlin and Bennett, 2014, par. 3.2 )

- **entailment does not suffice for supervenience**, since there are cases where $B$ entails $A$ but $A$ does not supervene on $B$; for example, possessing the property of being a brother entails possessing the property of being a sibling, but being a sibling does not supervene on being a brother;
- **supervenience does not suffice for entailment**, since there are cases where $A$ supervenes on $B$ but $B$ does not imply $A$; for example, thermal conductivity properties do not entail electrical conductivity properties.

The above approach is apparently irrefutable, but much depends on what we mean by the concept of *entailment*: if it corresponds to the consequence relation of classical logic, we agree with the mainstream literature on the idea that supervenience is not entailment. However, logicians are familiar with the idea that more logical paradigms can be developed, and that logical pluralism is a suitable option for analysing hard concepts (cf. Beall and Restall, 2000):

**Definition 1 (Logical pluralism (Informal)).** Logical pluralism is the thesis that there is more than one correct logical consequence relation.

From the operational viewpoint, this approach amounts to

1. creating a suitable logical system that is able to capture our philosophical intuitions about the concept of NS, and
2. checking whether conclusions and properties of the system are reasonable in our theory.

This is what we will do in this essay by considering two different philosophical aspects related to the idea of normative supervenience and thus developing two different suitable logical systems.

Our methodological perspective is not new. Humberstone (1993, 2002) offered a route where a non-classical consequence relation does the job.

**Definition 2.** [Inference-determined vs supervenience-determined consequence relation (Humberstone, 1993, 2002)] Let $\mathscr{V}$ be a set of valuations.

- **A consequence relation inference-determined by** $\mathscr{V}$ is as follows:

$$\Gamma \models^{I}_{\mathscr{V}} A$$

  iff, if for all $v \in \mathscr{V}$ and all $B \in \Gamma$, $v(B) = T$ then $v(A) = T$;
- **A consequence relation supervenience-determined by** $\mathscr{V}$ is as follows:

$$\Gamma \models^{S}_{\mathscr{V}} A$$

  iff, for all $v, z \in \mathscr{V}$ and all $B \in \Gamma$, if $v(B) = z(B)$ then $v(A) = z(A)$.

[2] We will discuss later some complexities behind reflexivity.

The second type of consequence relation, among the other things, is meant to capture the idea of co-variance, something which characterises the concept of supervenience, but which seems not correctly expressed by classical logical entailment.

We will not directly work on Definition 2, but our analysis falls indeed within the same general approach.

## 2 Introduction – Assumption II: Supervenience of the Normative vs Normative Supervenience

Let us assume that normative properties are generically those properties that constitute being what one ought to do[3]. If so, a simple preliminary definition relating supervenience and normativity can be the following:

**Definition 3 (Supervenience and normativity (Version I)).** NS is a binary relation between properties such that normative properties of kind $A$ supervene on non-normative properties of kind $B$.

From the logical point of view, this definition has the advantage of relating NS to the general relation of supervenience: what makes NS *normative* is simply the fact that the supervenient properties are normative. Definition 3 is pictorially illustrated by Figure 1. This would formally mean to distinguish two sorts of predicates. However, such a definition can be seen as problematic for those who dispute about the plausibility of normative properties or about the fact that such properties are intuitively, and fundamentally distinguishable, e.g., from the natural ones.

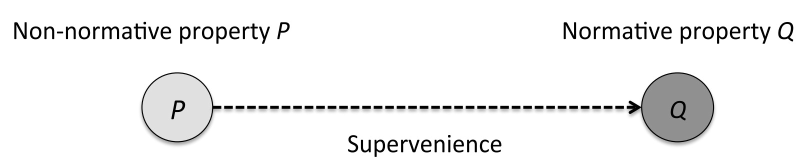


**Fig. 1** Supervenience and normativity (Version I: Definition 3)

A rather different way for defining NS does not amount to partitioning the set of predicates into two sorts, but to identifying a *normative way* through which possible

[3] Notice that this assumption is generic and it does not necessarily mean that individual normative properties directly provide grounds for single prescriptive statements. We simply assume that such properties are normative insofar as they are true in the context of ideal worlds. For instance, one can argue that the sentence "This piece of paper counts as a five euro bill" is normative without directly stating that something here is obligatory: indeed, this sentence presupposes the existence of a certain human institution which (i) refers to a set of ideal worlds where some properties apply to certain individuals, (ii) is globally oriented to guide human behaviour (see Section 4).

worlds (where predicates are made true of individuals) are related—as standardly done in possible-world semantics for deontic logic (cf. Gabbay et al., 2013).

**Definition 4 (Supervenience and normativity (Version II)).** NS is a logical entailment that makes properties normative and that corresponds to ways for identifying the set of normative possible worlds (normative necessity).

We can thus distinguish two concepts of NS:

- NS in a broad sense, or **supervenience of the normative**, as proposed in Definition 3;
- NS *strictu sensu*, **normative supervenience**, as proposed in Definition 4.

We will technically explain in the subsequent sections how the idea in Definition 4 can be framed to reconstruct the concept of NS. Here it suffices to say that normative properties in any world *w* are nothing but those properties that are true of individuals in worlds that are selected as the most-preferred (or ideal) ones with respect to *w*: hereafter, NS will be denoting the concept of normative supervenience of Definition 4.

The basic philosophical and logical move behind NS is pictorially rendered in Figure 2.

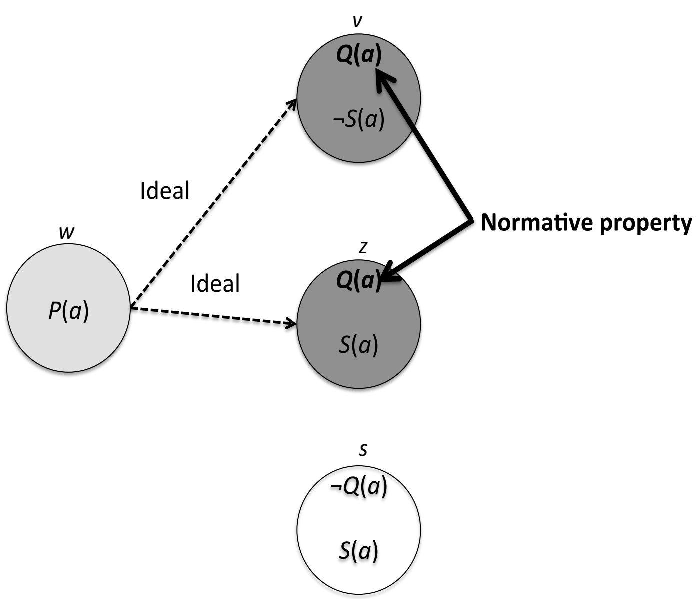


**Fig. 2** Supervenience and normativity (Version II: Definition 4)

Figure 2 shows four possible worlds *w*, *v*, *z*, and *s*. The worlds *v* and *z* are normatively ideal with respect to *w*, while *s* is not. In *v* and *z* the individual *a* is *Q*, thus

$Q$ can be seen as a normative property with respect to $w$ and $a$. We will show how this idea is suitable for defining $Q$ as supervenient, e.g., with respect to $P$.

## 3 Layout of This Essay

In this essay we will see how alternatives of classical logic can grasp specific aspects of NS.

- Our assumptions are:
  - Formal aspects of supervenience can be captured by suitable forms of non-classical entailment;
  - Supervenience-based entailment is modelled as NS and not as supervenience of the normative.
- We will examine two cases:
  - The idea of institutional supervenience corresponding to the *counts-as relation*;
  - A meta-theory of NS and different *modal logics for jumping or generating the normative dimension of supervenient properties*.

## 4 The Counts-as Relation

### *4.1 Introduction*

John Searle famously introduced the counts-as relation in the context of his theory of the rule-based nature of social institutions (Searle, 1969, 51–52):

> A marriage ceremony, a baseball game, a trial, and a legislative action involve a variety of physical movements, states, and raw feels, but a specification of one of these events only in such terms is not so far a specification of it as a marriage ceremony, baseball game, a trial, or a legislative action. The physical events and raw feels only count as parts of such events given certain other conditions and against a background of certain kinds of institutions. Such facts as are recorded in my above group of statements I propose to call institutional facts. They are indeed facts; but their existence, unlike the existence of brute facts, presupposes the existence of certain human institutions. [. . . ] These "institutions" are systems of constitutive rules. Every institutional fact is underlain by a (system of) rule(s) of the form "X counts as Y in context C." Our hypothesis that speaking a language is performing acts according to constitutive rules involves us in the hypothesis that the fact that a man performed a certain speech act, e.g. made a promise, is an institutional fact.

In other words, institutions emerge from an independent ontology of "brute", natural facts through *constitutive rules* of the mentioned counts-as form "$X$ counts as Y in context C", where $X$ is any object satisfying certain conditions and $Y$ is a label that qualifies $X$ as being something of an entirely new sort. In Searle's terms, these rules may be seen as assigning a status to $X$, and with it, a function that $X$ does not already have before just in virtue of its being an $X$. Systems of such kind of rules, which are more or less implicit ingredients of stable social practices, "create the very possibility of certain activities" within those systems (Searle, 1995, 28)[4].

To sum up:

**Definition 5 (Informal 1 (Searle, 1969, 1995)).** The *counts-as* (constitutive) *rules* have the following canonical form:

$$X \text{ COUNTS AS } Y \text{ in the context } C$$

where, typically, $X$ denotes a "brute" fact and $Y$ an "institutional" fact.

**Definition 6 (Informal 2 (Searle, 1969, 1995)).** The *counts-as rules* establish a *counts-as relation* that assigns a status $Y$ to $X$, and with it, a function that $X$ does not already have before just in virtue of its being an $X$.

Two paradigmatic examples of counts-as relations are

$$\text{This piece of paper counts as a five euro bill} \quad (1)$$

$$\text{X counts as a presiding official in a wedding ceremony} \quad (2)$$

The most we formally learn from Searle (1995, 28, 44, 45) is that

1. counts-as contexts are *intensional* in the usual sense of failing the substitutivity test, and
2. the $X$ and $Y$ terms in the counts-as relation are *causally unrelated*.

These properties seem to capture crucial features of Searle's counts-as construction. For example, if $X$= 'US president's declaration' and this counts as a certain $Y$, it is not hard to understand that 'Barack Obama's declaration' in itself may not count as $Y$ since Obama's declaration is institutionally relevant insofar as Barack Obama is the president of US. Also point 2 is reasonable, especially when we consider the peculiarities of institutional ontology as opposed to the domain of brute (empirical) facts. In this perspective, a US president's declaration is not a causal reason

[4] As Searle (1995) strongly emphasises, in all such cases the ascription of a status-function through the appropriate rule is not enough to establish such types of facts. To establish these facts we have to believe, or otherwise accept, acknowledge, collectively intend, etc., that $X$ has the status-function assigned by the corresponding rules. This means that, in order to do their job these rules have to be agreed upon or believed in by members of the relevant community *qua* members of a collective. The reason for this is that it is a merely contingent fact that some $X$ stands in a certain counts-as relation to some $Y$, because the only connection between $X$ and $Y$ obtains in virtue of collective belief or acceptance, and intention—or, to put it in Searle's words, satisfying the $X$ term *is not by itself sufficient* for being money, and the X term does not specify causal features that would be sufficient to enable the stuff to function as money. In order to function as money, human agreement has to be involved. Roughly, this is nothing but the idea that the ontology of institutional facts relies on a kind of "epistemic objectivity".

for obtaining $Y$. In fact, Searle does not seem to argue that the occurrence of $X$ determines the existence of $Y$ along with a broad causal interpretation of constitutive rules (Searle, 2001, sec. II): these simply specify the "constitutive features of the *actuality* [...] of [...] institutional facts"[5].

This analysis offers some general directions for clarifying the logical nature of counts-as link:

- First, since the counts-as relation is intensional, it can be reconstructed in the context of modal logics; this thesis is in line with most literature on supervenience and, more specifically, with the intuition we have proposed in Section 2, according to which NS corresponds to a normative way through which we select ideal worlds;
- Second, since the counts-as relation does not correspond to a causal link, its logical reconstruction cannot enjoy the same formal properties that causal relations usually have; we will see that Searle's view is just an option and that alternative philosophical (and logical) views are possible. Such alternatives are perhaps better for reconstructing the counts-as relation as NS.

### *4.2 Is the Counts-as Relation a Type of NS?*

The counts-as link exhibits some intuitive similarities with supervenience. Consider this legal example:

| | | |
|---|---|---|
| Electronic signature | COUNTS AS | handwritten signature |
| | IN CONTEXT | Italian contract law |

Indeed, for the sake of illustration let us now assume the following standard definitions:

**Definition 7 (Indiscernibility and Weak Supervenience (Kim, 1993)).**

**Indiscernibility:** If $B$ is a set of properties, any two individuals $x$ and $y$ are $B$-*indiscernible* iff $P(x) \rightarrow P(y)$ for all $P$'s belonging to $B$.

**Weak supervenience:** $B$-properties weakly supervene on $A$-properties iff, for any two individuals $x$ and $y$ that belong to the same possible world $w$, if $x$ and $y$ are $A$-indiscernible in $w$, then they are also $B$-indiscernible in $w$.

The just mentioned legal example illustrates well at what extent counts-as relations are similar to (weak) supervenience (McLaughlin, 1995). Indeed, if we consider any two individuals $x$ and $y$, we can conceptually admit the following cases

[5] It would sound perhaps more reasonable to emphasise the causal role that mental attitudes, such as collective intentionality, play in bringing into existence institutional reality. But even in this case, Searle (2001, sec. II) argues that "collective intentionality is not something which just causes institutional reality, it is constitutive of that reality precisely because it is constitutive of the ontology according to the constitutive rules".

(*Esign* and *Hsign* stand for electronic signature and handwritten signature, respectively):

1. $Esign(x), Esign(y) \quad Hsign(x), Hsign(y)$
2. $\neg Esign(x), \neg Esign(y) \quad Hsign(x), Hsign(y)$
3. $\neg Esign(x), \neg Esign(y) \quad \neg Hsign(x), \neg Hsign(y)$

Given Definition 7, these cases are *formally* admissible.
What about the following?

4. $Esign(x), \neg Esign(y) \quad Hsign(x), Hsign(y)$
5. $\neg Esign(x), \neg Esign(y) \quad \neg Hsign(x), Hsign(y)$

These cases should be in principle ruled out, as they do not meet Definition 7. In addition, individuals $y$ (case 4.) and $x$ (case 5.) either

(a) are true instances of handwritten signatures: they are not electronic signatures because they are just brute handwritten signatures; or
(b) are not even brute instances of handwritten signatures (but, e.g., smoke signals): i.e., the fact that they count as handwritten signatures depends on a different counts-as relation (e.g., stating that smoke signals count as handwritten signatures).

The second case does not help here, as it simply refers to another counts-as rule. The first case, instead, suggests that we should reject that, for any property $P$, $P$'s count as $P$'s, especially if $P$ is not an institutional property. Hence, under the analysis above, case 2., too, should be ruled out: if so, the remaining cases for the counts-as relation satisfy Definition 7[6].

In the specific legal example, if we use $\Rightarrow$ to denote the counts-as link

$$Hsign(x) \Rightarrow Hsign(x)$$

must be assumed to be invalid. In other words, under the hypothesis that the counts-as link is NS, whenever brute facts are related with institutional facts we have to reject the general view (McLaughlin and Bennett, 2014, par. 3.2 ) that NS is reflexive as the classical logical entailment is: if the counts-as must ensure co-variance and thus is genuine NS, it may be argued that counts-as relations do not enjoy the following schema:

$$A \Rightarrow A. \qquad \text{(Reflexivity)}$$

[6] In addition to the intuitive observation that cases 2., 4., and 5. speak of being a handwritten signature as a brute fact, we should also recall that NS, in the sense of Definition 4, does not rely on distinguishing in the formal language different sorts of predicates or propositional letters.

## 4.3 A Formal Analysis of the Counts-as Relation

The logical nature of counts-as rules has been investigated following several directions (for e general overview, cf. Grossi and Jones, 2013). Here, we will consider two options that express the counts-as link as a non-classical (modal) conditional[7].

In their seminal paper, Jones and Sergot (1996) develop an analysis of the notion of institutionalised power by introducing a new conditional connective '$\Rightarrow_s$'. This connective expresses the counts-as connection holding in the context of an institution $s$. In short, this approach is roughly in line with Goldman's theory of actions generating actions (Goldman, 1970). In this perspective, it may be argued that the generation of institutional facts via counts-as rules is quite close to the idea of a causal relation—contrary to Searle's argument—and assumes that reflexivity does not hold—as required above.

A second formalisation, though openly inspired by Jones and Sergot, proposes some substantial changes in the light of a different philosophical interpretation of the counts-as relation (Gelati et al., 2004). Counts-as rules are meant to capture the constitutive, but classificatory character of institutional ontology. Accordingly, their function is to represent the constitutive ingredients of institutional facts, whose nature is conceptually distinct from that of the empirical facts. On the other hand, counts-as rules have a normative status. They are norms insofar as their conditional nature exhibits some basic properties enjoyed by the usual normative links.

### 4.3.1 Counts-as Link as NS: A Generative (Dynamic) Relation

Jones and Sergot developed a formal approach to the notion of institutionalised power by introducing a conditional connective $\Rightarrow_s$ to express the counts-as connection holding in the context of an institution $s$. Accordingly, an expression like $A \Rightarrow_s B$ means that $A$ counts as $B$, where $A$ is viewed a sufficient condition for obtaining $B$ within $s$.

Jones and Sergot characterise the logic for $\Rightarrow_s$ as a classical conditional logic (RCEA, RCEC) (Chellas, 1980), plus the axioms

$$((A \Rightarrow_s B) \wedge (A \Rightarrow_s C)) \rightarrow (A \Rightarrow_s (B \wedge C)) \tag{3}$$

$$((A \Rightarrow_s B) \wedge (C \Rightarrow_s B)) \rightarrow ((A \vee C) \Rightarrow_s B) \tag{4}$$

$$(A \Rightarrow_s B) \rightarrow ((B \Rightarrow_s C) \rightarrow (A \Rightarrow_s C)) \tag{5}$$

In addition, Jones and Sergot's analysis is integrated by introducing the normal **KD** modality $D_s$, such that $D_sA$ means that $A$ is a "constraint on the institution $s$". More precisely, this is suggested to capture all (logical, causal, deontic, etc.) constraints on $s$ which include the counts-as connection. Accordingly, a formula like $D_s(A \rightarrow B)$

[7] According to Section 2, a formal analysis of NS may require to use predicate logics. For the sake of simplicity, we will work in this section with a propositional language, referring the reader to Delgrande (1998)'s investigations on quantification in conditional logics.

means "it is a constraint of (operative in) institution $s$ that if $A$ then $B$" or "it is incompatible with the constraints operative in $s$ that $A$ and not-$B$". When linked to $D_s$ through the following schema

$$(A \Rightarrow_s B) \rightarrow D_s(A \rightarrow B) \tag{6}$$

counts-as links thus express institutional constraints (on $s$) to the effect that within $s$ the realisation of $A$ (e.g., performing certain acts as a presiding official in a wedding ceremony) counts as a sufficient condition of creating $B$ (the status of married people).

It is important to note that this approach guarantees a restricted form of detachment of "institutional consequents" from antecedents in the form: if $A \Rightarrow_s B$ holds and it is the case that $A$, then it is the case in $s$ that $D_sB$ according to the constraints operative in $s$. This is done by adopting also the following schema:

$$(A \Rightarrow_s B) \rightarrow (A \rightarrow D_sA). \tag{7}$$

#### 4.3.2 Counts-as Link as a Classificatory Relation

Following Gelati et al. (2004), we can argue that the counts-as link—in Searle's sense—is a normative classificatory relation involving institutional facts. As such, it enjoys, among others, Reflexivity: if $\Rightarrow_s$ is a classificatory relation, how can we reject that some $A$ holds as itself in a given institution (i.e., $A \Rightarrow_s A$)? If so, there is at least another way to model counts-as relations:

- let us introduce a generic, normative, and classificatory, conditionality $\Rightarrow$; in other words, any expression $A \Rightarrow B$ means that $A$ is normatively falling within type $B$; and
- let us use a non-normal "institutional" modality $D_s$, to strictly denote the domain of institutional facts. More precisely, an expression such as $D_sA$ is to be read as "it is an institutional fact within $s$ that $A$".

The machinery reframes the counts-as link as follows:

$$(A \Rightarrow_s B) =_{def} (A \Rightarrow D_sB) \wedge (D_sA \Rightarrow D_sB) \tag{8}$$

This statement accounts for the structuring of institutional facts, with regard to an institution $s$, in a hierarchy of counts-as relations linking (a) brute facts with institutional facts ($s$-facts) and (b) $s$-facts with other $s$-facts.

The formal theory of the counts-as conditional is provided by a logic corresponding at least to (Kraus et al., 1990)'s system of nonmonotonic cumulative logic.

The axiomatisation adopted for '$\Rightarrow$' is as follows:

$$A \Rightarrow A \tag{9}$$

$$(A \Rightarrow B) \wedge (A \wedge B \Rightarrow C) \rightarrow (A \Rightarrow C) \tag{10}$$

$$(A \Rightarrow B) \wedge (A \Rightarrow C) \rightarrow (A \wedge B) \Rightarrow C) \tag{11}$$

As expected, the logic for $\Rightarrow$ is closed under RCEA and RCK (Artosi et al., 2002). In addition, it is possible to add

$$(A \Rightarrow C) \wedge (B \Rightarrow C) \rightarrow (A \vee B) \Rightarrow C) \tag{12}$$

Finally, the logic for $D_s$ is closed under logical equivalence and contains the following schemata:

$$D_s A \rightarrow \neg D_s \neg A \tag{13}$$

$$(D_s A \wedge D_s B) \rightarrow D_s(A \wedge B) \tag{14}$$

Since this modality is meant to strictly represent the institutional facts holding within $s$, the necessitation rule is not adopted. In fact, it would sound strange that $\top$ may be viewed as an institutional fact for any institution $s$.

### 4.3.3 Semantics for $\Rightarrow$

A rather standard semantics for conditionals is based of selection-function models (Stalnaker, 1968, 33–34):

> Consider a possible world in which *A* is true, and which otherwise differs minimally from the actual world. "If *A*, then *B*" is true (false) just in case *B* is true (false) in that possible world.

From a normative point of view, we can reframe the idea of minimal difference between possible worlds into the one of normative most-preferred worlds:

**Definition 8 (Selection function semantics).** Selection function semantics is based on the following notions:

- A *selection function frame* is a tuple $\mathscr{F} := \langle W, f \rangle$ where $W$ is a non empty set and $f$ is a a function assigning to each sentence and world in $W$ a subset of normatively most-preferred worlds in $W$.
- A *selection function model* is a tuple $\mathscr{M} := \langle W, f, V \rangle$ where $\langle W, f \rangle$ is a selection function frame and $V$ is an evaluation function.
- The condition to evaluate $A \Rightarrow B$-formulae is as follows. For any $w \in W$: $w \models_V A \Rightarrow B$ iff $f(A, w) \subseteq ||B||$ ($||B||$ is the set of all worlds making $B$ true).

The section function thus picks up the best normative states with respect to a certain world and a true propositional condition. This normative interpretation of conditional is far from new: see the discussion in (Nute, 1997).

The basic system for $\Rightarrow$ of (Gelati et al., 2004) is characterised by the following semantic conditions (Artosi et al., 2002):

**Definition 9 (Selection function semantics (cont'd)).** For all $w \in W$ and any $A$ and $B$, conditions for schemata are

- if $||A|| = ||B||$ then $f(A, w) = f(B, w)$

- if $f(A,w) \subseteq ||B||$ then $f(A \wedge B,w) \subseteq f(A,w)$
- if $f(A,w) \subseteq ||B||$ then $f(A,w) \subseteq f(A \wedge B,w)$

We can similarly proceed with Jones and Sergot's system and identify suitable semantic conditions characterising it (see Jones and Sergot, 1996, sec. 4).

### 4.3.4 Discussion

Jones and Sergot are clearly inspired by Goldman's theory of action generation. As is well-known, the action generation may be characterised as a kind of conditionality where the occurrence of the antecedent $A$, which corresponds, typically, to an action description, generates the occurrence of a different action description $B$, if some background conditions are satisfied. In particular, $A$ and $B$ are modelled as simultaneous and $A$ is not strictly part of doing $B$. Goldman (1970, 25ff.) considers, among other things, a specific case where the consequent $B$ is generated by $A$ by convention. This type is quite close to the idea of "counts as", since we may speak of conventional generation exactly when, for example, we state that a chess player wins the game by checkmating her opponent.

(Jones and Sergot, 1996)'s connective $\Rightarrow_s$ is standing for a peculiar kind of conventional generation. In fact, such a connective is stated to be non-reflexive and transitive, two properties that, among others, are typically assigned to characterise any conditionality expressing forms of causality or generation (Shoham, 1990). In particular, rejecting or adopting reflexivity of counts-as relations constitutes perhaps one of the most decisive aspects that seem to differentiate the two logical approaches recalled in above. As noted by Jones and colleagues, with regard to the counts-as link, "it is precisely the property of non-reflexiveness that distinguishes a generation relation as such". Reflexivity affects the meaning ascribed to the counts-as link. First of all, as is well-known, if '$A$ counts as $A$' holds we clearly cannot argue in favour of transitivity since its presence plus reflexivity imply monotonicity (Kraus et al., 1990). The problem is then to decide whether reflexivity must prevail over transitivity or the other way around. This inevitable choice cannot be avoided insofar as non-monotonicity is a crucial feature of counts-as conditionals:

*Example 1.* Suppose that in an auction if the agent $x$ raises one hand, this may count as making a bid. It is clear that this does not hold if $x$ raises one hand *and* scratches his own head. If '$x$ scratches his own head' is true, there are good reasons to conclude that $x$ does not make any bid.

Whether normative relations enjoy reflexivity can be a thorny question (cf. the discussion in Parent, 2001). For example, arguments against reflexivity are quite serious when we deal with dyadic obligations (Hansen, 2005). On the other hand, one may tend to view a normative conditional by integrating a generic connective $\Rightarrow$ with suitable monadic operators and, especially, with a suitable institutional classificatory operator. According to Gelati et al., a formula like $A \Rightarrow A$ simply states the systematic claim that any formula $A$ is a consequence of itself. Also, notice that this view, according to (8), does not lead to full reflexivity, but only to $D_sA \Rightarrow D_sA$, which means nothing but that an institutional fact is an institutional fact.

In fact, if the counts-as link means that "*A is to be classified as B* within *s*", rejecting reflexivity may be problematic: Defeasible classificatory relations, such as typicality, normally enjoy reflexivity. Actually, counts-as rules are not only regulative; they are primarily constitutive insofar as they express the constitutive elements of institutional reality. As we have seen, this is not only a well-known thesis developed by Searle, but seems to correspond to the idea that such rules may encode as well classificatory relations between categories within any institution.

Of course there are sound arguments to accept transitivity (instead of reflexivity). If we know that raising one hand counts as making a bid, and bidding counts as buying a good, then raising one hand counts as buying a good. This is basically Jones and Sergot's perspective, which is based on the idea that the occurrence of the antecedent of a counts-as relation is a (defeasible) *sufficient* reason for getting the consequent. But, as in the case of causality, some pathological examples may be put forward. In Jones and Sergot's view, counts-as sufficient conditions are defeasible. If so, given a rule such as $r_1 : X \Rightarrow_s Y$, we may say that $X$ is sufficient to imply $Y$ if some implicit background conditions $S_1$ are satisfied. Each rule of this kind is conceptually linked to a number of implicit conditions. Now imagine we have another rule $r_2 : Y \Rightarrow_s Z$ where the background conditions are $S_2$. The acceptability of $r_3 : X \Rightarrow_s Z$ depends on the compatibility of $S_1 \cup S_2 \cup X$ with respect to $Z$. In fact, when $S_1$ and $S_2$ are made explicit conjunctively in the corresponding antecedents, we have $X \wedge S_1 \Rightarrow_s Y$ and $Y \wedge S_2 \Rightarrow_s Z$. If so, since '$\Rightarrow_s$' is defeasible, nothing prevents us from having that $S_1 \cup S_2 \cup \{X\} \Rightarrow_s \neg Z$. In a different perspective, when $S_1 \cup S_2$ is inconsistent, we would get the disruptive conclusion to infer trivially any formula. To be sure, these are a logical possibilities that may in theory jeopardise the adoption of full transitivity for counts-as relation. Let's see an example. Consider the following rules:

$r_1$: $x$'s electronic signature *counts as* $x$'s handwritten signature

$r_2$: $x$'s handwritten signature *counts as* evidence of $x$'s handwriting.

Of course, we cannot conclude that

$r_3$: $x$'s electronic signature *counts as* evidence of $x$'s handwriting.

The intuitive background presuppositions behind rules $r_1$ and $r_2$ are clearly and conjunctively incompatible with regard to $Z$. So, if counts-as is applied to fact-descriptions, full transitivity cannot in general be accepted.

If the argument above is correct, what we can do is just to drop Jones and Sergot's (5) and adopt at most restricted transitivity (cumulativity), namely (10).

### 4.3.5 Conclusions

The previous discussion has shown that the counts-as relation can be logically viewed as a conventional type of NS. In particular, to sum up:

- **Option 1:**
  - If the counts-as link does not enjoy reflexivity, then
    - it is a generative (dynamic) relation;
    - it is a form of conventional NS, which semantically refers to the best normative worlds specific given institutions.
- **Option 2:**
  - If we do not drop reflexivity and transitivity, then
    - we validate the following schema:

$$(A \Rightarrow B) \rightarrow ((A \wedge C) \Rightarrow B) \qquad \text{(Strengthening of the antecedent)}$$

    - Thus, the counts-as relation amounts in fact to classical implication (classificatory, non-generative/non-dynamic, based on subset inclusion), and so it is not NS (see Section 1).
- **Option 3:**
  - If we drop transitivity but not reflexivity, then
    - we obtain a defeasible classificatory (non-generative, non-dynamic) institutional relation;
    - the counts-as only partially meet the formal requirements of weak supervenience.

## 5 Meta-theory for NS: Which Modal Logic?

### *5.1 Introduction*

In the previous section we showed that NS can be analysed in terms of suitable consequence relations and non-classical entailments. In this perspective, under suitable conditions the counts-as link is an interesting type of NS-relation.

Let us now move to a more abstract level of analysis and consider two well-known logical reconstructions for weak and strong supervenience (cf. Kim, 1993):

**Definition 10. Weak supervenience$_m$ ($WS_m$):** A set of properties $A$ weakly supervenes$_m$ on a set of properties $B$ iff

$$\Box\forall x(\bigwedge_{A_j \in A=\{A_1,\ldots,A_n\}} (A_j(x) \rightarrow \bigvee_{B_i \in B=\{B_1,\ldots,B_m\}} (B_i(x) \wedge \forall y(B_i(y) \rightarrow A_i(y)))))$$

**Strong supervenience$_m$:** ($SS_m$) A set of properties $A$ strongly supervenes$_m$ on a set of properties $B$ iff

$$\Box\forall x(\bigwedge_{A_j\in A=\{A_1,\dots,A_n\}}(A_j(x)\to\bigvee_{B_i\in B=\{B_1,\dots,B_m\}}(B_i(x)\land\Box\forall y(B_i(y)\to A_i(y)))))$$

Whatever meta-theoretical model we are inclined to accept, a crucial question arises: What is the logics for $\Box$? If universal quantification over possible worlds is taken to be equivalent to $\Box$, then at least the following system, S5, must be adopted, which traditionally corresponds to a form of *metaphysical necessity* (cf. Forbes, 1985):

$$\Box(A\to B)\to(\Box A\to\Box B) \quad \textbf{(K)}$$

$$\Box A\to A \quad \textbf{(T)}$$

$$\Box A\to\Box\Box A \quad \textbf{(4)}$$

$$\Diamond A\to\Box\Diamond A. \quad \textbf{(5)}$$

However, S5 raises a number of problems. Consider Bacon (1986)'s objection:

*Remark 1 ((Bacon, 1986)).* Under the assumption of diagonal closure (given any set $B$ of properties, if for every world $w$, $\psi$ is coextensive in $w$ with some property in $B$, then $\psi\in B$), weak supervenience entails strong supervenience whenever $\Box$ in the definitions of supervenience is based on standard modal logic S4 consisting of $\mathbf{K}\oplus\mathbf{T}\oplus\mathbf{4}$.

This analysis shows that the choice for an appropriate logic for $\Box$ is not irrelevant. Indeed, that weak supervenience entails strong supervenience can be viewed as counterintuitive: given two individuals $x$ and $y$, strong supervenience clearly rules out the case where $x$ and $y$ are $A$ and $B$ in a world $w$ but, in another world $v$, $x$ is still $A$ and $B$ while $y$ is $A$ and not $B$, something that weak supervenience in principle admits.

For this reason, several philosophers maintained that some axiom schemata of S4 (and so of S5 as well) do not make sense in the perspective of reconstructing supervenience (cf. Schmitt and Schroeder, 2011). Wedgwood (2007), among others, presented a number of reasons for rejecting **4**, a choice which blocks Bacon's argument.

In general, suppose to work in the context of the quantified modal logic $\mathsf{FOL}\oplus X$—where $X$ is some modal system—and thus assume to have constant domains of individuals across possible worlds—i.e., in normal modal logics, that Barcan schemata are valid (cf., e.g., Corsi, 2002):

- Assume $X=\mathsf{T}$ and, for simplicity, $A=\{A_j\}$, $B=\{B_i\}$. For any world $w$ and evaluation $V$, and for some individuals $b$ and $c$:

$$\{WS_m\}\models^w_V(A_j(b)\to(B_i(b)\land(B_i(c)\to A_j(c)))).$$

- Assume $X=\mathsf{K}$ (the minimal modal logic based on Kripke semantics) and, for simplicity, $A=\{A_j\}$, $B=\{B_i\}$. For for some individuals $b$ and $c$, any evaluation $V$ and some world $w$:

$$\{SS_m\} \models^w_V (\Box A_j(b) \rightarrow (\Box B_i(b) \wedge (\Box\Box B_i(c) \rightarrow \Box\Box A_j(c))))).$$

- Assume S5:

$$\models SS_m \equiv \forall x \Box ( \bigwedge_{A_j \in \{A_1,\dots,A_n\}} (A_j(x) \rightarrow \bigvee_{B_i \in \{B_1,\dots,B_m\}} (B_i(x) \wedge \forall y \Box y (B_i(y) \rightarrow A_i(y))))).$$

Do these cases make sense for NS? Although they look harmless, they in fact rely on a logic for $\Box$ that does not fit a sound idea of normativity, something that is required according to the interpretation of NS outlined in Definition 4. Indeed, it is well known, for instance, that schemata such as **T** are not of any use for characterising the idea of ought (see the discussion in McNamara, 2014).

In general, it seems that the choice for the best logic for NS requires to identify suitable systems of quantified modal logics for the ought. However, if we do not want to commit to any strong logical system (i.e., those including the schemata mentioned above), one rather basic option is developing a non-normal quantified deontic logic[8] using, e.g., machineries such as the ones recently studied by Calardo and Rotolo (2016), which we briefly and partially recall in the next section.

Another condition that we could relax is the assumption of constant domains of individuals. On the one side, we may have good arguments to keep this assumption as valid, since

- the distinction between *de dicto* and *de re* normative (deontic) statements looks controversial, namely, between formulae with and without free occurrences of variables within the scope of the ought operator **Ought** (von Wright, 1951; Castañeda, 1981);
- Why does the truth of formulae such as $\exists x \mathbf{Ought}(x = a)$ should vary from world to world (Goble, 1973)? In other words, what does it mean that an individual exists in some deontically preferred worlds but does not in other ideal worlds?

The above arguments, however, could be rejected. In particular:

- Why should we assume that exactly the same individuals populate all (normatively) ideal worlds? Indeed, it is known that the assumption of constant domains is often associated with precise metaphysical views, such as—but not only—logical atomism (cf. Cocchiarella, 1984); it is far from obvious whether this is required from the normative viewpoint.
- The equivalence between *de dicto* and *de re* statements is not in general guaranteed by the assumption of constant domains:
  - Barcan schemata alone are not sufficient in general to eliminate *de re* modalities, namely, to prove that, given any modal logic S, for each formula $\phi$, there exists a *de dicto* formula $\phi'$ such that $\mathsf{S} \vdash \phi \equiv \phi'$. This can be done only adding some extra-conditions and within strong modal systems such as S5 (see, e.g., Fine, 1978; Kaminski, 1997);

[8] Non-normal deontic logics have been considered a solution to avoid many drawbacks of standard deontic logic (i.e., deontic **KD**), which does not tolerate deontic conflicts and gives rise to a number of paradoxes and puzzles (Goble, 2005; Jones and Carmo, 2002).

– Barcan schemata are not in general characterised in non-normal modal logics by constant domains (Calardo and Rotolo, 2016).

## 5.2 Quantified Non-normal Modal Logics for the Ought

Let us recall a piece of Calardo and Rotolo (2016)'s machinery and define an appropriate semantics for quantified non-normal modal logic.

**Definition 11 (Multi-relational frames).** A multi-relational frame is a tuple $\mathscr{F} := \langle W, \mathscr{R}, D, U \rangle$ where:

- $W$ is a non empty set of worlds;
- $\mathscr{R}$ is a (possibly infinite) set of binary relations over $W$
- $D$ is a function associating to each world $w \in W$ a set $D_w$ of individuals (the inner domain of $w$);
- $U$ is a function associating to each world $w \in W$ a set $U_w$ of individuals (the outer domain of $w$) such that for any $w \in W$, $U_w \neq \emptyset$ and $D_w \subseteq U_w$ and if $wRv$ for some $R$, then $U_w \subseteq U_v$.

Models, assignments, and the concepts of *satisfaction, truth, validity* are defined in the standard way.

**Definition 12 (Multi-relational models).** A multi-relational model is a tuple $\mathscr{M} := \langle W, \mathscr{R}, D, U, I \rangle$ where $\langle W, \mathscr{R}, D, U \rangle$ is a multi-relational frame and $I$ is a function $I : \mathscr{L} \times W \mapsto \bigcup_{w \in W} U_w$ such that:

- $I_w(P^n) \subseteq (U_w)^n$
- $I_w(c) \in U_w$.

**Definition 13 (Assignments).** For any $w \in W$, a $w$-assignment $\sigma$ is a function $\sigma : Var(\mathscr{L}) \mapsto U_w$.
An $x$-variant $\tau$ of a $w$-assignment $\sigma$ is a $w$-assignment which may differ from $\sigma$ for the value assigned to $x$.

**Definition 14 ($\sigma$-interpretation).** Given a $w$-assignment $\sigma$, define

(a) $I^\sigma_w(c) = I_w(c)$, and
(b) $I^\sigma_w(x) = \sigma(x)$.

**Definition 15 (Truth conditions).** Let $\mathscr{M} := \langle W, \mathscr{R}, D, U, I \rangle$ be any multi-relational model, $\sigma$ any assignment, and $w \in W$. Truth evaluation clauses are as follows:

- $\mathscr{M} \models^\sigma_w P^n(t_1, \dots, t_n)$ iff $\langle I^\sigma_w(t_1), \dots, I^\sigma_w(t_n) \rangle \in I_w(P^n)$
- $\mathscr{M} \not\models^\sigma_w \bot$
- $\mathscr{M} \models^\sigma_w \forall x A$ iff for every $x$-variant $\tau$ of $\sigma$ such that $\tau(x) \in D_w$, $\mathscr{M} \models^\tau_w A(x)$
- $\mathscr{M} \models^\sigma_w \mathbf{Ought} A$ iff $\exists R_i \in \mathscr{R} :\ \forall v \in W (wR_i v \Rightarrow \mathscr{M} \models^\sigma_v A)$.

A model $\mathcal{M}$ *satisfies* a set of formulae $\Delta$ iff for some world $w$ and some $w$-assignment $\sigma$, $\mathcal{M} \models_w^\sigma A$ for all $A \in \Delta$. A formula $A$ is *true in a world* $w$ of a model $\mathcal{M}$, $\mathcal{M} \models_w A$, iff for any $w$-assignment $\sigma$, $\mathcal{M} \models_w^\sigma A$. A formula $A$ is *true in a model* $\mathcal{M}$, $\mathcal{M} \models A$, iff for all $w$, $\mathcal{M} \models_w A$. A formula $A$ is *valid* on a frame $\mathcal{F}$, $\mathcal{F} \models A$, iff for any model $\mathcal{M}$ on $\mathcal{F}$, $\mathcal{M} \models A$. Given a class of frames $\mathbb{F}$, a formula $A$ is $\mathbb{F}$-*valid*, $\mathbb{F} \models A$, iff for any frame $\mathcal{F} \in \mathbb{F}$, $\mathcal{F} \models A$. $\mathcal{M}$ is a model for a logic $\mathsf{L}$ iff $\mathcal{M} \models A$ for all $A \in \mathsf{L}$.

We assume all individual terms to be rigid designators.

The above semantics characterises the system $\mathsf{Q}^\circ.\mathsf{MN}$, which contains the following axioms and inference rules:

- Propositional tautologies;
- $\mathbf{UI}^\circ := \forall y(\forall x A(x) \to A(y/x))$
- $\forall x \forall y A \leftrightarrow \forall y \forall x A$
- $A \to \forall x A$, $x$ not free in $A$
- $\forall x(A \to B) \to (\forall x A \to \forall x B)$
- $\mathbf{M} := \mathbf{Ought}(A \land B) \to (\mathbf{Ought}A \land \mathbf{Ought}B)$
- $\mathbf{N} := \mathbf{Ought}\top$
- $\mathbf{MP} := A \to B, A/B$
- $\mathbf{RE} := A \equiv B/\mathbf{Ought}A \equiv \mathbf{Ought}B$
- $\mathbf{UG} := A/\forall x A$

We must notice that the propositional modal schemata and inference rules in $\mathsf{Q}^\circ.\mathsf{MN}$ amount to a deontic system proposed by Goble (2001, 2004).

### *5.3 Discussion about NS*

Multi-relational semantics was originally proposed by Schotch and Jennings (1981) and Goble (2001, 2004) in the domain of deontic logic. Semantic structures consist of

- **a plurality of worlds**, with possibly infinitely many deontic alternatives,
- **a plurality of accessibility relations**, with possibly infinitely many normative standards (codes, individual or collective preferences, ...) that select the most-preferred (or ideal) worlds.

In deontic logic the Kripke accessibility relation selects for each world those states of affairs that are (morally, legally, etc.) ideal with respect to it: hence, if $\mathbf{Ought}A$ is true in a world $w$, this simply means that $A$ is the case in all ideal alternatives to $w$. The interpretation of multi-relational models, as given in deontic logics, is thus that each accessibility relation corresponds to a particular "standard of value" or a set of norms that selects those ideal worlds; however, it is not guaranteed that such worlds are still ideal according to different standards of value or norms, namely, according to different accessibility relations. In this perspective, different relations correspond to different deontic standards or that conflicting norms are obtained from otherwise consistent different systems of norms. As Goble argues, when both $\mathbf{Ought}A$ and $\mathbf{Ought}\neg A$ "are true it is because $A$ is prescribed by one set of norms or regulations while $\neg A$ is prescribed by another, distinct set. [...] Each set of norms or regulations is presumed to be internally consistent, and conflicts only emerge as a result of rivalry between sets of norms" (Goble, 2001). In short, multi-relational models can provide a semantic analysis of *normative pluralism*, according to which different normative systems may generate obligations.

The assumption of a plurality of relations, in particular, allows us to provide some useful comments on NS and offers interesting conceptual tools for distinguishing weak and strong NS. Let us recall them using **Ought** and working two sets of predicates consisting of singletons:

$$\textbf{Ought}\forall x(A(x) \rightarrow (B(x) \wedge \forall y(B(y) \rightarrow A(y)))) \qquad (WNS_1)$$

$$\textbf{Ought}\forall x(A(x) \rightarrow (B(x) \wedge \textbf{Ought}\forall y(B(y) \rightarrow A(y)))) \qquad (SNS_1)$$

Let us consider the following model:[9]:

*Example 2.*

$$\mathscr{M} := \langle W, \mathscr{R}, D, U, I \rangle$$

where

- $W = \{w_1, w_2, w_3, w_4, w_5\}$;
- $\mathscr{R} = \{R_1 = \{\langle w_1, w_2\rangle\}, R_2 = \{\langle w_1, w_2\rangle, \langle w_1, w_3\rangle\}, R_3 = \{\langle w_3, w_4\rangle\}, R_4 = \{\langle w_2, w_5\rangle\}\}$;
- $D_x = U_x = \{a, b\}$, $\forall x \in \{w_1, w_2, w_3, w_4, w_5\}$.

$I$ is defined as follows:

- $I_{w_1}(A) = \{a,b\}$, $I_{w_2}(A) = \{a,b\}$, $I_{w_3}(A) = \{a\}$, $I_{w_4}(A) = \{b\}$, $I_{w_5}(A) = \{a,b\}$;
- $I_{w_1}(B) = \{a,b\}$, $I_{w_2}(B) = \{a,b\}$, $I_{w_3}(B) = \{b\}$, $I_{w_4}(B) = \{a\}$, $I_{w_5}(B) = \{a,b\}$;
- $I_x(a) = a$, $I_x(b) = b$, $\forall x \in \{w_1, w_2, w_3, w_4, w_5\}$.

It is straightforward to verify that both $WNS_1$ and $SNS_1$ are true at world $w_1$: see Figure 3. In particular, let us examine $WNS_1$: indeed, there exists a normative system/standard, i.e., the accessibility relation $R_1$, which selects a set of worlds (just consisting of $w_2$) at which all predicates are made true of all individuals[10]. Notice that the other relation $R_2$, departing from $w_1$, does not do the job: this means that, at $w_1$, we only need to have one normative system/standard for saying that $A$ weakly supervenes on $B$. Things are partially similar, at $w_1$, in regard to $SNS_1$: here, the evaluation works as before in regard to the $x$, while for $y$ we have to consider another (nested) occurrence of **Ought**, which semantically states that there exists at least one normative system/standard, not necessarily $R_1$, selecting worlds that make true the formula in the scope of this second **Ought**. In fact, $R_4$ does the job, thus validating $SNS_1$.

[9] We assume, for the sake of simplicity, to work with constant domains. This assumption makes things simpler but is not conceptually required and thus is not essential for our purposes. It is a rather straightforward, but pedantic exercise to extend the analysis to varying domains.

[10] Of course, we could have other models that make trivially true the conditionals by falsifying the antecedent.

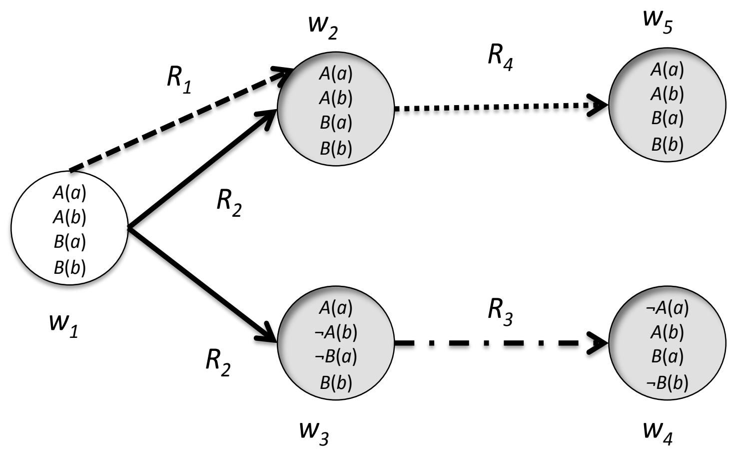


**Fig. 3** Model from Example 2

*Example 3 (Normative pluralism and NS).* Normative pluralism is a philosophical view according to which, either there is a fundamental plurality of ways of being good that cannot be reduced to something they all have in common, or, less radically, that there exists a plurality of bearers of value (see Mason, 2015). Hence, that something is morally due might change depending on whether we examine different moral standards. E.g., if we consider (i) religious morality, and (ii) liberal morality, then we could plausibly have two normative standards.

Suppose now

- to change the model in Example 2 by imposing that $I_{w_5}(A) = \{a\}$;
- that

  (i) $R_1$ = liberal morality,
  (ii) $R_4$ = religious morality;

- that

  (a) $A$ = valuable
  (b) $B$ = desired

In the revised model, $WNS_1$ is still true at $w_1$, whereas $SNS_1$ is falsified: there, being valuable weakly—but not strongly—supervenes on being desired, because this relation does not hold for religious moral standards.

In conclusion, we should observe that the iteration of **Ought** in $SNS_1$ requires to possibly consider more normative standards; under this assumption, if we assume to work with normative pluralism, strong NS exhibits interesting differences with respect to weak NS. Accordingly, the relative strength of $SNS_1$, in comparison with $WNS_1$, is not based on a strong modality, but conceptually depends on the fact that supervenient properties must be checked against more normative systems.

Finally, while the axiom schema **4**, i.e., **Ought**$A \rightarrow$ **OughtOught**$A$, does not look as appropriate in logics for the ought, it is also partially useless to make $SNS_1$ and $WNS_1$ closer, since the semantic property charactering in multi-relational semantics axiom **4** is not standard transitivity; this schema requires that whenever there are two relations $R_1$ and $R_4$ connecting respectively any world $w_1$ with any world $w_2$ and any such $w_2$ with any world $w_5$, then we must ensure that $w_5$ is always reachable from $w_1$ with *any* relation (and not necessarily with those which connect $w_1$ with $w_2$, and $w_2$ with $w_5$ ) (Calardo and Rotolo, 2014). Hence, we reiterate here the same limit we have just noticed above.

## 6 Summary

In this essay we proposed a logical discussion on NS. Despite the idea that supervenience is taken sometimes to be different from entailment, we argued much depends on what we mean by this last concept. In particular, there is room for an analysis that sees alternatives of classical logic which can grasp specific aspects of NS. We discussed

- the distinction between normative supervenience and supervenience of the normative, the former selecting ideal worlds where properties hold, the latter selecting normative properties holding in all possible worlds;
- how NS can be analysed in terms of suitable consequence relations and non-classical entailments, remarking that the counts-as relation is an interesting, but *sui generis* type of NS, and that weak, but suitable modal logics for the Ought may affect the formal and conceptual behaviour of standard definitions of weak and strong supervenience.

## References

Artosi, A., G. Governatori, and A. Rotolo (2002). Labelled tableaux for nonmonotonic reasoning: Cumulative consequence relations. *Journal of Logic and Computation 12*(6), 1027–1060.

Bacon, J. (1986). Supervenience, necessary coextension, and reducibility. *Philosophical Studies 49*(2), 163–176.

Beall, J. and G. Restall (2000). Logical Pluralism. *Australasian Journal of Philosophy 78*, 475–493.

Calardo, E. and A. Rotolo (2014). Variants of multi-relational semantics for propositional non-normal modal logics. *Journal of Applied Non-Classical Logics 24*(4), 293–320.

Calardo, E. and A. Rotolo (2016). Quantification in some non-normal modal logics. *Journal of Philosophical Logic*, 1–36.

Castañeda, H.-N. (1981). The paradoxes of deontic logic: The simplest solution to all of them in one fell swoop. In R. Hilpinen (Ed.), *New Studies in Deontic Logic*, pp. 37–86. Dordrecht: D. Reidel.

Chellas, B. F. (1980). *Modal Logic*. Cambridge University Press.

Cocchiarella, N. B. (1984). Philosophical perspectives on quantification in tense and modal logic. In D. Gabbay and F. Guenthner (Eds.), *Handbook of Philosophical Logic: Volume II: Extensions of Classical Logic*, pp. 309–353. Dordrecht: Reidel.

Corsi, G. (2002). A unified completeness theorem for quantified modal logics. *Journal of Symbolic Logic 67*(4), 1483–1510.

Delgrande, J. P. (1998). On first-order conditional logics. *Artificial Intelligence 105*(1), 105 – 137.

Fine, K. (1978). Model theory for modal logic - Part II: The elimination of *de re* modality. *Journal of Philosophical Logic 7*, 277–306.

Forbes, G. (1985). *The metaphysics of modality*. Clarendon library of logic and philosophy. Clarendon Press.

Gabbay, D., J. Horty, X. Parent, and L. van der Torre (Eds.) (2013). *Deontic Logic Handbook*. London: College Publications.

Gelati, J., A. Rotolo, G. Sartor, and G. Governatori (2004). Normative autonomy and normative co-ordination: Declarative power, representation, and mandate. *Artif. Intell. Law 12*(1-2), 53–81.

Goble, L. (1973). Opacity and the Ought-To-Be. *Noûs 7*(4), 407–412.

Goble, L. (2001). Multiplex semantics for deontic logic. *Nordic Journal of Philosophical Logic 5*(2), 113–134.

Goble, L. (2004). Preference semantics for deontic logic — Part II: Multiplex models. *Logique et Analyse 47*, 113–134.

Goble, L. (2005). A logic for deontic dilemmas. *J. Appl. Log. 3*(3-4), 461–483.

Goldman, A. (1970). *A theory of human action*. Prentice-Hall.

Grossi, D. and A. Jones (2013). Constitutive rules and counts-as conditionals. In X. P. D. Gabbay, J. Horty and L. van der Torre (Eds.), *Deontic Logic Handbook*. London: College Publications.

Hansen, J. (2005). Conflicting imperatives and dyadic deontic logic. *Journal of Applied Logic 3*(3-4), 484–511.

Humberstone, L. (1993). Functional dependencies, supervenience, and consequence relations. *Journal of Logic and Computation 2*, 309–36.

Humberstone, L. (2002). The modal logic of agreement and noncontingency. *Notre Dame Journal of Formal Logic 2*, 95–127.

Jones, A. and J. Carmo (2002). Deontic logic and contrary-to-duties. In D. Gabbay and F. Guenthner (Eds.), *Handbook of Philosophical Logic* (2nd ed.). Dordrecht ; Boston: Kluwer Academic Publishers.

Jones, A. J. I. and M. J. Sergot (1996). A formal characterisation of institutionalised power. *Logic Journal of the IGPL 4*(3), 427–443.

Kaminski, M. (1997). The elimination of *de re* formulas. *Journal of Philosophical Logic 26*, 411–422.

Kim, J. (1993). *Supervenience and Mind: Selected Philosophical Essays*. Cambridge Studies in Philosophy. Cambridge University Press.

Kraus, S., D. Lehmann, and M. Magidor (1990, July). Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence 44*(1-2), 167–207.

Mason, E. (2015). Value pluralism. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Summer 2015 ed.). Metaphysics Research Lab, Stanford University.

McLaughlin, B. (1995). Varieties of supervenience. In E. Savellos and U. Yalcin (Eds.), *Supervenience: New Essays*. Cambridge University Press.

McLaughlin, B. and K. Bennett (2014). Supervenience. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Spring 2014 ed.).

McNamara, P. (2014). Deontic logic. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Winter 2014 ed.). Metaphysics Research Lab, Stanford University.

Nute, D. (Ed.) (1997). *Defeasible Deontic Logic*. Dordrecht: Kluwer.

Parent, X. (2001). Cumulativity, identity and time in deontic logic. *Fundam. Inform. 48*(2-3), 237–252.

Schmitt, J. and M. Schroeder (2011). Supervenience arguments under relaxed assumptions. *Philosophical Studies 155*(1), 133–160.

Schotch, P. K. and R. E. Jennings (1981). Non-Kripkean deontic logic. In R. Hilpinen (Ed.), *New Studies in Deontic Logic*, pp. 149–162. Reidel.

Searle, J. (1969). *Speech Acts: An Essay in the Philosophy of Language*. Cambridge: Cambridge University Press.

Searle, J. (1995). *The Construction of Social Reality*. Harmondsworth: Penguin.

Searle, J. (2001). Neither phenomenological description nor rational reconstruction: Reply to Dreyfus. *Revue Internationale de Philosophie 55*, 277–284.

Shoham, Y. (1990). Nonmonotonic Reasoning and Causation. *Cognitive Science 14*, 213–252.

Stalnaker, R. (1968). A theory of conditionals. In *Studies in Logical Theory*, Volume 2, pp. 98–112. Blackwell.

von Wright, G. H. (1951). *An Essay in Modal Logic*. Amsterdam: North-Holland.

Wedgwood, R. (2007). *The Nature of Normativity*. Oxford: Oxford University Press.