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# Performance-based seismic design of multi-storey frame structures equipped with Crescent-Shaped Brace

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## ABSTRACT

The primary objective of the *Performance-Based Seismic Design* (PBSD) is to provide stipulated seismic performances for building structures. However, a certain degree of design freedom is needed for matching a specific seismic response. This design freedom is not obtainable by the conventional lateral resisting systems because their stiffness and strength are coupled. Here, we put emphasis on the role of the unconventional lateral resisting systems in adding more flexibility to the design. In this paper, we seek to explore the seismic design of moment resisting frame structures equipped with an innovative hysteretic device, known as *Crescent-Shaped Brace* (CSB). One conspicuous feature of this device is its distinctive geometrical configuration, which is responsible for the enhanced nonlinear force-displacement behavior exhibited by the device. A new performance-based approach for the seismic design of the CSB is proposed. The performance of the device is evaluated and its application in multi-storey shear-type structures is investigated. Two case studies were established to illustrate the design methodology. The first is a new two-storey RC structure and the second is an existing three-storey RC structure. Nonlinear time history and pushover analyses are performed to evaluate the behavior of the controlled structures. The analyses show that for each of the two case studies the acceleration-displacement capacity spectrum conforms to the performance objectives curve. This finding confirms the validity of the proposed design approach and the effectiveness of the new hysteretic device in resisting lateral forces.

**Keywords:** *Crescent Shaped Brace, Design method, Dynamic analysis, Performance Based Seismic Design.*

## 1 INTRODUCTION

Recent development in earthquake engineering has resulted in the emergence of new structural design approaches such as the Performance-Based Seismic Design (PBSD) [1]. PBSD is still deemed as a new approach even though its origin can be traced back as far as the late 20th century. The design efficiency of PBSD is the main reason behind its emergence [2]. The Performance-Based Design specifies the main objectives that should be attained by the structure and gives the standards for accepting a specified performance [3]. Today, structures are designed with the goal of achieving a predefined functionality. This is because the challenge is no longer limited to protecting human lives, but extended to minimizing damages and disruption down to reasonable levels. Nevertheless, matching a defined seismic response necessitates additional design freedom that is unable to be achieved by the traditional structural components, such as beams and columns. Here, it is necessary to emphasize the role of the unconventional lateral resisting systems in making the design more flexible and thus allowing to reach specific seismic performances.

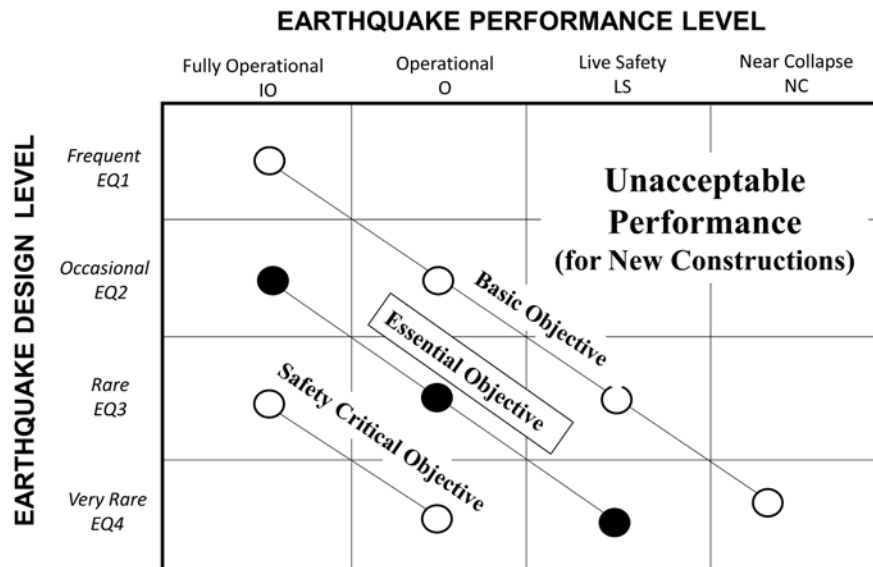
Lately, several efforts in the earthquake engineering discipline could find their ways into various advanced lateral resisting systems. These systems can provide enhanced performances to the structure under particular ground motion levels. Examples of such systems include: (a) seismic isolation systems, which disengage the superstructure from its substructure, thereby giving rise to a “conceptual separation between the horizontal and vertical resisting systems” [4]; (b) tuned mass damping systems, which are practically employed to reduce the vibration level of the structure resulted from high lateral excitations [5]; (c) active and semi-active systems, which use the actual seismic vibration to modify the mechanical properties of the structure accordingly [6]; (d) dissipative systems, which are integrated into the superstructure to reduce the damage in the structure through their energy dissipation capability [7]. Whilst the listed systems have been nicely

52 incorporated into practice and literature, none of them could completely fulfil the intended seismic  
 53 objectives of structures as outlined by the PBSD.

54 In this paper, we focus on a new innovative lateral resisting device, the *Crescent Shaped*  
 55 *Brace* (CSB). CSB is a hysteretic device that is grouped under the ‘energy dissipation devices’  
 56 classification. The device enables the structure to have prescribed multiple seismic  
 57 performance through its passive resisting capability [8]. Up to the present time, the design of multi  
 58 storey buildings equipped with Crescent Shaped Braces has not been exposed to wide-ranging  
 59 research. The application of the CSBs is restricted to a single case study of a steel structure in which  
 60 the braces were inserted at the ground floor. The objective of that study was to obtain a controlled  
 61 soft-storey response. The upper storeys were braced with conventional concentric steel diagonal  
 62 braces in order to conceptually model the system as a single degree of freedom (SDOF) system [4].

63 The work presented in this study proposes a comprehensive method for the seismic design of  
 64 multi storey shear-type-structures strengthened with CSB devices. In this study, the geometrical and  
 65 mechanical properties of the controlled structure are assumed to be given, as in the case of existing  
 66 structures; therefore, there is no control on the structure’s stiffness and strength. This implies that  
 67 the CSB system is the only variable in the design. In the case of designing new structures, more  
 68 design freedom is added as the properties of the structure can be chosen in accordance with the  
 69 desired performance objectives. The design method proposed in the study involves: (i) sizing the  
 70 CSB devices in the elastic field; (ii) verifying the behavior of the braces in the plastic field. The first  
 71 part of the method is to design the braces in the elastic field with reference to a predefined  
 72 performance point. Then, the post yielding behavior of the CSB is determined numerically using the  
 73 FEM software ‘SeismoStruct V.7.0.6’ [9]. In the second part of the method, the post yielding  
 74 behavior of the controlled system (i.e. structure equipped with the designed braces) is verified by  
 75 means of nonlinear pushover and time history analyses.

76 To illustrate the procedure in all the details, the methodology has been applied to two case  
 77 study structures. The controlled structures are designed to satisfy the ‘Essential Objectives’ shown in  
 78 Figure 1 [1]. Non-linear pushover and time-history analyses are performed to verify the performance  
 79 of the controlled system under a given seismic input. The outcome of the study proved the validity of  
 80 the proposed design method and the efficiency of the hysteretic device.



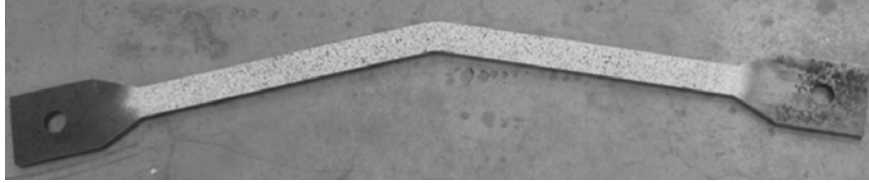
81  
 82 Figure 1. Performance-based seismic design goals. Adopted from [1]

83

84 **2 THE CRESCENT SHAPED BRACES**

85 **2.1 Overview**

86 The Crescent-Shaped brace (CSB) (Figure 2) is a unique hysteretic lateral resisting device  
 87 that provides additional design freedom to frame structures. Its geometrical configuration, as shown  
 88 in Figure 3, permits the structure to have predefined multiple seismic performances [8]. The CSB  
 89 enables the designer to have full control over the design because its yielding strength and lateral  
 90 stiffness are not coupled.



91  
92 Figure 2. A sample of the Crescent Shaped Brace

93  
94 **2.2 Analytic model of the CSB**

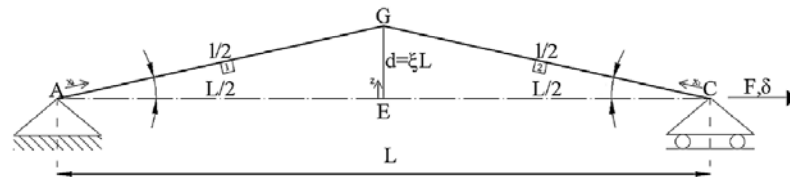
95 Previous work conducted on the Crescent-Shaped Braces by Palermo et al. (2015) led to the  
 96 derivation of analytical formulations for sizing the device based on a target stiffness and a target  
 97 yielding strength. Eqs. (1) and (2) represent a simplified version of the original equations developed  
 98 in [8]. Strength and stiffness are initially imposed according to the predefined performance  
 99 objectives that to be achieved. The process involves a consideration of the structural and non-  
 100 structural responses of the studied system. Equation (1) allows obtaining the arm ratio of these  
 101 devices, which is the ratio between the arm of the device  $d$  and the diagonal length  $L$  (see Figure 3).  
 102 This ratio can be assumed as 0.1 for preliminary designs. The arm ratio is subsequently replaced in  
 103 Eq. (2) to get the target moment of inertia of the CSB device.

104 
$$\xi = \frac{M_{pl}}{F_y \cdot L} \quad (1)$$

105 where  $\xi = d / L$  represents the arm ratio of the device,  $d$  is the device arm,  $M_{pl} = W_{pl} \cdot f_y$  is the  
 106 plastic bending resisting moment of the cross section,  $W_{pl}$  is the plastic section modulus,  $f_y$  is the  
 107 yield strength,  $\bar{F}_y$  is the target yield strength,  $L$  is the diagonal length (i.e. the line connecting both  
 108 extremities of the device).

109 
$$J = \frac{L^3 \cdot \bar{K} \cdot \xi^2}{3 \cdot E \cdot \cos^2 \theta} \quad (2)$$

110 where  $J$  represents the cross-section inertia,  $\bar{K}$  is the target initial lateral stiffness,  $E$  is the  
 111 modulus of elasticity of the steel section,  $\theta$  is the angle formed between the applied force and the  
 112 device diagonal (i.e.  $\theta = 0$ ).



114  
115 Figure 3. The geometric configuration of the studied device. Adopted from [8]

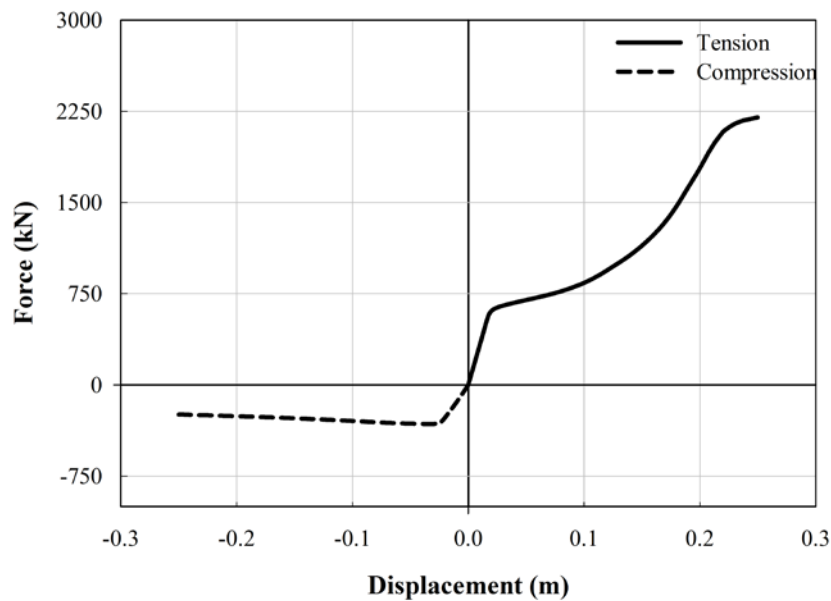
116

117 **2.3 Mechanical behavior of the CSB**

118 The post-yielding behavior of a random CSB device has been numerically studied using the  
119 fiber-based software ‘SeismoStruct V.7.0.6’, which considers both geometric nonlinearities and  
120 material inelasticity. First, a sample of the bracing device ‘HEB200 European profile’ was subjected  
121 to a monotonic rising tension load. The result of the numerical analysis is displayed in Figure 4 (the  
122 solid segment of the curve). At the beginning, the CSB responds in flexure, acting linearly until first  
123 yielding is reached at the knee section. Then, the device encounters a plastic behavior due to the  
124 spread of plasticity (pseudo-horizontal part). This is followed by a second remarkable hardening  
125 behavior as the device’s arm  $d$  decreases. At this stage, the device mainly reacts through its axial  
126 stiffness capacity, like a conventional brace or a truss in a tensile layout.

127 The same specimen was subjected to a monotonically increasing compressive loading. Figure  
128 4 (the dotted segment of the curve) is a graphical representation of the constitutive law of the device  
129 in compression. It is very important to note that unlike traditional concentric braces, the CSB  
130 device does not suffer from sudden Eulerian in-plane buckling when exposed to a compressive  
131 force, and this is due to its unique shape. Regarding the out-of-plane buckling, the appropriate  
132 selection of the cross section is highly effective in preventing such a problem [8] (e.g. choosing  
133 balanced inertias along weak and strong axes). Another solution is to include longitudinal ribs in  
134 correspondence to the neutral axis fiber.

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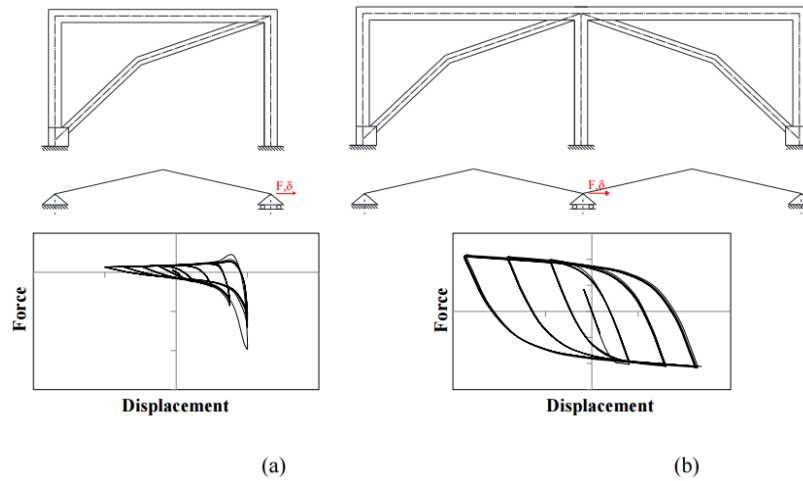
138 Figure 4. Monotonic behavior of a single CSB in tension and compression

139

140 The hysteretic behavior of the CSB is that of typical steel bracings given that the device is  
141 nothing more than a steel member having a curved configuration. The numerical studies conducted  
142 on the device has demonstrated a good hysteretic response [8]. The simulated hysteretic responses  
143 have been also confirmed by experimental tests conducted by some of the authors (the test results  
144 will be available soon [10]) and by other researchers [11].

145 The hysteretic force-displacement response of the single CSB device is strongly asymmetric  
146 due to the non-linear geometrical effects [8][10]: significant hardening response under lateral loads  
147 inducing tension in the brace, and softening response under lateral loads inducing compression in  
148 the braces (Figure 5a). On the contrary, when two CSB devices inserted in a two-span frame  
149 structure, the overall behavior becomes symmetric given that one works in compression while the  
150 other one works in tension (Figure 5b).

151



152  
 153 Figure 5. (a) A bilinear CSB device inserted in a frame and its asymmetric force-displacement  
 154 response; (b) two mirrored disposed bilinear CSB devices inserted in two frames and their  
 155 symmetric force-displacement response. Adopted from[10].  
 156

### 157 3 METHOD: PERFORMANCE-BASED DESIGN OF A MULTI-STOREY SHEAR-TYPE 158 FRAME EQUIPPED WITH CSB DEVICES

159 The design philosophy behind the use of CSBs as enhanced bracings is grounded on the  
 160 concept of actively designing a structure behaving according to a so called ‘‘Building–Target  
 161 Capacity (B–TC) curve’’ that is then translated into a ‘‘Building–Actual Capacity (B–AC)  
 162 curve’’[4]. The B–TC curve is the graphical representation of the idealized seismic behavior of the  
 163 building that we expect to achieve by imposing preselected multiple performance objectives, while  
 164 the B–AC curve is the graphical representation of the effective seismic behavior of the building,  
 165 once all structural members are designed. The use of CSBs at all storey levels is the design strategy  
 166 here adopted to achieve the performance design objectives.

167 Given that CSBs can be used in different configurations, several design strategies can be  
 168 identified to achieve the desired performance objectives. In the literature, the behavior of an SDOF  
 169 steel structure equipped with this device has been investigated [4]. In this section, we propose a  
 170 general procedure for the seismic design of multi-storey shear-type frame structures equipped with  
 171 Crescent-Shaped Braces (CSB). The proposed method can be used to design or strengthen  
 172 structures that do not satisfy particular performance objectives. The design method proposed in the  
 173 study involves: (i) designing (sizing) the CSB devices in the elastic field; (ii) verifying the behavior  
 174 of the braces within the global system in the plastic field.

175 (i) Designing the CSB devices is done with reference to the performance point corresponding  
 176 to the earthquake level *occasional* (EQ2) and the performance level *fully operational* (IO) (Figure  
 177 1). This point belongs to the *Essential Objective* performance line, not the ordinary *Basic Objective*  
 178 performance line. The reason to choose a high seismic demand is to show the capability of the  
 179 braces in achieving a predefined performance level. The sizing method comprises 6 steps. In the first  
 180 step, an initial global stiffness matrix for the controlled structure (i.e. structure equipped with  
 181 braces) is imposed based on certain criteria, which are described in section 4. The global stiffness  
 182 matrix is refined as more iterations are executed. In the second step, a modal analysis for the system  
 183 is performed. The building’s drift obtained from the modal analysis is compared to the design drift  
 184 that is set according to the desired performance point (i.e. EQ2-IO). The global stiffness matrix is  
 185 continuously modified through several iterations until the structure’s drift meets the target drift.  
 186 Once the actual drift matches the design drift, we move to step four and we compute the stiffness of  
 187 the CSB bracing system. This is done by subtracting the stiffness matrix of the naked structure from  
 188 the global stiffness matrix. In step five, the structural configuration (i.e. position and number of  
 189 braces) of the CSB system is defined and hence the stiffness of each brace is computed. Finally, by

190 knowing the stiffness of each device, the moment of inertia and the arm of the devices are evaluated  
191 in step 6, and this allows choosing a cross-section for the device from a wide range of cross-  
192 sections that satisfy the inertia demand. Once the cross-section is known, the post-yielding behavior  
193 of the brace is obtained by means of a static nonlinear pushover analysis using the fiber-based FEM  
194 software “SeismoStruct V.7.0.6”. SeismoStruct considers the geometric nonlinearity of the model  
195 based on the corotational formula [12], and the material nonlinearity in accordance to Menegotto  
196 Pinto law, with adequate focus on the isotropic hardening as given in [13]. The stiffness of the  
197 device is computed at each step of analysis, and then updated automatically in the following  
198 analysis step. Generally, the post yielding behavior of the device is greatly affected by its section  
199 profile; therefore, different section profiles must be compared and the one that conforms most to the  
200 predefined performance is chosen.

201 (ii) The behavior of the CSB system within the global system is obtained by means of  
202 nonlinear static pushover (PO) and dynamic time-history (TH) analyses using the FEM software  
203 SAP2000 [14]. The behavior of the equipped structure is verified against the performance points  
204 ‘EQ3-O’ and ‘EQ4-LS’ shown in Figure 1. The CSB devices are introduced in the model as multi  
205 linear links (NL) by importing the force-displacement curves (backbone curves) of the braces  
206 obtained from SeismoStruct software. Using the backbone curves of the braces, SAP2000 updates  
207 the stiffness of the device at each analysis step according to the displacement exhibited by the  
208 device. The force-displacement curves obtained from SeismoStruct are calibrated in order to  
209 account for the structural configuration (inclination) of the devices in the structure. Moreover, the  
210 kinematic hysteresis model, which is the default hysteresis model for all metal materials in the  
211 program, is considered in the analysis as it is very appropriate for ductile materials. The above  
212 mentioned implies that the actual nonlinear stiffness of each device is effectively considered in the  
213 analysis. The nonlinearity of the structure is considered using concentrated plastic hinges. The  
214 results of both PO and TH analyses are plotted together in order to verify the analysis performed.  
215 Finally, the nonlinear pushover curve (i.e. capacity curve) is compared with the predefined  
216 performance curve, according to which the devices were initially designed, to check if the target  
217 performances are met. Although the nonlinear behavior of the structure equipped with the CSB  
218 braces is not designed for ‘automatic’, previous studies suggested that the system would perform in  
219 a good way with respect to severe earthquakes [4][15][16][17]. This is mainly due to the shape of  
220 the brace (the peculiar mechanical behavior) (Figure 2) and to its hysteretic dissipation properties. In  
221 the following section, we introduce the first part of the methodology (i.e. the design of the CSB  
222 system), and in section 5 we cover the second part by means of a case study (i.e. the post yielding  
223 verification of the braces within the global system).

224

#### 225 **4 DESIGN OF THE CSB SYSTEM**

226 The dimensioning procedure of the braces is illustrated in Figure 6. The purpose of this  
227 design procedure is to obtain a target lateral stiffness for the single CSB device. The stiffness output  
228 is then used in the previously delivered design formulas (Eqs. (1) and (2)) to get the inertia demand of  
229 the brace. Once securing the moment of inertia, the cross-section profile of the device can be  
230 selected from a broad range of cross-sections. In the following, the design procedure of the CSB is  
231 described in all details.

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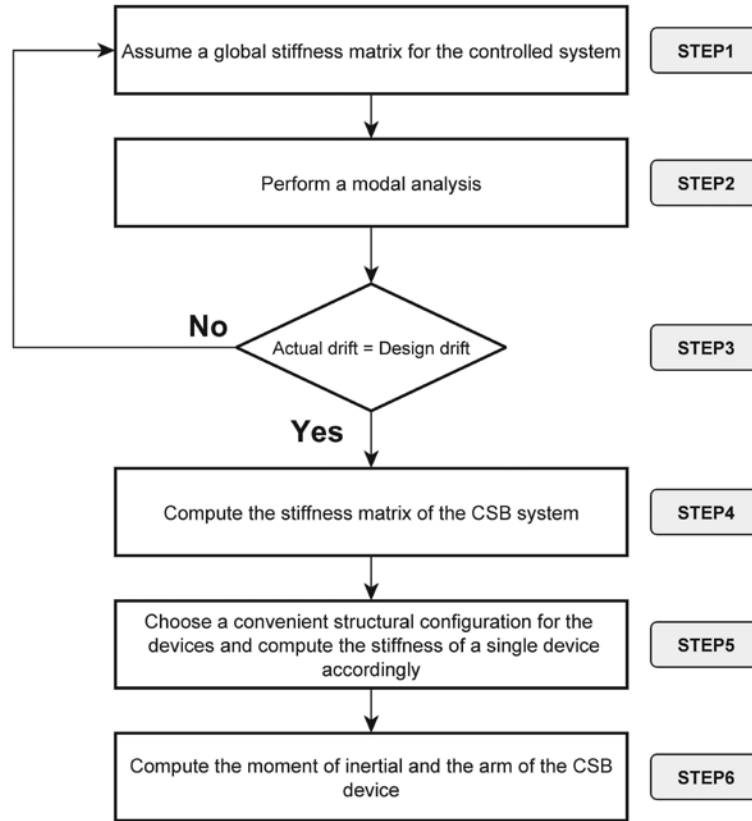


Figure 6. Flowchart of the CSB design scheme

#### 4.1 Step 1: Global stiffness matrix

The global stiffness matrix defines the rigidity of the controlled system. This matrix is determined by summing (as they act in parallel) the stiffness matrices of the bare structure and the bracing system.

$$[K^*] = [K] + [K_b] = \begin{pmatrix} k_1^* + k_2^* & -k_2^* & & & & \\ -k_2^* & k_2^* + k_3^* & \ddots & & & \\ & \ddots & \ddots & & & \\ & & & -k_{N-1}^* & & \\ & & & -k_{N-1}^* & k_{N-1}^* + k_N^* & -k_N^* \\ & & & & -k_N^* & k_N^* \end{pmatrix} \quad (3)$$

where  $[K^*]$  denotes the stiffness matrix of the controlled system,  $k_1^*$ ,  $k_2^*$ , ...,  $k_N^*$  represent the stiffness terms of the controlled system at the different storey levels. These stiffness terms are mathematically represented as follows:

$$k_i^* = k_i + k_{bi} \quad (4)$$

where  $k_i^*$  is the stiffness of the controlled system at storey  $i$ ,  $k_i$  is the stiffness of the uncontrolled system at storey  $i$ ,  $k_{bi}$  is the stiffness of the bracing system at storey  $i$ . From the mathematical illustrations above, we see that the global stiffness matrix  $[K^*]$  consists of  $N$  unknowns, denoted as  $k_1^*$ ,  $k_2^*$ , ...,  $k_N^*$ . The number of unknowns, however, can be reduced by





## 323 4.6 Step 6: Moment of inertia of the CSB

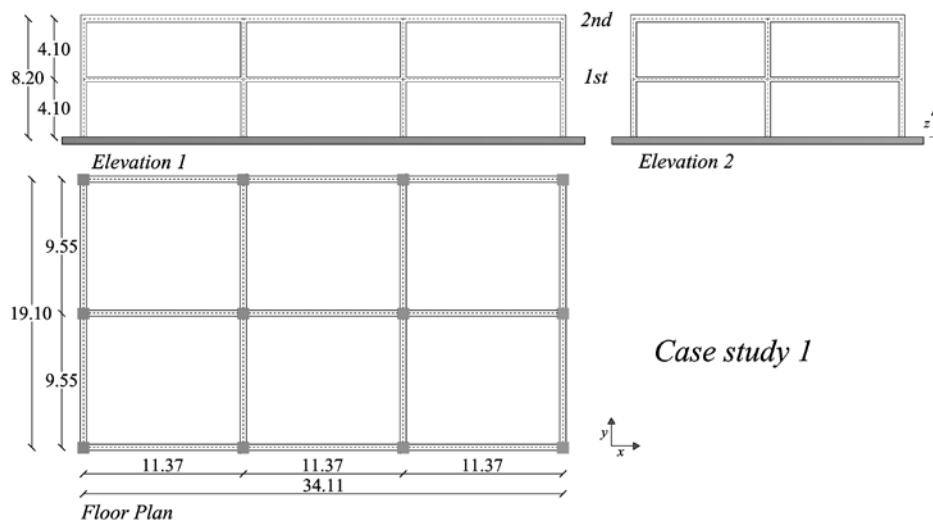
324 At this stage, Eqs. (1) and (2) are used to calculate the moment of inertia of the devices. In  
325 these two formulas,  $\bar{K}$  is set equal to  $K_{CSB,i}$ , which is the target stiffness that we seek to  
326 achieve, while  $\bar{F}$  represents the target yielding strength at which the device goes inelastic. Once  
327 securing a moment of inertia for each CSB unit, a cross-section profile for the CSB is chosen from a  
328 broad range of cross sections satisfying the target inertia. It is important to note that the cross-  
329 section profile choice may dominate the post yielding behavior of the bracing device. This can have  
330 a significant impact on the post yielding behavior of the whole structure [8]. Thus, it is necessary to  
331 evaluate different cross-section profiles in order to fulfil the inelastic performance objectives  
332 (i.e. performance points corresponding to EQ3-O and EQ4-LS shown in Figure 1).  
333

## 334 5 POSTYIELDING VERIFICATION OF THE CSB SYSTEM: CASE STUDIES

### 335 5.1 The reference structures

336 The first case study structure (CS1) is a new commercial building situated in Gubbio city,  
337 Italy. Gubbio is a city located in the far north-eastern area of the Italian province of Perugia, which  
338 is in a comparatively high seismic zone. The building was designed according to the Italian seismic  
339 standard [18]. Therefore, the building satisfies the operational and the life safety seismic objectives  
340 under occasional and rare earthquake levels, respectively. Figure 7 shows the geometry of the building  
341 structure. The building is rectangular with dimensions equal to 34.11 m x 19.10 m. It consists of two  
342 storey levels with 4.1 m height each. The backbone forming the structure consists of three bays in the  
343 y-direction (Elevation 1) and two bays in the x-direction (Elevation 2).

344 The second case study structure (CS2) is an existing elementary school built in 1983. It is  
345 located in Bisignano city, Italy, which is also a high seismic zone. As shown in Figure 8, the building  
346 structure has a rectangular planar geometry with dimensions equal to 21.39 m x 15.00 m. It is made  
347 up of three storey levels with a roof pavilion on the top. The backbone forming the structure  
348 consists of four bays in the y-direction and three bays in the x-direction. The mechanical properties  
349 of the concrete were determined by the presidency of the council of ministers and the department of  
350 civil protection in Italy, who performed ultrasonic and rebound hammer tests on a set of columns  
351 and beams. The mechanical and geometrical properties of the concrete elements of both case  
352 studies are listed in Table 1.  
353



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Figure 7. Elevations and plans of the first case study

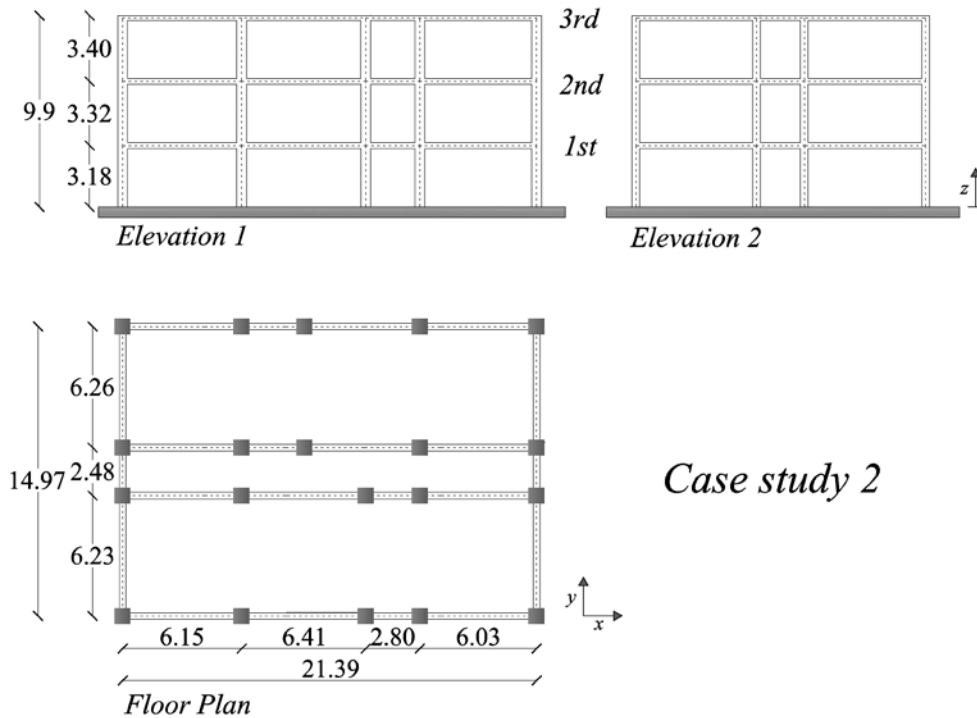


Figure 8. Elevations and plans of the second case study

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360

Table 1 at the end of the paper

## 362 5.2 Types of analysis

363 Two types of non-linear analysis are performed to verify the performance of the case study  
364 structures. A three-dimensional model was built using the commercial software SAP2000 in order to  
365 perform the analysis. As recommended by the Italian seismic standard, the loads applied to the  
366 structure are: (a) the live loads multiplied by a combination factor ( $\psi_E$ ); (b) the dead loads without  
367 any combination factor. The P- $\Delta$  effect was neglected given the small height and the high regularity  
368 of the structures. The nonlinear behavior of the frames is modelled using concentrated plastic  
369 hinges. Flexural Hinges (type Moment M3) were applied to the beam elements, while flexural  
370 hinges (type P-M2-M3) were applied to the columns. The hinge force-deformation relationship,  
371 also known as the 'backbone curve', is obtained using the concentrated plasticity model indicated by  
372 FEMA 356 [19].

373 After designing the CSB devices as introduced in section 4, the force-displacement curve of  
374 each device is obtained using SeismoStruct software by performing a nonlinear static pushover  
375 analysis. The Braces are then introduced in the SAP model as multi linear links (NL) by importing  
376 the force-displacement curves of the braces. The kinematic hysteresis model is considered in the  
377 analysis as it is very appropriate for ductile materials (Figure 9).

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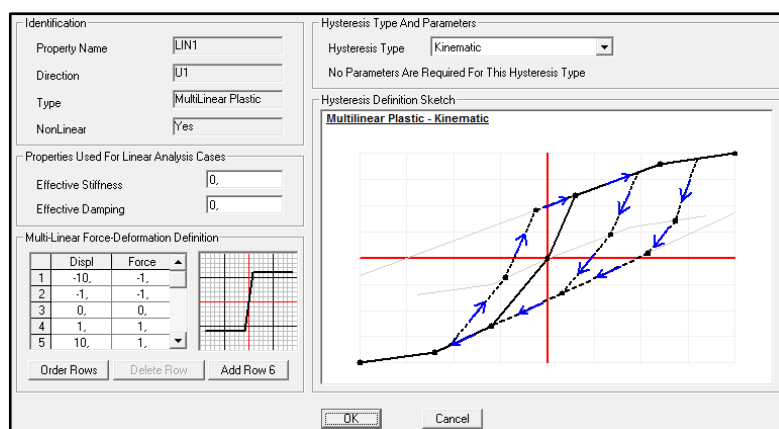


Figure 9. Nonlinear plastic link with kinematic hysteresis type to model the behavior of the CSB in SAP2000.

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The first type of analysis is the static pushover analysis, which yields the capacity curve of the structure starting from rest until the failure point [20]. In this analysis, two displacement shapes were applied 'linear' and 'uniform', whose average is considered. The pushover curve was obtained in terms of the base shear and the roof (top) displacement. The second type of analysis is the dynamic time-history analysis, which was performed using the non-linear direct integration method with a damping ratio of 5%. The analysis was conducted by scaling a set of seven accelerograms to the four design values of PGA at the fundamental period of the structure. The ground motion accelerograms needed for the time-history analysis have been obtained using the software *SIMQKE\_GR* [21]. The accelerograms are consistent with the design spectra of the structure given by the Italian seismic standard. The Earthquake design levels and the corresponding response spectra parameters are indicated in Table 2. In the table,  $T_y$  represents the return period of the design earthquake,  $PGA$  is the peak ground acceleration,  $F_0$  is the maximum spectral dynamic amplification,  $T_c^*$  is the characteristic period at the beginning of the constant velocity branch of the design spectrum. As shown in the table, the design requirements of the school (CS2) are more stringent than the commercial structure (CS1). The reason is that schools are generally more vulnerable than other types of structures.

Table 2 at the end of the paper

### 5.3 Structural configurations and local optimization of the CSB devices

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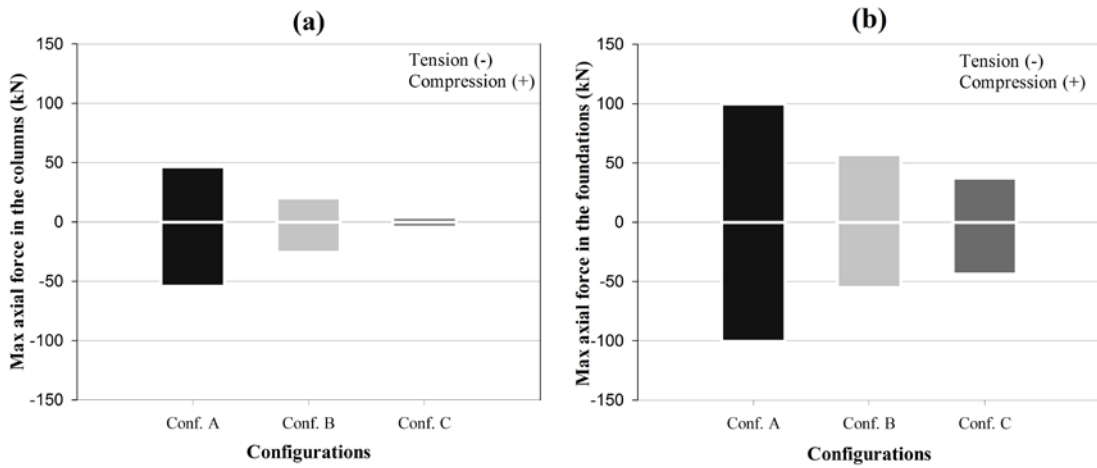
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The structural configuration of the bracing devices defines their effectiveness level. A proper arrangement of the bracing devices in the structure would maximize the lateral resistance capacity while decrease the internal forces in the structural elements. This also leads to a reduction in the devices' cross sections [22]. In addition, high axial force levels can dramatically decrease the moment capacity of columns; therefore, large axial forces should be avoided.

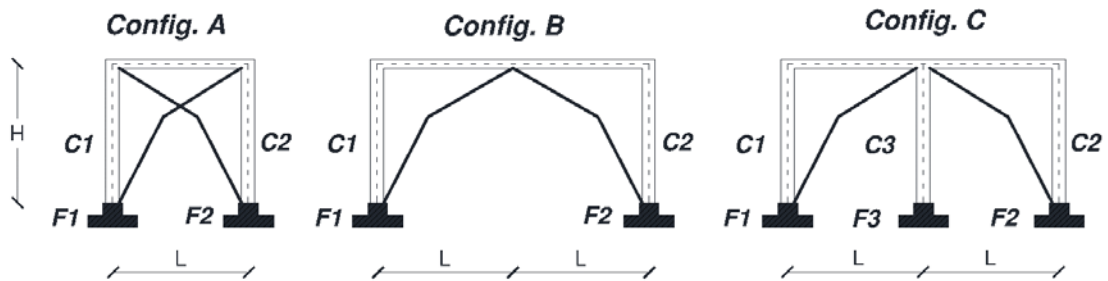
Choosing the right configuration depends on several factors, such as the architectural constraints, the beam span length, and the axial and moment capacities of the columns and foundations. The latter is very important especially if the structure is an existing structure where the structural elements capacities are predetermined. In the design case, on the other hand, the designer can design the columns to stand the additional axial forces coming from the bracing system, and thus this problem can be prevented.

In this section, three possible configurations of the bracing devices (see Figure 10) are investigated by performing a time-history analysis.



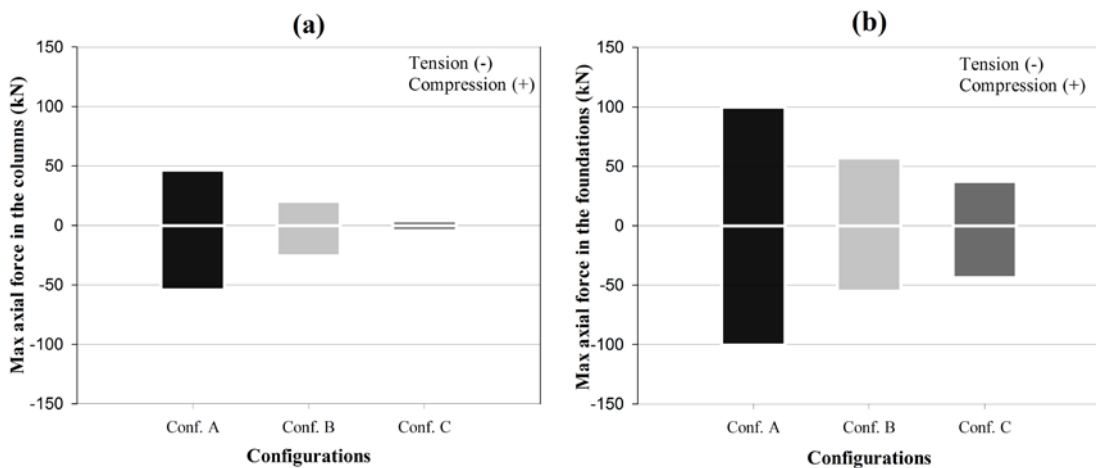
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Figure 11 shows the results of the time history analysis in terms of the axial force transmitted into column (C1) and foundation (F1) for each of the configurations. Config. A indicates the highest axial forces in C1 and F1 compared to the other two configurations, whereas Config. B shows small axial forces in columns and foundations. The third configuration Config. C induces almost no axial force in column C1, while it causes the least amount of forces in foundation F1. Among all three configurations, Config. C is the best configuration regarding the internal stresses in columns and foundations; however, this comes at the cost of the resistance efficiency. On the other hand, although Config. A produces the highest amount of forces in the columns and foundations, the resistance efficiency is very high. Finally, Config. B seems to be less problematic in the architectural point of view, as it leaves sufficient area in the façade for windows installation; nevertheless, it is less resistant than the previous two configurations and it causes concentrated stress in the mid span of the beam.



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Figure 10. CSB configurations



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436 Figure 11.(a) Maximum axial force in column (C1) for each of the three configurations;(b)  
437 maximum axial force in foundation (F1) for each of the three configurations

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## 439 5.4 Performance Objectives

440 As we mentioned earlier, the first case study (commercial structure)has been designed in  
441 compliance with the Italian seismic standard;therefore,the building satisfies the basic design  
442 objectivescorresponding to the two earthquake design levels‘occasional’ and ‘rare’ indicated in  
443 Figure 1 and Table 2. The second case study (school), on the other hand, is an existing structure;  
444 thus, we need first to verify its performance. This is done byperforming a pushover analysis to  
445 capture the base shear level at which the building yields.

446 In this work, higher demands are set to be attained by the structures. The *Essential*  
447 *Objectives* specified in Figure 1 are considered instead of the *Basic Objectives*according to which  
448 the structures were designed in the first place. The‘Essential Objectives’ require the structure to  
449 remain in a fully operational condition under *occasional* earthquake design level (EQ-2), to stay in  
450 an operational condition with limited yielding and damages under *rare* earthquake design level  
451 (EQ-3), and to have some degree of damage while preventing life losses under *very rare* earthquake  
452 design level (EQ-4).

453 The Performance Objectives are usually set depending on the client’s requirements,building’s  
454 destination, building’s importance, and building’s typology[15]. A study conducted by Bertero et al.  
455 established applicable performance limits on the basis of some structural and non-structural damage  
456 criteria, such as structural damage indexes (DM), storey drift indexes (IDI), and rate of  
457 deformations (floor velocity, acceleration)[1]. Those performance objectives, however, correspond  
458 to the *Basic Objectives* (Figure 1); therefore, they cannot be used in our design because our desire is  
459 to fulfil higher requirements. Table 3 reveals the basic objectives corresponding to each of the four  
460 earthquake levels, as proposed by Bertero et al. (2002). The table also showstwo proposed sets of  
461 performance limits (for the two case studies) belonging to the *EssentialObjectives*. Selecting the  
462 new performance limits was done by firstly settingthe inter-storey drift indexcorresponding to EQ-3  
463 (PO-3) to a value that insures no structural or nonstructural damage in the structure. The IDI  
464 corresponding to PO-3 of the first case study structure is 0.005while it is 0.0045for the second one.  
465 The second case study structure was found to yield at a low IDI and this is the reason we set a more  
466 stringent performance demand (i.e. IDI=0.0045). Other objective points (PO-1, PO-2, and PO-4)  
467 were set proportionally to the corresponding values of PGA at the fundamental period of the  
468 structure.

469

470 Table 3at the end of the paper

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## 472 5.5 Design of the CSB devicein the x-direction

473 Following the CSB design methodology presented in section 4,Table 4 shows the  
474 methodology applied to the two case study structures. The reason of considering two case studies is  
475 to show the stability of the design method when applied to structures with different occupancies and  
476 different seismic demands. Another reason is to stress that existing structures do not always satisfy  
477 the seismic standards. For instance, the second case study structure (existing school) yielded at an  
478 inter-storey drift index of 0.0045 (PO-3), which does not comply with the Italian seismic standard  
479 that requires the building to yield at a higher drift ratio.

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Table 4at the end of the paper

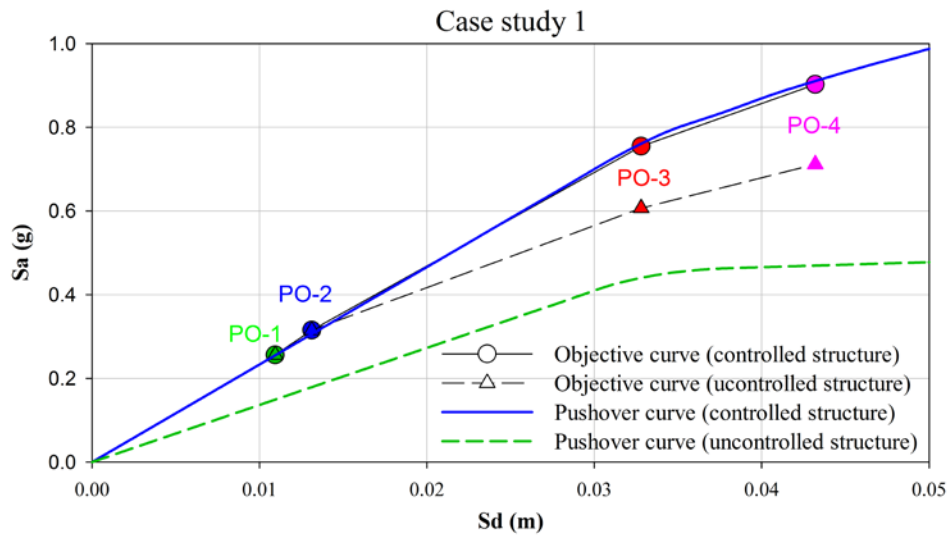
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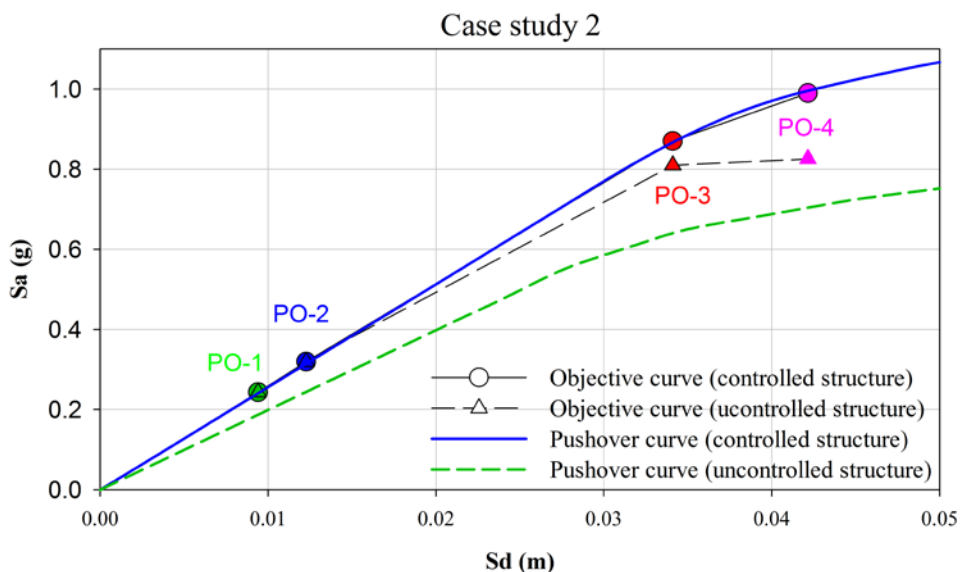
482 **5.6 Numerical verification**

483 In this section, the fulfilment of the pre-defined seismic performance objectives is verified.  
 484 This was done through a numerical simulation of the seismic behavior of the two case studies. With  
 485 this purpose, a finite element model for each case study has been developed using SAP2000. The fiber-  
 486 based software “SeismoStruct V.7.0.6” was used to obtain the constitutive laws of the designed CSB  
 487 bracing elements, which were then imported to SAP2000 as non-linear links (NL).

488 First, a non-linear pushover analysis was conducted using two displacement shapes (linear  
 489 and uniform), whose average was considered. The base shear and the roof (top) displacement were  
 490 used to signify the force and displacement respectively. Figure 12 and Figure 13 show the  
 491 capacity spectra of the controlled and uncontrolled structures with their corresponding objective  
 492 curves in  $S_{ad}$  format for the case studies 1 and 2 respectively. Investigation of the graphs reveals  
 493 that for each of the two case studies the capacity spectrum (i.e. pushover curve) of the  
 494 *controlled* structure matches the corresponding predefined target curve (i.e. objective curve). On the  
 495 other hand, the capacity spectrum of the *uncontrolled* structure was not able to match the  
 496 corresponding objective curve.  
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 499 Figure 12. The performance objectives and the results of the pushover analyses in  $S_{ad}$  format  
 500 of the controlled and uncontrolled structures (Case study 1)  
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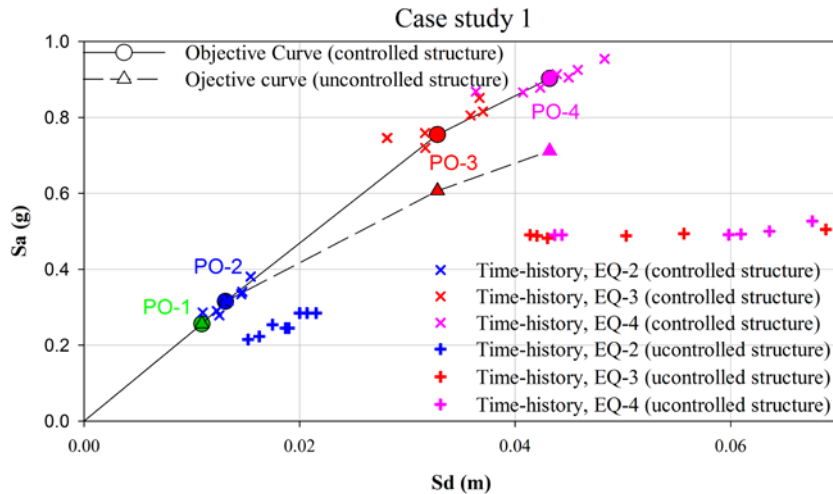
Figure 13. The performance objectives and the results of the pushover analyses in  $S_{ad}$  format of the controlled and uncontrolled structures (case study 2)

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Another type of analysis, nonlinear time-history, was performed to assess the seismic performance of the structure. Four groups of spectrum-compatible accelerograms were considered in agreement with the EQ levels reported in Table 2. Each group consists of seven ground motion records scaled to the PGA of the corresponding EQ level at the fundamental period of the structure. The results of the time-history analyses for the two case studies are plotted in Figure 14 and Figure 15 respectively, where each point represents the maximum base shear and ultimate displacement of the corresponding time-history analysis. Investigation of the graph allows observing that the seismic response of the uncontrolled structure fails to achieve the predefined performances, unlike the controlled structure whose time-history analyses results show a large agreement with the prescribed objectives.

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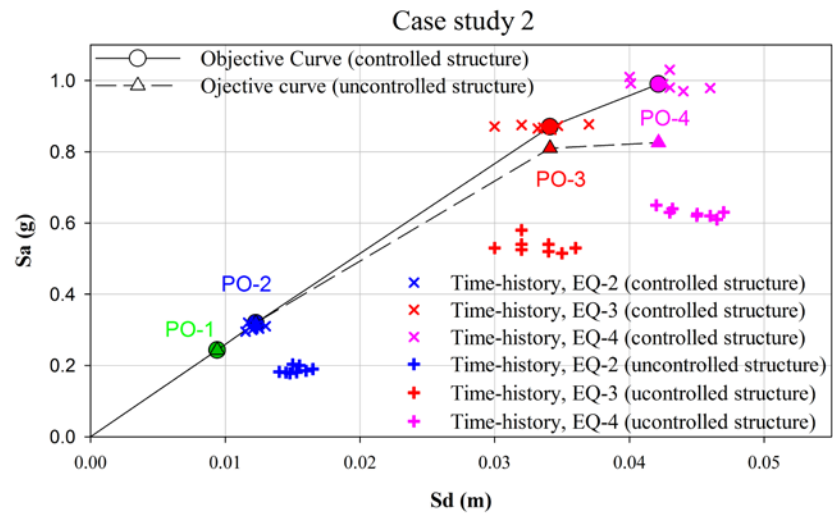
It is important to note that the nonlinear behavior of the structure equipped with the CSB braces is not designed for in this study 'automatic'; however, this good behavior is expected due to the shape of the brace (the peculiar mechanical behavior) (Figure 2) and to its hysteretic dissipation properties, and this is verified in this study.



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Figure 14. The performance objectives and the results of the time-history analyses in  $S_{ad}$  format of the controlled and uncontrolled structures (case study 1)

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Figure 15. The performance objectives and the results of the time-history analyses in  $S_{ad}$  format of the controlled and uncontrolled structures (case study 2)

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## 526 6 CONCLUSION

527 In this paper, a comprehensive procedure for the seismic design of multi-storey frame  
528 structures equipped with an energy dissipation device “Crescent Shaped Brace” is proposed. The  
529 procedure falls within the Performance-Based Seismic Design (PBSD) approach. The first part of  
530 the method is to design the braces in the elastic field with reference to the performance point IO-  
531 EQ2. Then, the post yielding behavior of the CSB is determined numerically using the FEM  
532 software SeismoStruct. In the second part of the method, the post yielding behavior of the  
533 controlled system (i.e. structure equipped with the designed braces) is verified by means of  
534 nonlinear pushover and time history analyses.

535 The validity of the method was determined by analyzing two reinforced concrete frame  
536 structures equipped with crescent-shaped braces (CSB). First, the performance objectives are chosen.  
537 The performance objectives have been expressed in terms of the storey drift index (IDI), which is a  
538 measure of the non-structural damage in the structure. Then, the CSB devices have been designed by  
539 implementing the proposed design procedure. Static pushover and dynamic time-history analyses  
540 were conducted on the case study structures to validate the nonlinear behavior of the CSB within  
541 the global system. The analyses performed showed a good behavior of the devices when applied to  
542 both case studies although the two structures were of different occupancies and different seismic  
543 demands. This confirms the validity of the proposed design approach and the effectiveness of the  
544 new hysteretic device in resisting lateral forces regardless of structure’s mechanical properties and  
545 the seismic demands.

546 It is important to point out that all prior efforts to design the CSB were majorly based on SDOF  
547 structures. The present design procedure is applicable to both SDOF and MDOF shear-type  
548 structures. Future research will be aimed at generalizing the method to be applicable to other types of  
549 structures.

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553 RESCUE— Integrated DEsign and control of Sustainable CommUnities during Emergencies.  
554

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557 "Definizione di metodi di progetto, procedure e software dedicati ai sistemi di dissipazione di  
558 energia e proposte di normativa sviluppate nell’ambito del presente progetto") is gratefully  
559 acknowledged.  
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Table 1. Mechanical and geometrical properties of the structural elements

Characteristics	CS1 (commercial building)	CS2 (school)
Concrete average cubic strength, $R_{ck}$	C45/55, $R_{ck}=55$ Mpa	C20/25, $R_{ck}=24.6$ MPa
Steel yield strength, $f_y$	B540C, $f_y=450$ Mpa	FeB38K, $f_y=375$ Mpa
Modulus of elasticity, E	E=36000 Mpa	E=25150 Mpa
Columns cross-sections	1 <sup>st</sup> level 60cmx60cm 2 <sup>nd</sup> level 50cmx50cm	1 <sup>st</sup> level 50cmx40cm 2 <sup>nd</sup> level 50cmx40cm 3 <sup>rd</sup> level 50cmx40cm
Beams cross-sections	x-direction 50cmx40cm y-direction 50cmx40cm	x-direction 60cmx40cm y-direction 50cmx40cm

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Table 2. Earthquake design levels with corresponding response spectra parameters for the two case studies

Earthquake design level	Earthquake performance level	$T_r$ [years]		PGA [g]		$F_0$		$T_c^*$ [s]	
		CS1	CS2	CS1	CS2	CS1	CS2	CS1	CS2
EQ1: frequent	Fully operational-IO	30	45	0.071	0.089	2.39	2.27	0.27	0.29
EQ2: occasional	Damage-O	50	75	0.093	0.116	2.34	2.28	0.27	0.32
EQ3: rare	Life safety-LS	475	712	0.230	0.323	2.39	2.45	0.31	0.38
EQ4: very rare	Near collapse-NC	975	1462	0.293	0.426	1.27	2.49	0.32	0.41

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Table 3. Quantification of the Basic and the Essential performance objectives

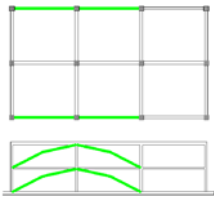
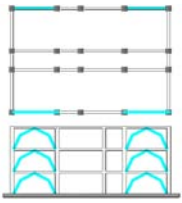
Limit state (Basic objectives)	IDI[1] (Basic objectives)	Limit state (Essential objectives)	IDI (Essential objectives) CS1	IDI (Essential objectives) CS2
EQ1: Fully operational	0.003	EQ1: Fully operational	PO-1 = 0.0015	PO-1 = 0.0013
EQ2: Damage	0.006	EQ2: Fully operational	PO-2 = 0.0020	PO-2 = 0.0018
EQ3: Life safety	0.015	EQ3: Damage	PO-3 = 0.0050	PO-3 = 0.0045
EQ4: Near collapse	0.020	EQ4: Life safety	PO-4 = 0.0067	PO-4 = 0.0055

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Table 4. Application of the proposed design methodology to the two case studies

First case study:	Second case study:
<i>Step 1: Global stiffness matrix</i>	
<ul style="list-style-type: none"> <li>❖ Mass matrix:  <math>[M] = \begin{pmatrix} m_1 &amp; 0 \\ 0 &amp; m_2 \end{pmatrix} = \begin{pmatrix} 8781.55 &amp; 0 \\ 0 &amp; 7035.165 \end{pmatrix} (kN)</math> </li> <li>❖ Initial stiffness matrix:  <math>[K] = \begin{pmatrix} 338474 + 163230 &amp; -163230 \\ -163230 &amp; 163230 \end{pmatrix} \left( \frac{kN}{m} \right)</math> </li> <li>❖ Initial global stiffness matrix for the first iteration:  <math>[K^*] = \begin{pmatrix} 1 + 0.615 &amp; -0.615 \\ -0.615 &amp; 0.615 \end{pmatrix} \cdot k_1 \left( \frac{kN}{m} \right)</math> </li> </ul> For the first iteration: $k_1^* = k_1 = 338474$ kN/m	<ul style="list-style-type: none"> <li>❖ Mass matrix:  <math>[M] = \begin{pmatrix} m_1 &amp; 0 &amp; 0 \\ 0 &amp; m_2 &amp; 0 \\ 0 &amp; 0 &amp; m_3 \end{pmatrix} = \begin{pmatrix} 3799.5 &amp; 0 &amp; 0 \\ 0 &amp; 3470.1 &amp; 0 \\ 0 &amp; 0 &amp; 3153.08 \end{pmatrix} (kN)</math> </li> <li>❖ Initial stiffness matrix:  <math>[K] = \begin{pmatrix} 362800 + 318810 &amp; -318810 &amp; 0 \\ -318810 &amp; 318810 + 189340 &amp; -189340 \\ 0 &amp; -189340 &amp; 189340 \end{pmatrix} \left( \frac{kN}{m} \right)</math> </li> <li>❖ Initial global stiffness matrix for the first iteration:  <math>[K^*] = \begin{pmatrix} 1 + 0.942 &amp; -0.942 &amp; 0 \\ -0.942 &amp; 1.396 &amp; -0.454 \\ 0 &amp; -0.454 &amp; 0.454 \end{pmatrix} \cdot k_1^*</math> </li> </ul> For the first iteration: $k_1^* = k_1 = 362800$ kN/m
<i>Step 2: Modal analysis (LS response spectrum)</i>	

<ul style="list-style-type: none"> <li>❖ Inter-storey drifts: <math>\delta_{01} = 2.63cm</math> <math>\delta_{12} = 3.46cm</math></li> </ul>	<ul style="list-style-type: none"> <li>❖ Inter-storey drifts: <math>\delta_{01} = 2.11cm</math> <math>\delta_{12} = 1.90cm</math> <math>\delta_{12} = 1.84cm</math></li> </ul>
<i>Step 3: Matching the design drifts</i>	
<ul style="list-style-type: none"> <li>❖ Design drifts: <math>\delta_{01,d} = 0.005.h = 0.005 * 410 = 2.05cm</math> <math>\delta_{12,d} = 0.005.h = 0.005 * 410 = 2.05cm</math></li> <li>❖ Global stiffness matrix at the final iteration: <math display="block">[K^*] = \begin{pmatrix} 826650 &amp; -312290 \\ -312290 &amp; 312290 \end{pmatrix} \left(\frac{kN}{m}\right)</math></li> </ul>	<ul style="list-style-type: none"> <li>❖ Design drifts: <math>\delta_{01,d} = 0.0045.h = 0.0045 * 318 = 1.43cm</math> <math>\delta_{12,d} = 0.0045 * 332 = 1.49cm</math> <math>\delta_{23,d} = 0.0045 * 340 = 1.53cm</math></li> <li>❖ Global stiffness matrix at the final iteration: <math display="block">[K^*] = \begin{pmatrix} 923770 &amp; -401980 &amp; 0 \\ -401980 &amp; 631000 &amp; -229020 \\ 0 &amp; -229020 &amp; 229020 \end{pmatrix} \left(\frac{kN}{m}\right)</math></li> </ul>
<i>Step 4: Stiffness of the CSB system</i>	
<ul style="list-style-type: none"> <li>❖ Stiffness matrix of the bracing system: <math display="block">[K_b] = [K^*] - [K] = \begin{pmatrix} 324950 &amp; -149060 \\ -149060 &amp; 149060 \end{pmatrix} \left(\frac{kN}{m}\right)</math> <math>k_{b1} = 175890 \frac{kN}{m}</math> <math>k_{b2} = 149060 \frac{kN}{m}</math></li> </ul>	<ul style="list-style-type: none"> <li>❖ Stiffness matrix of the bracing system: <math display="block">[K_b] = [K^*] - [K] = \begin{pmatrix} 242160 &amp; -83170 &amp; 0 \\ -83170 &amp; 122850 &amp; -39680 \\ 0 &amp; -39680 &amp; 39680 \end{pmatrix} \left(\frac{kN}{m}\right)</math> <math>k_{b1} = 158990 \frac{kN}{m}</math> <math>k_{b2} = 83170 \frac{kN}{m}</math> <math>k_{b3} = 39680 \frac{kN}{m}</math></li> </ul>
<i>Step 5: Stiffness of the single CSB device</i>	
<ul style="list-style-type: none"> <li>❖ Structural configuration of the CSB in the commercial building</li> </ul>  <p><math>N_{CSB,1} = N_{CSB,2} = 4</math></p> <p><math>k_{CSB,1} = \frac{175890}{4} = 43972.5 \frac{kN}{m}</math> <math>k_{CSB,2} = \frac{149060}{4} = 37265 \frac{kN}{m}</math></p>	<ul style="list-style-type: none"> <li>❖ Structural configuration of the CSB in the school building</li> </ul>  <p><math>N_{CSB,1} = N_{CSB,2} = N_{CSB,3} = 8</math></p> <p><math>k_{CSB,1} = \frac{158990}{8} = 19873.7 \frac{kN}{m}</math> <math>k_{CSB,2} = \frac{83170}{8} = 10396.2 \frac{kN}{m}</math></p> <p><math>k_{CSB,3} = \frac{39680}{8} = 4960 \frac{kN}{m}</math></p>
<i>Step 6: Moment of inertia and cross section profile</i>	
<ul style="list-style-type: none"> <li>❖ Arm ratio: <math>\xi = 0.1</math></li> <li>❖ Moments of inertia: <math>J_1 = 139684.3 cm^4</math> <math>J_2 = 118377 cm^4</math></li> <li>❖ Cross sections: <math>CSB_1</math>: rect. <math>48cm \times 15cm</math> <math>CSB_2</math>: rect. <math>45cm \times 15cm</math></li> </ul>	<ul style="list-style-type: none"> <li>❖ Arm ratio: <math>\xi = 0.1</math></li> <li>❖ Moments of inertia: <math>J_1 = 5580.3 cm^4</math> <math>J_2 = 3277.8 cm^4</math> <math>J_3 = 1671.5 cm^4</math></li> <li>❖ Cross sections: <math>CSB_1</math>: rect. <math>20cm \times 8.4cm</math> <math>CSB_2</math>: rect. <math>18cm \times 6.8cm</math> <math>CSB_3</math>: rect. <math>14cm \times 7.3cm</math></li> </ul>