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Price equilibrium and willingness to pay in a vertically differentiated mixed duopoly

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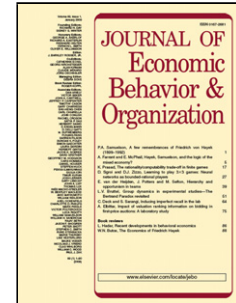
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# Price equilibrium and willingness to pay in a vertically differentiated mixed duopoly

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## Abstract

In the framework of a vertically differentiated mixed duopoly, with uncovered market and costless quality choice, we study the existence of a price equilibrium when a welfare-maximizing public firm producing low quality goods competes against a profit-maximizing private firm producing high quality goods. We show that a price equilibrium exists if the quality spectrum is wide enough *vis à vis* a measure of the convexity of the distribution of the consumers' willingness to pay, and that such equilibrium is unique if this sufficient condition is tightened. Log-concavity of the income distribution is inconsistent with the existence of equilibrium.

**JEL classification:** D43, L13, L51.

**Keywords:** price equilibrium, vertical differentiation, mixed duopoly

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# Price equilibrium and willingness to pay in a vertically differentiated mixed duopoly

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21 December 17, 2015  
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## Abstract

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27 covered market and zero costs, we study the existence of a price equi-  
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29 goods competes against a profit-maximizing private firm producing  
30 high quality goods. We show that a price equilibrium exists if the  
31 quality spectrum is wide enough *vis à vis* a measure of the convexity  
32 of the distribution of the consumers' willingness to pay, and that such  
33 equilibrium is unique if this sufficient condition is tightened. Logcon-  
34 cavity of the income distribution is inconsistent with the existence of  
35 equilibrium.  
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45 Lorenzo Zirulia for their helpful comments on an earlier version of  
46 this paper, as well as two anonymous referees for helpful and detailed  
47 comments and suggestions.  
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# 1 Introduction

Mixed oligopolies can be observed in many countries and sectors, as mixed industries in advanced economies became particularly relevant in the last decades, following extensive privatization programmes of public monopolies in the 80's and 90's.<sup>1</sup> In mixed industries (e.g. public utilities, transportation, telecommunication, energy, postal services, education, health care, etc.) public firms compete with private firms in price, quantity and the quality of goods. It is frequently argued that public firms supply goods or services, the quality of which is lower than that provided by private firms: e.g., such is allegedly the case in many countries for education and health care, or in transportation and postal services. To be sure, the idea that public firms consistently supply lower quality can be challenged on empirical grounds – indeed, cases can even be found where the same industry is characterized by public firms supplying higher or lower quality, depending on the country or the sector one looks at (Epple & Romano 1998, Jofre-Bonet 2000, Sanjo 2009, Cremer & Maldonado 2013). However, cases where public firms do offer lower quality are many, and the literature of mixed oligopolies usually relies on such an assumption (see, e.g., Ishibashi & Kaneko 2008).<sup>2</sup>

A number of papers address the question of why this should be so, in the framework of a welfare-maximizing public firm competing with a profit-

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<sup>1</sup>In Europe several public utilities such as telecommunication, electricity, gas retailing, and postal services became mixed markets as private firms were allowed to compete with public firms. The same happened with many previously public industries, such as airlines, railways, energy, steel, banking, broadcasting, life insurance, health care, and education. On the main privatization programmes and the relevance of mixed industries see e.g., Cuervo & Villalonga (2000), Megginson & Netter (2001).

<sup>2</sup>Examples of mixed industries where public firms allegedly provide low quality are transport services (Dodgson & Katsoulacos 1988), postal services (Mizutani & Uranishi 2003), telecommunication (Ros 1999), and financial services (Barros & Modesto 1999). It should perhaps also be noticed that, according to Blackorby & Donaldson (1988) and Besley & Coate (1991), if the quality differential between public and private sectors is justified by a concern for accessibility, the quality offered by the public firm should be sufficiently low to make accessibility effective.

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5 maximizing private one;<sup>3</sup> however, the answer they provide is usually sought  
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7 by assuming away any role for the distribution of the willingness to pay  
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9 across consumers: either because the crucial feature of uncovered market  
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11 is ruled out, or because – while allowing for uncovered markets – the stan-  
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13 dard, uniform-distribution model of vertical differentiation is used.<sup>4</sup> This is  
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15 somewhat surprising on at least two counts: at a very general level, most  
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17 informal arguments justifying the very existence of public firms competing  
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19 with private firms rely on distributional concerns about inequality and pro-  
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21 viding the poor with access to goods and services; and, more to the point at  
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23 the analytical level, it is in general well known that the distribution of the  
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25 willingness to pay affects the firms' equilibrium choices and can in princi-  
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27 ple affect the very existence of equilibria (Grandmont 1993, Anderson et al.  
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29 1997).

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31 In this paper we focus on the existence of a short-run price equilibrium  
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33 in a vertically differentiated mixed duopoly with uncovered market, to con-  
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35 firm that the distribution of the willingness to pay affects equilibria. We  
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37 assume costless production, which allows us to concentrate upon the rele-  
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39 vant features of demand and hence the distribution of the willingness to pay;  
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41 and we model a mixed duopoly as a case where a welfare-maximizing, low-  
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43 quality producing public firm competes against a profit-maximizing, high-  
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45 quality producing private firm – which in principle might provide a first

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47 <sup>3</sup>For an overview of the theory of mixed oligopoly see De Fraja & Delbono (1990)  
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49 and for standard models of mixed oligopoly see, e.g., Harris & Wiens (1980), De Fraja  
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51 & Delbono (1989), Grilo (1994), Barros & Martinez-Giralt (2002), Cantos-S. & Moner-  
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53 Colonques (2006), Cremer & Maldonado (2013).

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55 <sup>4</sup>Thus, e.g., Ishibashi & Kaneko (2008) use the Hotelling model to argue that the in a  
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57 duopoly equilibrium the public firm would supply the lower quality, and the private firm  
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59 the higher (in fact, higher than is efficient) quality level. On the other hand, Delbono  
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61 et al. (1996) use the standard uncovered market model to show that an equilibrium where  
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63 the public (private) firm chooses the low (high) quality exists, though an equilibrium with  
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65 inverted quality allocations also exists, and market segmentation is exogenous (also, this  
is a framework where it is problematic to find analytical solutions).

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5 step in addressing the important general question of comparing the overall  
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7 performance of ‘mixed’ vs ‘pure’ oligopolies within vertically differentiated  
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9 markets.<sup>5</sup> In this framework, we show that for a price equilibrium to exist  
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11 the distribution of the willingness to pay cannot be logconcave, and that  
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13 sufficient conditions for existence and uniqueness place a lower bound on  
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15 the (given) quality spectrum – a lower bound which is higher, the higher the  
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17 given convexity bound on the income distribution.

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19 The paper is organized as follows. Section 2 presents the model and the  
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21 general framework of mixed duopoly with vertical differentiation; Section 3  
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23 gives the solution for a market price equilibrium and discusses existence and  
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25 uniqueness; Section 4 presents an example where the consumers’ willingness  
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27 to pay is supposed to be distributed as a Pareto distribution, while some  
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29 concluding remarks are gathered in Section 5.

## 30 31 32 **2 The model**

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35 We start from a standard model of duopoly competition with vertical dif-  
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37 ferentiation, uncovered market and costless quality choice, as developed  
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39 by Mussa & Rosen (1978), Shaked & Sutton (1982) and Tirole (1988).  
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41 There are two competing firms,  $i = H, L$ , playing a non-cooperative game  
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43 on price. Each firm  $i$  produces a good of quality  $s_i \in \{s_H, s_L\}$ , where  
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45  $0 < s_L < s_H < \infty$  and  $\Delta = s_H - s_L > 0$  denotes the quality differential.  
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47 We crucially assume that  $L$  is a public firm producing low quality goods,  
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49 while  $H$  is a profit-maximizing firm producing high quality goods; produc-  
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51 tion costs are normalized to zero.<sup>6</sup> The firms’ profits are  $\Pi_i = p_i D_i$ , where

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53 <sup>5</sup>As we argue in subsection 3.2, the focus on short-run price equilibria with given  
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55 qualities makes it easier to think of marginal costs as constant.

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57 <sup>6</sup>This amounts to marginal costs being constant and independent of quality (see our  
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59 discussion in Section 3.2). Notice that in this framework a high-quality producing pub-  
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61 lic firm would serve the whole market at a price equal to marginal cost, and no profit  
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5  $p_i$  and  $D_i$ ,  $i = H, L$ , denote prices and demands: higher quality  $s_H$  sells at  
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7 a price  $p_H$ , and lower quality  $s_L$  at a price  $p_L$ .

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9 Each consumer is identified by her marginal willingness to pay for quality,  
10  $\theta$ , and has a utility  $U_i(\theta) = \theta s_i - p_i$  if she buys a unit of good from firm  $i$ ,  
11 and 0 otherwise. The marginal consumer, who is indifferent between buying  
12 the high and the low quality, has utility  $U_H(\theta) = U_L(\theta)$ , and is accordingly  
13 identified by  $\theta_H = (p_H - p_L) / \Delta$ ; the marginal consumer who is indifferent  
14 between purchasing the low quality commodity and nothing at all has utility  
15  $U_L(\theta) = 0$ , and is identified by  $\theta_L = p_L / s_L$ .<sup>7</sup> Clearly,  $\theta_L$  and  $\theta_H$  denote the  
16 positions of these marginal consumers along the "income" scale: for later  
17 reference, it is useful to derive the price elasticities of  $\theta_L$  and  $\theta_H$ , which are  
18 given by  $\varepsilon_H = \frac{\partial \theta_H}{\partial p_H} \frac{p_H}{\theta_H} = \frac{p_H}{p_H - p_L} > 1$  and  $\varepsilon_L = \frac{\partial \theta_L}{\partial p_L} \frac{p_L}{\theta_L} = \frac{-p_L}{p_H - p_L} < 0$ , such  
19 that  $\varepsilon_H + \varepsilon_L = 1$ .

20  
21 Normalizing the consumers' population to 1 and assuming that the will-  
22 ingness to pay  $\theta$  is continuously distributed over some nonnegative support  
23  $\Theta \subseteq \mathbb{R}_+$ , we define the density function  $f(\theta)$  such that the implied cumula-  
24 tive distribution is  $F : \Theta \rightarrow [0, 1]$ . Using primes to denote derivatives, it is  
25 convenient for our purposes to define also the following elasticities:  
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$$30 \quad \eta(\theta) = \frac{\theta f'(\theta)}{1 - F(\theta)}, \quad (1)$$

$$31 \quad \pi(\theta) = \lim_{h \rightarrow 0} \frac{d \log \left( \frac{1}{\mu} \int_y^{\theta+h} x f(x) dx \right)}{d \log \theta} = 1 + \frac{\theta f'(\theta)}{f(\theta)}, \quad (2)$$

32 where definition (1) is the (positive) elasticity of  $1 - F(\theta)$  and definition (2)  
33 is the Esteban elasticity of the density  $f(\theta)$ .<sup>8</sup> We use these definitions to

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maximizing firm could produce low-quality goods in equilibrium.  
<sup>7</sup>These are the basic features of the standard vertical differentiation model (Mussa & Rosen 1978); as is well known, the marginal willingness to pay  $\theta$  can be looked at as a proxy for income (Gabszewicz & Thisse 1979).

<sup>8</sup>Esteban (1986) defines the function  $\pi(\cdot)$  as per our definition (2) and shows that it

gather our basic assumptions on  $F$  in the following assumption:

**Assumption 1** *The distribution  $F$  is such that:*

- (a) *the lower bound of the support  $\Theta$  ( $\theta_{\min}$  say) obeys  $\theta_{\min} = 0 = \eta(\theta_{\min})$ , and the upper bound ( $\theta_{\max}$  say) is such that  $\lim_{\theta \rightarrow \theta_{\max}} \eta(\theta) > 1$ ;*
- (b) *there exists some  $\alpha \in (0, 1]$  such that,  $\forall \tilde{\alpha} \in [\alpha, 1]$ ,  $(1 + \tilde{\alpha})\eta(\theta) + \pi(\theta) - 1 > 0, \forall \theta \in \Theta$ ;*
- (c) *let  $\tilde{\theta}$  be the smallest value such that  $\eta(\cdot) = 1$ : then there exists a (unique) value  $\theta^\circ, 0 < \theta^\circ \leq \tilde{\theta}$  such that  $\pi(\theta^\circ) = 0$ , and such that  $\pi(\theta^\circ) > 0$  for  $\theta < \theta^\circ$  and  $\pi(\theta^\circ) < 0$  for  $\theta > \theta^\circ$ .*

Assumption 1(a) implies that at equilibrium the market cannot be completely covered, and that the value of  $\theta$  at which  $\eta(\theta) = 1$  (which is pivotal in what follows) lies strictly within  $\Theta$ . Assumption 1(b) implies that  $(1 - F)^{-\alpha}$  is a convex function, which in turn limits in some way the convexity of the relationship between the size of the covered market,  $1 - F(\cdot)$ , and the consumers' willingness to pay  $\theta$ .<sup>9</sup> It should be stressed that by excluding the extreme value  $\alpha = 0$  we are ruling out log-concavity, while  $\alpha$  being finite rules out the uniform distribution, which one would get as  $\alpha \rightarrow -\infty$  (e.g., Caplin & Nalebuff (1991), p. 3). The same assumption also implies that:

$$\eta(\theta) + \pi(\theta) > 1 - \alpha\eta(\theta), \quad (3)$$

stands in a one-to-one relationship with the underlying density  $f(\cdot)$ : accordingly, it gives an alternative representation of the density itself, which in some circumstances may be useful, especially so as some regularity features are apparently supported by empirical evidence. See (Benassi & Chirco 2006) for the relationship between the Esteban elasticity and stochastic dominance, and Majumder & Chakravarty (1990) for some related empirical evidence.

<sup>9</sup>Indeed, it is easily seen that  $\frac{d^2}{d\theta^2}(1 - F)^{-\alpha} = \frac{\alpha}{\theta} \frac{f(\theta)}{(1 - F(\theta))^{\alpha+1}} [(1 + \alpha)\eta(\theta) + \pi(\theta) - 1] > 0$ . Following Caplin & Nalebuff (1991), we can say that function  $(1 - F)$  is  $\rho$ -concave (with  $\rho = -\alpha < 0$ ), which is equivalent to saying that  $-(1 - F)^\alpha$  is concave. Moreover, a  $\rho$ -concave function is also  $\tilde{\rho}$ -concave for all  $\tilde{\rho} < \rho$ , which means that what is true for a given  $\alpha \in (0, 1]$  is also true for every  $\tilde{\alpha} > \alpha$  included in the same interval.

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5 which in turn means that  $\eta$  is monotonically increasing for  $\eta(\theta) \leq 1$ , i.e.  
6 over  $[0, \tilde{\theta}]$ . Also, this places a restriction on the function  $\pi(\cdot)$  to the effect  
7 that, for  $\eta(\theta) \leq 1$ , i.e. over  $[0, \tilde{\theta}]$ ,

$$11 \quad \pi(\theta) > -\alpha \geq -1. \quad (4)$$

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15 All this should clarify Assumption 1(c), which rules that the function  
16  $\pi$  (surely positive by condition (3) as  $\theta$  nears zero) changes sign only once  
17 within  $[0, \tilde{\theta}]$ .<sup>10</sup>

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19 Since we look for the Nash equilibrium of the game, we first have to  
20 determine the demand functions faced by firms  $L$  and  $H$ :  $D_H = 1 - F(\theta_H)$ ,  
21  $D_L = F(\theta_H) - F(\theta_L)$ , where  $F(\theta_j)$  represents the fraction of consumers with  
22 a taste parameter less than  $\theta_j$ ,  $j = L, H$  (Tirole 1988). The corresponding  
23 profit functions are given by  $\Pi_H = p_H D_H$ ,  $\Pi_L = p_L D_L$ . Finally, we define  
24 the social welfare function as the sum of the consumers' surplus:  $W =$   
25  $s_H \int_{\theta_H}^{\infty} \theta f(\theta) d\theta + s_L \int_{\theta_L}^{\theta_H} \theta f(\theta) d\theta$ , and crucially assume that the public firm  
26 sets the price of low quality goods  $p_L$  to maximize the social welfare function  
27  $W$ .<sup>11</sup>

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39 <sup>10</sup>This is the case with many widely used distributions, such as the Gamma and Pareto  
40 distributions.

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65 <sup>11</sup>In principle, however, if one looks at the public firm's objective as justified in terms of  
the median voter theorem, and the income distribution is asymmetric, the policy makers  
may be driven to look at the marginal willingness to pay of the median consumer, instead of  
that of the average consumer (as required by social welfare maximization). For alternative  
models of mixed oligopoly with non-welfare-maximizing behavior, see, e.g., Fershtman  
(1990), Cremer et al. (1991), Barros (1995), Estrin & De Meza (1995). In particular, to  
analyze mixed oligopoly equilibria when the firms' objectives are endogenous, De Donder  
& Roemer (2009) study a vertically differentiated mixed market where one firm is profit-  
maximizing while the other maximizes revenues, but one firm becomes welfare-maximizing  
when the government takes a participation in it.

### 3 Price equilibrium

In this Section, we take up Nash equilibria in prices: we first study existence, and then enquire about uniqueness.<sup>12</sup>

#### 3.1 Existence of the price equilibrium

Given the price  $p_L$  set by the public firm on the ‘low-quality’ goods,  $p_H$  is charged by firm  $H$  maximizing its profit  $\Pi_H$ . The corresponding first order conditions (FOCs) in terms of elasticity are given by:

$$\eta(\theta_H) \varepsilon_H = 1, \quad (5)$$

which implies  $\eta(\theta_H) < 1$ . The second order conditions (SOCs) can be similarly characterized in elasticity terms as:

$$2\eta(\theta_H) + \pi(\theta_H) > 0, \quad (6)$$

which implies, given that  $\eta(\theta_H) < 1$  by condition (5), the necessary condition  $\pi(\theta_H) > -1$ , consistently with (4).<sup>13</sup>

To set the price  $p_L$ , the public firm maximizes the social welfare  $W$ . The corresponding FOCs are:

$$\frac{\eta(\theta_H)}{\eta(\theta_L)} = \frac{1 - F(\theta_L)}{1 - F(\theta_H)} > 1, \quad (7)$$

<sup>12</sup>Given a quality pair  $(s_H, s_L)$ , existence and uniqueness can clearly be established with reference to a (or the) pair of marginal consumers along the ‘income’ scale,  $(\theta_H^*, \theta_L^*)$ , as it will be  $p_H^* = \theta_H^* (s_H - s_L) + \theta_L^* s_L$ , and  $p_L = \theta_L^* s_L$ .

<sup>13</sup>The FOCs and SOC for firm  $H$  can be written out as:  $\frac{\partial \Pi_H}{\partial p_H} = 1 - F(\theta_H) - \frac{p_H}{\Delta} f(\theta_H) = 0$ ;  $\frac{\partial^2 \Pi_H}{\partial p_H^2} = -2f(\theta_H) - \frac{p_H}{\Delta} f'(\theta_H) < 0$ , from which (5) and (6) can easily be derived by using definitions (1) and (2).

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5 from which  $\theta_H > \theta_L$  implies  $\eta(\theta_H) > \eta(\theta_L)$ . The SOCs are given by:

$$6 \quad (1 - \eta(\theta_H)) \pi(\theta_H) + \eta(\theta_H) \pi(\theta_L) > 0, \quad (8)$$

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10 which again are set in elasticity terms.<sup>14</sup>

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12 As a result, at a price equilibrium for given  $s_H$  and  $s_L$ ,  $p_H$  and  $p_L$  are  
13 identified by the twin FOCs (5) and (7), such that the twin SOCs (6) and  
14 (8) hold.  
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19 Before enquiring about the existence of equilibrium, it is perhaps worth  
20 stressing that – irrespective of our assumptions on the distribution of the  
21 willingness to pay and indeed justifying them – the basic framework we  
22 are using (though indeed quite standard) is inconsistent with a logconcave  
23 distribution of the consumers' willingness to pay – including the limit case  
24 of the uniform distribution. Intuitively, this is so because of the way a  
25 marginal changes in prices affect the positions of the marginal consumers.  
26 An increase in  $p_L$  pushes the marginal consumers nearer each other, by  
27 shifting linearly one to the right ( $\theta_L$ ) and the other to the left ( $\theta_H$ ) – that  
28 is, the set of middle-class consumers patronizing low-quality gets smaller,  
29 and that of the high-income consumers patronizing high quality gets larger:  
30 since this has opposite effects on overall welfare, the latter is maximized  
31 when the marginal contribution to welfare of enlarging the set of high-quality  
32 consumers is equal to the marginal cost of pricing out the poor. This however  
33 (a) requires that the income density falls sharply enough as we move from  $\theta_L$   
34 to  $\theta_H$ , and (b) has to be consistent with the high-quality firm maximizing  
35 its profits. The latter obviously calls for the price elasticity of demand  
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52 <sup>14</sup>The FOCs and SOCs for the public firm are respectively  $\frac{\partial W}{\partial p_L} = \theta_H f(\theta_H) - \theta_L f_L(\theta_L) =$   
53  $0$ , and  $\frac{\partial^2 W}{\partial p_L^2} = -\frac{f(\theta_H)}{\Delta} \pi(\theta_H) - \frac{f(\theta_L)}{s_L} \pi(\theta_L) < 0$ . In equilibrium, the latter is equivalent  
54 to condition (8), as can be seen by multiplying through by  $p_L > 0$ , substituting for  
55  $\varepsilon_L = 1 - \varepsilon_H$ , and taking advantage of the FOCs (5) and (7).  
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5 for high-quality be one: given the structure of preferences (such that a  
6 small increase in  $p_H$  has a big effect on the location of the high-quality  
7 marginal consumer:  $\varepsilon_H > 1$ ), this in turn dictates that  $\eta(\theta_H) < 1$  as from  
8 (5). Logconcavity, by constraining the relationship between  $\eta(\cdot)$  and  $\pi(\cdot)$   
9 as defined in (1) and (2), is inconsistent with both requirements holding at  
10 once: if the distribution is logconcave, high-quality demand being sufficiently  
11 rigid is inconsistent with the density falling rapidly enough around  $\theta_H$ , which  
12 under logconcavity would mean high demand elasticity from the marginal  
13 high-quality consumer.<sup>15</sup>

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22 We can now state the following proposition on the existence of a price  
23 equilibrium.

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27 **Proposition 1** *Let  $(s_H, s_L)$  be a given pair of qualities, such that  $0 < s_L <$   
28  $s_H < \infty$ , and let  $k = s_L/\Delta$  such that  $\eta(\theta^\circ) < \frac{1}{1+k}$ . Then under Assumption  
29 1 a price equilibrium exists.*

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34 **Proof.** See Appendix 6A ■

35  
36 Proposition 1 establishes that a price equilibrium exists, if some con-  
37 straints are satisfied concerning the distribution of the willingness to pay *vis*  
38 *à vis* the quality spectrum being offered. First notice that welfare maximiza-  
39 tion by firm  $L$  leads to  $\theta_H$  lying on a downward portion of the density  $f(\theta)$ .  
40 Indeed, the FOCs (7) boil down to  $\theta_L f(\theta_L) = \theta_H f(\theta_H)$ : analytically, this is  
41 inconsistent with both marginal consumers being on an upward sloping por-  
42 tion of the density itself, while economically it amounts to the marginal gain  
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49 <sup>15</sup>Log-concavity amounts to the constraint  $\pi(\theta) > 1 - \eta(\theta)$  for all  $\theta$ , such that  $\eta(\theta_H) < 1$   
50 is inconsistent with  $\pi(\theta_H) < 0$ . On the other hand,  $\pi(\theta_H)$  has to be negative, if welfare  
51 has to be maximized. This condition, which dictates that the density should be sufficiently  
52 (and negatively) steep around  $\theta_H$ , can be seen by observing that  $\theta f(\theta)$  is the marginal  
53 contribution to social welfare of the consumers whose willingness to pay is  $\theta$ , and that its  
54 derivative is  $f(\theta)\pi(\theta)$ : the former cannot be increasing around  $\theta_H$  if FOCs (7) is to be  
55 satisfied (see also footnote 13).

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5 in welfare due to a marginal increase in  $p_L$  being nil. In other words, welfare  
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7 maximization leads to an ‘aggressive’ behaviour by the low quality (public)  
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9 firm which expands output, driving the ‘high-quality’ indifferent consumer  
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11 (identified by  $\theta_H$ ) towards the right tail of the distribution.<sup>16</sup> This in turn  
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13 accounts for our Assumption 1(a) ruling out complete market coverage, as  
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15 this would imply  $\theta_H f(\theta_H) = 0$ , which is inconsistent with firm  $H$  maxi-  
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17 mizing its profits.<sup>17</sup> It also accounts for Assumption 1(b), which rules out  
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19 log-concavity: as already remarked, log-concave distributions (as well as the  
20  
21 uniform distribution) are inconsistent with a price equilibrium of this game.

22 Secondly, the condition  $\eta(\theta^\circ) < \frac{1}{1+k}$ , together with Assumption 1(b),  
23  
24 implies  $k < \alpha$ , i.e.

$$\frac{s_H}{s_L} > 1 + \frac{1}{\alpha}, \quad (9)$$

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28 which again is consistent with ruling out log-concavity ( $\alpha = 0$ ), and links  
29  
30 the width of the admissible quality spectrum to the degree of convexity of  
31  
32 the distribution of the consumers’ willingness to pay. In fact, the lower  $\alpha$ ,  
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34 the higher the lower bound on the (given) quality differential consistent with  
35  
36 the existence of a price equilibrium, while the upper limit case where  $\alpha = 1$   
37  
38 yields the constraint  $s_H > 2s_L$ .<sup>18</sup> Intuitively, this happens because *ceteris*  
39  
40 *paribus* the width of the quality spectrum affects the concavity of the firm’s  
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42 payoff: if the two products are close substitutes, the demand for (say) firm  
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44  $L$ ’s product will be very elastic, and indeed too much for firm’s  $L$  payoff  
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46 (welfare) function to be well behaved.<sup>19</sup> A minimum quality spread ensures  
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48 <sup>16</sup>Notice that, in the ‘ordinary’ case of both firms being profit-maximizers, both marginal  
49 consumers will be on the left of the mode when the density is symmetric and unimodal.  
50 See, e.g., Benassi et al. (2006).

51 <sup>17</sup>Given  $p_L = 0$ , firm  $H$  would set a price  $p_H$  such that  $\eta(p_H/\Delta) = 1$  so that  $\theta_H f(\theta_H) >$   
52  $0$ .

53 <sup>18</sup>In this case  $[1 - F(\theta)]^{-1}$  would be a convex function. Notice that if  $1 - F(\theta)$  is  
54 log-concave,  $[1 - F(\theta)]^{-1}$  is convex, but not viceversa.

55 <sup>19</sup>Take, e.g., the SOCs for firm  $L$  from footnote (9): since  $\pi(\theta_H)$  will be negative at  
56 equilibrium, this expression cannot be negative if  $\Delta$  is too small, and more generally, if  
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5 that vertical product differentiation survives in equilibrium, and that welfare  
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7 is not maximized by setting the price equal to marginal cost: the welfare  
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9 gain associated to complete market coverage is less than the welfare loss  
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11 associated with lower profits for both the high and the low quality firms.<sup>20</sup>

12  
13 Finally, from the existence proof reported in Appendix 6A, it turns out  
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15 that a *necessary* condition for existence is that  $\theta_L^* < \theta^\circ < \theta_H^*$ : i.e., along the  
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17 distribution of the willingness to pay,  $\theta^\circ$  is a sort of pivotal point around  
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19 which the positions of the marginal consumers arrange themselves. This in  
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21 turn implies that at equilibrium one necessarily has:

$$22 \quad \eta(\theta_H) - \eta(\theta_L) > \frac{1}{\frac{s_H}{s_L} \left( \frac{s_H}{s_L} - 1 \right)}, \quad (10)$$

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27 which means that the minimum (elasticity) distance between the two marginal  
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29 consumers (and hence the market for the low quality commodity) is higher,  
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31 the lower the quality ratio.<sup>21</sup> In some sense there is a trade-off between  
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33 how steeply demand rises with the willingness to pay going from  $\theta_L$  to  $\theta_H$ ,  
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35 and the quality differential: if the latter is low, equilibrium with vertical  
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37 differentiation requires that ‘middle-class’ consumers are very willing to pay  
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39 for even a modest quality premium.

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41 While Assumption 1 and the condition  $\eta(\theta^\circ) < \frac{1}{1+k}$  are sufficient to  
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43 ensure the existence of a price equilibrium, one is naturally interested in  
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45 looking at the circumstances under which such an equilibrium is unique.  
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47 Indeed, since we have to rule out log-concavity, we cannot use the well-

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products were too close welfare would be a convex function of  $p_L$ .

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51 <sup>20</sup>Under this respect, Assumption 1(b) plays a key role, as it amounts *ceteris paribus* to  
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53 a lower boundary on  $\eta(\cdot)$ : if the covered market is sufficiently elastic wrt the consumers’  
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55 willingness to pay, the marginal gain in welfare from a price reduction will be low.

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57 <sup>21</sup>Under our assumptions  $\eta(\theta)$  is monotonically increasing in the relevant interval: equa-  
58  
59 tion (10) then follows by noting that in equilibrium it must be  $\eta(\theta_L) < 1/(1+k)$  and  
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61 substituting for the definition of  $k$ . The income level  $\theta^\circ$  is such that  $\pi(\theta^\circ) = 0$ , i.e. the  
62  
63 elasticity of the density equals  $-1$ .

known result by Caplin & Nalebuff (1991) to the effect that log-concavity implies uniqueness. This is the issue we take up next.

### 3.2 Uniqueness of the price equilibrium

Our main result on uniqueness is the following:

**Proposition 2** *Let the assumptions of Proposition 1 hold, and assume further that:*

$$(a) k \leq \frac{\alpha^2}{1-\alpha^2}, \text{ and}$$

$$(b) \alpha \leq 1/2,$$

*then the price equilibrium is unique.*

**Proof.** See Appendix 7B ■

Both sufficient conditions can be read as strengthening the looser conditions which ensure existence. Indeed, it is easily seen that condition (a) amounts to:

$$\frac{s_H}{s_L} \geq \frac{1}{\alpha^2}, \quad (11)$$

and that, comparing this with constraint (9),  $1/\alpha^2 > 1+1/\alpha$  for  $\alpha \leq 1/2$ , i.e. condition (b). In this sense uniqueness is delivered when the lower bound on the quality differential is higher than that which is sufficient to ensure existence, so that broadly speaking for the price equilibrium to be unique, the quality levels should be sufficiently far apart, by an amount which is determined by the concavity of the distribution.

As a final remark, however, this broad conclusion should be qualified by a discussion of our twin assumptions of zero costs and given quality levels. Generally speaking, the former allows to focus on the firms' strategic choices as driven by demand, and hence to bring out more sharply the role of

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5 the distribution of the consumers' willingness to pay (Tirole 1988, Wauthy  
6 1996): the latter is in some sense the key to vertical differentiation – standard  
7 results point to the willingness to pay for quality upgradings as supporting  
8 equilibrium differentiation, even in the absence of differential quality costs.  
9 However, it should also be noticed that in many mixed markets one can  
10 arguably look at fixed costs as mainly quality driven, while variable unit  
11 costs may be thought of as constant (Lutz & Pezzino 2014); this is a situation  
12 where quality can be improved only by capital investments, and the focus  
13 on short-run price equilibria allows to normalize constant marginal costs to  
14 zero.<sup>22</sup> On the other hand, though the short-run focus is consistent with the  
15 assumption of given quality, the latter does beg the question of the likely  
16 outcome of a more general framework involving endogenous quality levels:  
17 since our model can be looked at as the first stage of a classical two-stage  
18 game, one naturally wonders what quality allocations would be supported  
19 by a perfect Nash equilibrium in prices and qualities.<sup>23</sup> Though no obvious  
20 answer presents itself, under the assumption of zero (low) marginal costs of  
21 quality upgrading, the high quality firm is likely to settle at the top quality  
22 level, as this allows to raise *ceteris paribus* the consumers' willingness to  
23 pay. By contrast, the public firm should tradeoff the welfare gain of higher  
24 quality in terms of larger market share, against the welfare loss in terms  
25 of lower consumers' surplus at the margin – the net effect of which is very  
26 likely to depend on the shape of the distribution of the willingness to pay.<sup>24</sup>

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<sup>22</sup>A similar assumption of constant unit costs with respect to quantity can be found in models with covered markets, such as Grilo (1994), though in general the covered-market case allows also to work with a richer setting – like cost functions differing across firms, and unit costs increasing in quantity and quality (Laine & Ma 2015).

<sup>23</sup>In even more general terms, one could also wonder how entry dynamics can be affected by the presence of a welfare maximizing public firm in the framework of uncovered-market vertical differentiation, where 'natural oligopolies' are likely to be found (Shaked & Sutton 1983).

<sup>24</sup>Recall that in the standard profit maximizing framework with zero costs, the high quality firm settles at the top quality (Benassi et al. 2006). As to the public firm, a

#### 4 An example: the Pareto distribution

In this Section we apply our model to the case of a Pareto distribution. While clearly limited to a specific case, we believe that this example can serve as an illustration of the way the distribution of the willingness to pay affects equilibrium outcomes. Suppose then that the consumers' willingness to pay is distributed as a Pareto distribution of the second kind (Johnson et al. 1995), so that the density and the cumulative distributions are respectively  $f(\theta, \gamma) = \gamma(1 + \theta)^{-(1+\gamma)}$  and  $F(\theta, \gamma) = 1 - \frac{1}{(1+\theta)^\gamma}$ , defined over the support  $\Theta = [0, \infty)$ , where  $\gamma > 1$  is a given parameter. It is then easily seen that  $\pi(\theta, \gamma) = \frac{1-\gamma\theta}{1+\theta}$  and  $\eta(\theta, \gamma) = \frac{\gamma\theta}{1+\theta}$  such that Assumption 1 is satisfied. In particular, Assumption 1(b) holds for any  $\alpha = 1/\gamma$ , so that  $1 - F(\theta)$  is  $\rho$ -concave with  $\rho = -1/\gamma$ ; notice also that in this case we have  $\tilde{\theta} = \frac{1}{\gamma-1} > \theta^\circ = \frac{1}{\gamma}$  such that  $\eta(\tilde{\theta}) = 1$  and  $\pi(\theta^\circ) = 0$ . In addition,  $\eta(\theta^\circ) = \gamma/(1 + \gamma)$ , such that the condition  $\eta(\theta^\circ) < 1/(1 + k)$  set out in Proposition 1 reduces to  $k < 1/\gamma$ . It should be remarked that in this framework  $\gamma$  is an inverse parameter of first order stochastic dominance, so that higher values of  $\gamma$  support lower mean values of the consumers' willingness to pay.<sup>25</sup>

We now perform a numerical simulation with different values of  $\gamma$ , say between  $\gamma = 2$  and  $\gamma = 3$ , to see the way a shift on the distribution of the willingness to pay affects equilibrium prices. To do so we set  $k = 1/8 < 1/\gamma$ , which is equivalent to  $s_H/s_L = 9$ . On the basis of Proposition 2 (and condition (11)), this quality ratio delivers a unique equilibrium for  $\alpha \geq 1/3$ , which is verified as  $\alpha = 1/\gamma \geq 1/3$ , while  $\alpha \leq 1/2$  as required by sufficient condition (b) of the same Proposition. Within this framework, we perform

generic consumer's surplus is given by  $\theta s_L - p_L$ : while higher  $s_L$  means for given prices lower  $\theta_L$  (and hence, widening the pool of consumers), it also means that many additional consumers will be "poor" (whose contribution to welfare in terms of surplus will be low).

<sup>25</sup>Mean willingness to pay is  $\mu = 1/(\gamma - 1)$ .

three simulations for  $\gamma = 2$ ,  $\gamma = 5/2$  and  $\gamma = 3$ . Table 1 reports equilibrium values of the positions of the marginal consumers,  $\theta_H^*$  and  $\theta_L^*$ , obtained for these different values of  $\gamma$ .

	$\gamma = 2$	$\gamma = 5/2$	$\gamma = 3$
$\theta_H^*$	0.93629	0.61392	0.45498
$\theta_L^*$	0.25484	0.25318	0.24011

**Table 1: Equilibrium marginal values of willingness to pay**

From Table 1, we see that in the case of a Pareto distribution, an increase in  $\gamma$  (i.e. lower mean income) leads to a leftward shift of both marginal consumers, together with a decrease in the distance between them. This would point to decreasing income leading to stiffer competition, which is confirmed by Table 2 below, such that relative prices decrease unambiguously with higher values of  $\gamma$ .

	$\gamma = 2$	$\gamma = 5/2$	$\gamma = 3$
$p_H^*/p_L^*$	30.392	20.399	16.159
$\frac{p_H^*}{s_H^*}$	0.86057	0.57384	0.43111
$\frac{p_L^*}{s_L^*}$	0.25484	0.25318	0.24011

**Table 2: Equilibrium relative and hedonic prices**

In Table 2 we also report the behaviour of hedonic prices, which would suggest that lower average income – at least in this example – puts a downward pressure on the price per ‘unit of quality’, which appears to be stronger for the (profit-maximizing) high quality firm. Both relationships (Table 2) are apparently monotone, suggesting that the competition of the public sector (or of the regulated industry) will be more intense in case of lower average

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5 income (i.e., the larger the parameter  $\gamma$ ).  
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## 8 **5 Conclusions**

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11 Starting by informal arguments that public firms competing with private  
12 firms rely on distributional concerns about inequality, and by formal reason-  
13 ing that the distribution of the willingness to pay affects the firms' equilib-  
14 rium choices, in this paper we show that a price equilibrium in a vertically  
15 differentiated mixed duopoly with uncovered market exists, if the quality  
16 spectrum is wide enough *vis à vis* a measure of the convexity of the dis-  
17 tribution of the consumers' willingness to pay, and that such equilibrium is  
18 unique if this sufficient condition is tightened. In particular, we show that for  
19 a price equilibrium to exist the distribution of the willingness to pay cannot  
20 be logconcave, and that sufficient conditions for existence and uniqueness  
21 place a lower bound which is higher, the higher the given convexity bound  
22 on the income distribution.  
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26 By way of example, we apply our basic model to a Pareto distribution,  
27 and find that a decrease of average income is (broadly speaking associated)  
28 with higher competitive pressure from the public firm, as signaled, e.g., by a  
29 decrease of the distance between the marginal willingness to pay for high *vs*  
30 low quality goods, and of relative prices – the price of high quality decreases  
31 relative to that of low quality; also, the decrease in hedonic prices appears  
32 to be stronger for the high quality goods. Though obviously constrained  
33 by the specific form of this example, these results confirm that assumptions  
34 about the distribution of the consumers' willingness to pay do play a key  
35 role in assessing the working of vertically differentiated markets.  
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39 Finally, while our results are based on assumptions (notably zero costs  
40 and given quality levels) which are obviously restrictive (though arguably  
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5 reasonable), our framework points to the important general question of com-  
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7 paring the overall performance of ‘mixed’ *vs* ‘pure’ oligopoly within vertical  
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9 differentiated markets. If this sets a general agenda for future research, such  
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11 a comparison should however take into account that the distributional as-  
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13 sumptions required to support the existence of a price equilibrium in the  
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15 ‘mixed’ case, are likely to be different from those required to support a per-  
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17 fect equilibrium in prices and qualities in the ‘pure’ case – as indeed is made  
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19 clear by the standard textbook assumption of uniform income distribution  
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21 being inconsistent with having one welfare maximizing firm.  
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yielding:

$$(1 + \alpha) \eta(\theta_H^*) \left( \frac{1}{1 - \eta(\theta_H^*)} + \frac{1}{\alpha^2} \right) < \frac{2\eta(\theta_H^*) - 1}{1 - \eta(\theta_H^*)} + \frac{1}{1 - \eta(\theta_H^*)} + \frac{1}{\alpha^2},$$

from which, after rearrangement,

$$\frac{(1 - \alpha) \eta(\theta_H^*)}{1 - \eta(\theta_H^*)} + \frac{1}{\alpha^2} > \frac{(1 + \alpha) \eta(\theta_H^*)}{\alpha^2}. \quad (\text{B.3})$$

We now invoke the condition  $k \leq \alpha^2 / (1 - \alpha^2)$ , which is consistent with  $k < \alpha$ , as  $\alpha^2 / (1 - \alpha^2) < \alpha$  for  $\alpha \leq 1/2$ . Under this assumption,  $1 - \eta(\theta_H^*) < \alpha^2$ : indeed, this is equivalent to  $\eta(\theta_H^*) > 1 - \alpha^2$ , which is true as  $\eta(\theta_H^*) > 1 / (1 + k)$  and:

$$\frac{1}{1 + k} - (1 - \alpha^2) = \frac{-k + \alpha^2 + \alpha^2 k}{1 + k} > 0.$$

There follows that  $\frac{(1 - \alpha) \eta(\theta_H^*)}{1 - \eta(\theta_H^*)} > \frac{(1 - \alpha) \eta(\theta_H^*)}{\alpha^2}$ , so that (b.3) holds if:

$$\frac{(1 - \alpha) \eta(\theta_H^*)}{\alpha^2} + \frac{1}{\alpha^2} > \frac{(1 + \alpha) \eta(\theta_H^*)}{\alpha^2},$$

i.e.

$$1 > 2\eta(\theta_H^*) \alpha,$$

which is surely true for  $\alpha \leq 1/2$ , as  $2\eta(\theta_H^*) \alpha < 1$ . ■

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### Highlights

- We study the existence of a price equilibrium in a vertically differentiated mixed duopoly with uncovered market exists when a welfare-maximizing public firm producing low quality goods competes against a profit-maximizing private firm producing high quality goods.
- We show that for a price equilibrium to exist the distribution of the willingness to pay cannot be log-concave, and that sufficient conditions for existence and uniqueness place on the quality spectrum a lower bound, which is higher, the higher the given convexity bound on the income distribution.
- We apply our model to a Pareto distribution, and find that a decrease of average income is associated with higher competitive pressure from the public firm, and the decrease in hedonic prices appears to be stronger for the high quality goods.

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