

Article

Task-Oriented Structural Health Monitoring of Dynamically Loaded Components by Means of SLDV-Based Full-Field Mobilities and Fatigue Spectral Methods

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Abstract: Expected lives of mechanical parts and structures depend upon the environmental conditions, their dynamic behaviours and the task-oriented spectra of different loadings. This paper exploits contactless full-field *mobilities*, estimated by Scanner Laser Doppler Vibrometry (SLDV), in the real manufacturing, assembling and loading conditions of the thin plate tested, whose structural dynamics can be described in broad frequency bands, with no distorting inertia of sensors and no numerical models. The paper derives the *mobilities* into full-field strain Frequency Response Functions (FRFs), which map, by selecting the proper *complex-valued* broad frequency band excitation spectrum, the surface strains. From the latter, by means of the constitutive model, dynamic stress distributions are computed, to be exploited in fatigue spectral methods to map the expected life of the component, according to the selected tasks' spectra and the excitation locations. The results of this experiment-based approach are thoroughly commented in sight of non-destructive-testing, damage and failure prognosis, Structural Health Monitoring, manufacturing and maintenance actions.



Keywords: structural health monitoring; failure prognosis; SLDV-based full-field FRFs; experiment-based receptances; structural dynamics; fatigue spectral methods; non-destructive testing

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1. Introduction

Any task – in which the mechanical components are used – brings its own dynamic signature, in terms of specific spectrum of loadings, the latter being external – as lumped forces or distributed pressures – or internal – as the result of motion laws or dynamics with accelerations and inertia forces. Therefore, it can be said that the specific duty a component, a mechanism or a structure, do fulfil brings a task-dependent excitation, characterised by its specific spectrum and location of energy injection. But, also, that the expected life of each mechanical part depends on both dynamic signature and location of loadings, together with the structural dynamics of the component that give responses to the specific excitation. Structural Health Monitoring (SHM) needs therefore to follow both the structural dynamics and the excitation, as they can change in time and can bring to new expected lives. In this paper, the structural dynamics is explored in one of the most adherent-to-reality approaches that is currently available, especially for lightweight structures, which starts from the broad frequency band optical full-field measurements. In this way, the experiment-based investigation deals exactly with the realisation of the structure, with the specific manufacturing and mounting conditions, with the materials & damping properties and broad band loading levels, while the above properties are hardly modellable in a numerical

tool such as a Finite Element Model (FEM), also when properly tuned as in [1–3], but still with approximation residuals. Instead of running any Experimental Modal Analysis (EMA) as in [4,5] – also in its Experimental Full-Field Modal Analysis (EFFMA) extension in [3] – the experiment-based *receptances* – or *displacement over force FRFs* – are used to avoid any modal identification uncertainty or modal base truncation errors. Thus, *full-field receptances* – here obtained from SLDV-based *mobilities*, or *velocity over force FRFs* – retain the best description of the actual operative deformation shapes due to the structural dynamics, especially in the spatial domain, later used to derive the dynamic strains and feed the fatigue life estimations with increased spatial sampling and detail. Due to their intrinsic nature, *full-field receptances* may easily present different frequency-domain relations between response displacements and excitations, depending on the energy injection locations. In particular, a *complex-valued* representation of the structural dynamics is ideal to retain all the real characteristics of the test. To exploit such a refined structural dynamics, also the excitation needs to be of a *complex-valued* nature, as it easily comes from the signal processing of any force cell data in a test: a broad frequency band complete spectrum, with *complex-valued* parts, to retain all the *complex amplitude* and *phase* relations in the frequency range of interest, without simplifications nor assumptions. Already introduced with the aid of *image-based receptances*¹, here the approach exploits instead the *non-synchronous scanning* of SLDV to maintain the following advantages, particularly relevant for the structural dynamics of lightweight components: (i) the real structural dynamics comes directly from testing, without the need of any tuning of the numerical modelling technique, e.g. FEM; (ii) the spatially dense grid of measurements overtakes the effort of finding the best location of traditional sensors, e.g. strain gauges; (iii) the measurements are contactless, without any distorting inertial contribution from attached sensors and their cabling. The expected task-oriented fatigue-life mapping, coming from SLDV-based *full-field receptances*, retains therefore the characteristic of the specific realisation of the component under test with good detail in a broad frequency band and spatial domain.

Among the optical – therefore contactless, not distorting because of sensors & cabling inertia – technologies, SLDV has permitted to add degrees of freedom (dofs) and discrete spatial resolution in established Noise and Vibration Harshness (NVH) approaches in broad frequency ranges, but it can be better defined as *non-native* full-field measurement technique, due to its *scanning* nature. SLDV has therefore become a reference in contactless measurements for standing vibrations, but not for transients, not sensing synchronously the whole mapping. SLDV is the full-field technology here adopted for the real-life *mobility* testing of the vibrating structural part, in order to assess its applicability to these extended data processing for fatigue life estimations. Instead, the techniques based on the sensing of photons across the whole sensor matrix can be called as optical *native* full-field measurements, or also *image-based*, with an even higher dofs' density and field-wise accuracy/continuity than that of SLDV, as proved extensively in [3,13–18], especially with stroboscopic ESPI. Although the latter is limited to standing vibrations in the frequency domain, ESPI can measure extremely detailed fields with a noise floor of the order of 50 nm and a ceiling of some μm , exploiting the interference phase maps and the precision of the constant wavelength of the coherent laser light, modulated up to several kHz in stroboscopic acquisitions. While quality-wise unsurpassed, for broad frequency band measurements ESPI has heavy drawbacks: in the time consuming tests, in coherent lighting requirements, in recording size and environmental vibration insulation needs, all hardly met outside a specific research laboratory. Another *native* full-field approach is that of DIC with good spatial detail (generally better than SLDV, but lower than ESPI) in the time

¹ See the approach deployed in [6–10] from Electronic Speckle Pattern Interferometry (ESPI)-based *receptances* and in [11,12] from Digital Image Correlation (DIC)-based ones.

histories of displacement maps, once proper high-speed cameras are used related to the frequency band of interest, although generally more limited in the upper frequency than SLDV; transients are acquired in the time domain with ease; furthermore, the evaluation of multiple series of correlated displacements might be computationally intensive. A relevant advantage of DIC systems is certainly in their usage flexibility – thus of increasing interest also in industrial environments – as some in-situ applications have proved, as in [19–23].

Concise notes about the author’s experience in optical full-field measurements follow. The first hints of the TEFFMA project appeared in 2014², consolidated by more enhanced proceedings in 2015³. The first works were briefly gathered in [17]; in [18], instead, extended notes on the *full-field receptances’ evaluation* followed; in [3] model updating exploited the full-field eigenshapes coming from the EFFMA. The quality of *native* (ESPI and DIC) full-field datasets – in broad frequency band dynamic testing – was underlined in [24]. Relevant achievements for rotational⁴ and strain FRF high resolution maps were compared in [6]. A risk index was firstly introduced in [7] to locate the areas mostly exposed to failure in a part subjected to dynamic load, while in [28] a damaged composite panel was tested in the same perspective. Recently, in [8], ESPI-based risk map variability was addressed by *real-valued* amplitude excitation signatures; the effect of the energy-injection location was investigated in [9]; both aspects were gathered in [10]. Although reduced – by interpolation in both spatial and frequency domains – to the more moderate resolutions of the SLDV references in the TEFFMA project, DIC-based full-field *receptances* were used instead in [11] for the risk index mapping. Instead, in [12] the *raw* datasets from DIC – with no numerical residuals due to the topology transforms and interpolations – boosted the risk index analyses; furthermore, the latter used *complex-valued* coloured noises for force signals, with potential randomness in the complex amplitude and phase. The contribution of full-field *receptances* was investigated also in vibro-acoustics⁵, with interest on the excitation dynamic signature and on the energy injection location. The author proved how – nowadays – full-field techniques already provide, even if not fully explored, clear improvements in EFFMA, but even more in advanced model updating, derivative calculations (rotational dofs, strains, stresses, risk index maps) and vibro-acoustics, thanks to the higher continuity and consistency of the fields in the datasets when compared to more traditional sensors. Therefore, a highly reliable experiment-based behavioural modelling of complex components – in their manufacturing and mounting state – may come from full-field optical *mobilities*, the latter retaining any modally dense structural dynamics and being able to deal with any dynamic signature of the excitation, under the linearity assumption.

² In Proceedings of the ISMA2014 including USD2014 - International Conference on Noise and Vibration Engineering, Leuven, Belgium, September 15–17, KU Leuven, 2014; see in *Dynamic testing: methods and instrumentation*, *On the estimation of frequency response functions, dynamic rotational degrees of freedom and strain maps from different full field optical techniques*; see in *Operational modal analysis, On the role of spatial resolution in advanced vibration measurements for operational modal analysis and model updating*.

³ In Proceedings of the ICoEV2015 International Conference on Engineering Vibration, Ljubljana, Slovenia, September 7–10, Univ. Ljubljana & IFToMM, 2015, symposium *Full Field Measurements for Advanced Structural Dynamics: Model updating from full field optical experimental datasets; Comparative studies on full field FRFs estimation from competing optical instruments; Accurate FRFs estimation of derivative quantities from different full field measuring technologies; Full field experimental modelling in spectral approaches to fatigue predictions*.

⁴ The reader is invited to explore in [1,4,5,25–27] the relevance that rotational dofs have in reliably modelling structures with high complexity and many components, although most times rotational dofs are completely disregarded because of their measurement burden.

⁵ See specifically [29,30], *On the approximation of sound radiation by means of experiment-based optical full-field receptances*, *Experiment-based Optical Full-field receptances in the Approximation of Sound Radiation from a Vibrating Plate*, and *On the use of full-field receptances in inverse vibro-acoustics for airborne structural dynamics*.

Coupled with the proper loading spectrum, the experiment-based mapping here proposed can provide the distribution of structural responses in the *whole* sensed domain – therefore, on the *macro-scale*, with respect to the linearity condition under which the *receptances* were estimated. This *macro-scale* distribution of results might be underrated by many fatigue and crack propagation researches. The latter are, instead, traditionally focused on what happens in a specific location, therefore, on the *micro-scale*, starting from an assumed dynamic stress distribution (e.g. [31–36]). The knowledge, about the potentially changing *full-field receptances* and loading spectra during the monitoring of an ageing structure, can become the key-tools of SHM inquiries, because this knowledge keeps the *macro-scale* mapping of the strain-stress distribution updated, on which to evolve the fatigue life estimations. Only then, the many works about failure progression in each specific material⁶ can find full deployment in a complete *macro-scale* mapping of the damage inflicted by the whole spectrum of the retained dynamics. In this paper, the fatigue life mapping is based on the suggestions from [43–45] about random loading, and – more specifically – from [46–50] about the fatigue spectral approaches in the frequency domain. The literature contemplates many other approaches⁷, but they go beyond the task of this work. But, it can be clearly underlined that any other estimation procedure – rather than what here formulated for the provided examples – of component’s fatigue life, or of failure progression, can be extended, by means of *full-field receptances* from experimental optical techniques, from the *micro-scale* – in which each estimation procedure was developed – to the *macro-scale* detailed mapping. As introduced in [7–12] supported by ESPI and DIC technologies, and later focused in Section 3 with examples exploiting SLDV-based datasets, the mapping of fatigue life estimations is greatly affected by the loading spectra⁸ and energy injection points, of which SHM is expected to be aware. SHM indeed can benefit from this extended knowledge into failure changing scenarios – together with the inspections, again also provided by full-field experiment-based Non-Destructive Testing (NDT) techniques as in [28,36,80–84] and in footnote 9, of externally added flaws from unexpected accidents (e.g. [41,42]) or with recordings of unpredicted/excessive loading – especially in better managing the life-long maintenance, inspections and operative costs. The NDT results might be matched, as already suggested in [8], with the specific task-oriented failure maps, to assess the risk related to each revealed defect. Potentially, therefore, such an enhanced SHM should be able to take better informed decisions on the maintenance actions, or allowed running conditions, for a pristine, but also a defected, component undergoing the specific task. This paper gives some further applicability proofs for SHM of SLDV – only restrained into native references, without interpolation errors from the topology transforms – in evaluations of dynamic strains & stresses, as well as in estimations of component’s fatigue life. The latter benefit of highly accurate derivative approaches – advanced in [6] – to first obtain dynamic strains, then stresses with a constitutive material model, and later the related quantities for the fatigue spectral methods here adopted.

This paper is organised as follows. After this introduction in Section 1, the materials and methods in Section 2 – enriched by Appendixes A–D – help to fully comprehend the theoretical background, which brought to the results proposed in Section 3 and further

⁶ Dependent on the defect types, e.g., as in [31–35,37]; on the micro-structure and sizes, e.g., as in [38]; on the ageing, e.g., as in [39]; on the external damaging factors, such as corrosion, e.g., as in [40], hitting debris or impacting objects, e.g., as in [41,42]. However, this broad discussion – about the *micro-scale* factors – cannot be part of this paper.

⁷ E.g., those based on strains [51,52], on strain energy [53–55], on loading-tailored criteria [56–63], and on safety regulations in design [48,64–67].

⁸ Pay attention also to the researches made by means of frequency based substructuring (FBS) [27,68–70], transfer path analysis (TPA) [71–76] and multi-input-multi-output (MIMO) vibration testing [77–79].

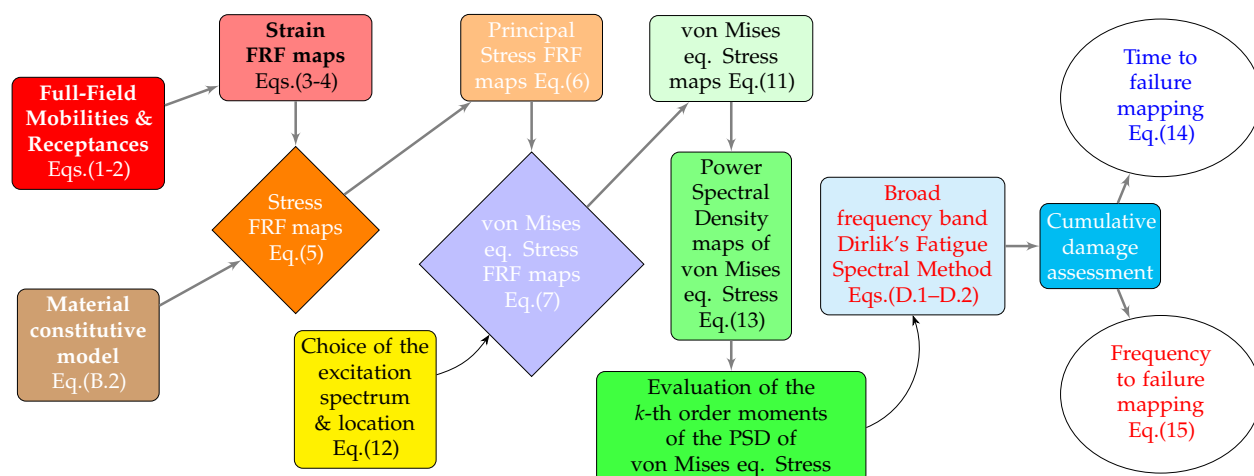


Figure 1. Schematic summary of the steps in the proposed experiment-based full-field approach to evaluate the cumulative damage by fatigue spectral methods as in Section 2.

discussed in section 4, before the conclusions in Section 5. “Abbreviations” finally gathers the nomenclature used in the manuscript without explicit explanations.

2. Materials and Methods

The backbone of the experiment-based approach is sketched in Figure 1, as here detailed. Specifically, in Section 2.1 the estimation of *experiment-based full-field FRFs* is addressed, with details on the testing set-up and data processing. Section 2.2 – with the aid of the three Appendixes A–C to complete the mathematical details that could have made the text heavier for the reader – gives the tools to estimate maps of full-field strains and stresses FRFs, of Principal stresses and of the von Mises equivalent stresses. Section 2.3 deals with the modelling of excitation forces by means of a *complex-valued spectrum*. Section 2.4, together with Appendix D, gives the tools to implement the Dirlik’s semi-empirical fatigue spectral method.

2.1. Full-Field SLDV-Based Mobilities & Receptances from the TEFFMA Project

At the University of Technology in Vienna, Austria, the author led the TEFFMA fundamental research project, where SLDV was taken as the reference technology in the comparison among state-of-the-art optical *native* full-field approaches (ESPI and DIC), in complex structural dynamics and NVH studies. Note that the TEFFMA project has useful roots for this work in ESPI-based testing⁹, specifically for enhanced structural dynamics and fatigue life assessments, by means of spectral methods. During the TEFFMA project, as can be seen in Figure 2, the measurement instruments (of SLDV, high-speed DIC and stroboscopic ESPI) were at the core of the activities; the test rig was mounted in a dedicated seismic floor room, on top of an anti-vibration air-cushioning table. These activities were favoured by the technicians of the mechanical & electronic workshop for customised tools. More detailed notes on the test campaign can be found in [3,6,18,24].

The common set-up – to make the unique comparison in measurements for NVH and structural dynamics of the 3 full-field technologies in the TEFFMA project – came after careful considerations on the limits of each approach. This unique set-up required a very

⁹ The *Speckle Interferometry for Industrial Needs* Post-doctoral Marie Curie Industry Host Fellowship project HPMI-CT-1999-00029 was held at Dantec Ettmeyer GmbH, Ulm, Germany. In particular, for the main achievements, see: [Full field ESPI measurements on a plate: challenging experimental modal analysis](#), [Fatigue life assessment by means of full field ESPI vibration measurements](#), [Full field ESPI vibration measurements to predict fatigue behaviour](#) and [Damage location assessment in a composite panel by means of electronic speckle pattern interferometry measurements](#).

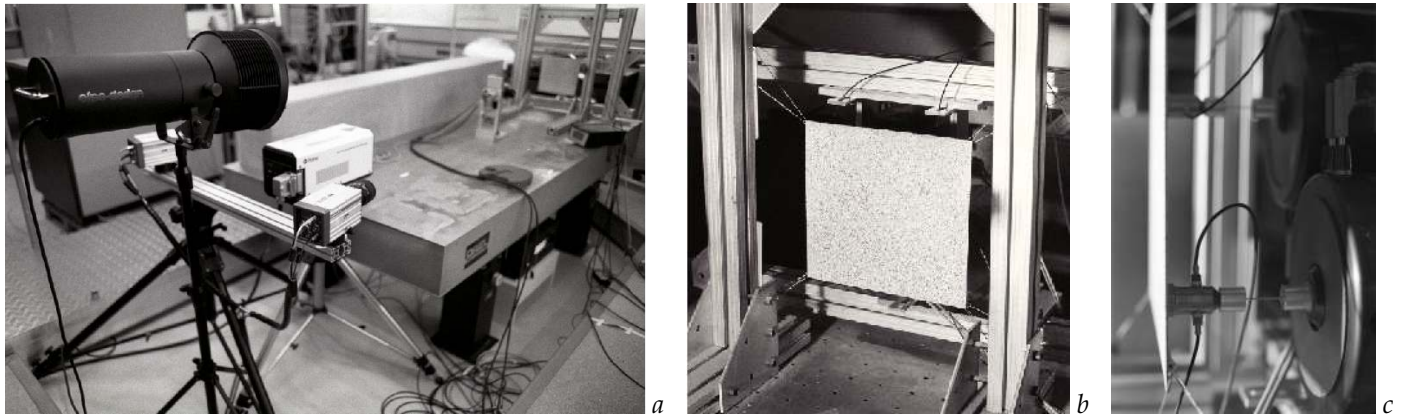


Figure 2. Full-field optical measurement instruments set-up in front of the specimen on the anti-vibration table with shakers on the backside: the instruments in (a) with SLDV head close to the border of the table, the restrained thin plate in (b) and the 2 shakers in (c).

fine tuning of the details, which brought the different approaches to a feasible performance overlapping. The same structural dynamics revealed itself in the time domain for DIC, but in the frequency domain for ESPI and SLDV. The qualitative comparison seemed certainly promising already from each instrument software, but certainly not precisely super-imposable. To obtain quantitative comparisons in the TEFFMA project – not here of specific interest – accurate topology transforms were designed and carefully coded, in order to have all the 3 different techniques’ datasets cast on the same physical (geometry and frequency) references, as hinted in the following pictures.

2.1.1. Direct Characterisation by means of Full-Field *Mobilities*: a Brief Recall

The *mobility* matrix $H_v(\omega)$ follows the formulation for the H1 estimator - noise only in the outputs - found in [4,5]: it is estimated as the spectral relation between multi-output velocities and multi-input forces. In this full-field extension, the inputs are just 2 shakers, while *thousands* (2907) of responses are acquired on the whole surface of interest. The structural domain is represented by the vibrating surface S of the plate, whose q -th point has global coordinates q_q , while q represents the whole-field’s coordinates of the active structural domain. Each element of the *mobility* matrix $H_v(\omega)$ can be expressed, in *complex-valued* representation, as:

$$H_{v_{qf}}(\omega) = \frac{\sum_{m=1}^N S_{\dot{X}_q F_f}^m(\omega)}{\sum_{m=1}^N S_{F_f F_f}^m(\omega)} \in \mathbb{C} \quad (1)$$

where $\dot{X}_q$ is the *output velocity* at q -th dof induced by the *input force* F_f at f -th dof, while $S_{\dot{X}_q F_f}^m(\omega)$ is the m -th cross-power spectral density between input and output, $S_{F_f F_f}^m(\omega)$ is the m -th auto-power spectral density of the input, evaluated in N repetitions.

The 3D *receptance map* $H_d(x, y, \omega) \in \mathbb{C}$ is easily obtained in the frequency domain, with i as the *imaginary unit*, as:

$$H_d(\omega) = -i/\omega H_v(\omega) \in \mathbb{C}. \quad (2)$$

2.1.2. Notes on the Plate under Test in the TEFFMA Project

The specimen was a thin 7075-T6 aluminium alloy¹⁰ rectangular plate – flat and in pristine conditions to avoid confusion of static deformations with dynamic Operative Deflection Shapes (ODSs) – with external dimensions of 250 mm for the width, 236 mm for

¹⁰ See [58] for the relevant properties, such as the elastic modulus $E = 71.7$ GPa and the Poisson ratio $\nu = 0.33$.

the height and 1.5 mm for the thickness s . It was fixed – to restrain any excessive rigid-body movement that could have impaired the ESPI technique – by wires to the rigid frame in Figures 2a, b, bolted on the air-spring optical table. A random noise pattern of black & white droplets was sprayed on the face towards the instruments, accordingly to DIC requirements. Two properly positioned shakers, as in Figure 2c, excited orthogonally the plate from the back side. At each shaker-structure interface the force signal from the impedance heads was recorded for the evaluation of the *mobility* FRFs $H_v(\omega)$ in Equation (1). Shakers were requested by the phase-shifting procedures in ESPI-based measurements, but also fully compatible with DIC and SLDV. The shakers were driven singularly, one at a time, by an external sine-waveform generator in ESPI, instead by LMS Test.Lab system in DIC and SLDV acquisitions.

2.1.3. Processing Notes about the Estimated Full-Field *Mobility* FRFs

By following the formulation of Equations (1),(2), accurate *mobility/receptance FRF* maps were estimated from all the techniques in the TEFFMA project. It was possible to appreciate, especially for ESPI, the enhanced consistency and continuity of the spatial data – with clean ODSs and sharp nodal lines – of the sensed structural dynamics, as also confirmed by the high-quality *Coherences* (see [4,5]). For each of the two shakers in different points of the structure, the SLDV-based dataset here retains $N_q=2907$ dofs ($N_q=57 \times 51$, about 4.39 – 4.64 mm grid spacing in the structural domain, taking into account that the grid was inside the plate borders of about 2 mm, thus sized around 246×232 mm); each contains 1285 frequency lines, detached by a frequency space of 0.78125 Hz; the frequency range is limited to [20-1023] Hz, the lower end as imposed by ESPI technique. The number of averages is $N=50$. SLDV-based datasets are here employed without any down-sampling nor interpolations due to missing topology transforms, in their *raw* estimation. Instead, the higher spatial resolution *image*-based techniques of the TEFFMA project (DIC & ESPI) are forced to contain additional computational noise from the topology transforms, if cast to the SLDV references.

2.2. Deriving New Quantities from Full-Field Receptances

Nowadays the optical full-field techniques bring high continuity and quality (see [3,17,18]) to the *receptance* fields, also obtainable from SLDV *mobilities* as in Equation (2). This allows the exploration (as from [6,8,30]) of novel derivative quantities, but limited to the plate surface. Note that, during the estimations of the TEFFMA project, only the *out-of-plane*-(z) displacements were above the sensitivity level of all the techniques, ESPI included – albeit the most sensitive. This means that the *in-plane* (x, y) components of the *receptance map* $H_d(x, y, \omega)$ were null.

2.2.1. Strain FRFs

In the case of full-field optical measurements – accurately mapping the whole spatial domain of interest with nearly-continuous data fields, but limited to the surface of the vibrating plate – the evaluation of the *total strain FRF tensor* $H_\epsilon(x, y, \omega) \in \mathbb{C}$ can be made, as detailed in Appendix A directly from the *receptance map* $H_d(x, y, \omega)$ in Equation (2), thanks to the functional relation $\mathcal{D}_\epsilon()$ in Equation (A.4), here recalled as:

$$H_\epsilon(x, y, \omega) = \mathcal{D}_\epsilon(H_d(x, y, \omega)) \in \mathbb{C}. \quad (3)$$

The diagonalisation¹¹ of $H_\epsilon(x, y, \omega)$, made at each frequency line, further brings the complex-valued *Principal Strain FRF maps* $H_{P_\epsilon}(x, y, \omega)$ ¹², which retain the phase relations from the excitations, as:

$$H_{P_\epsilon}(x, y, \omega) = \text{diag}[H_{P_{1\epsilon}}, H_{P_{2\epsilon}}, H_{P_{3\epsilon}}](x, y, \omega) \in \mathbb{C}. \quad (4)$$

The experiment-based characterisation of the strain FRF distribution, over the surface of interest, turns, therefore, to be highly effective in the frequency and spatial domains.

2.2.2. Estimating the Stress FRFs

The *Stress FRF tensor* $H_\sigma(x, y, \omega) \in \mathbb{C}$ can be evaluated only indirectly from the *total Strain FRFs* in Equation (3), once a functional relation $\mathcal{G}_{cm}()$ is used as the constitutive model¹³ of the specific material of the component. Note that the components of the *Stress*

¹¹ For further details see also Appendix C.

¹² $H_{P_{1\epsilon}}(x, y, \omega)$, $H_{P_{2\epsilon}}(x, y, \omega)$, $H_{P_{3\epsilon}}(x, y, \omega)$ are the *First*-, the *Second*-, and the *Third-Principal Strain FRF eigenvalue* maps, in descending magnitude order. In this testing, $H_{P_{3\epsilon}}(x, y, \omega)$ was orders of magnitude below $H_{P_{1\epsilon}}(x, y, \omega)$ and $H_{P_{2\epsilon}}(x, y, \omega)$, instead comparable.

¹³ Here the linear isotropic version in Appendix B is used.

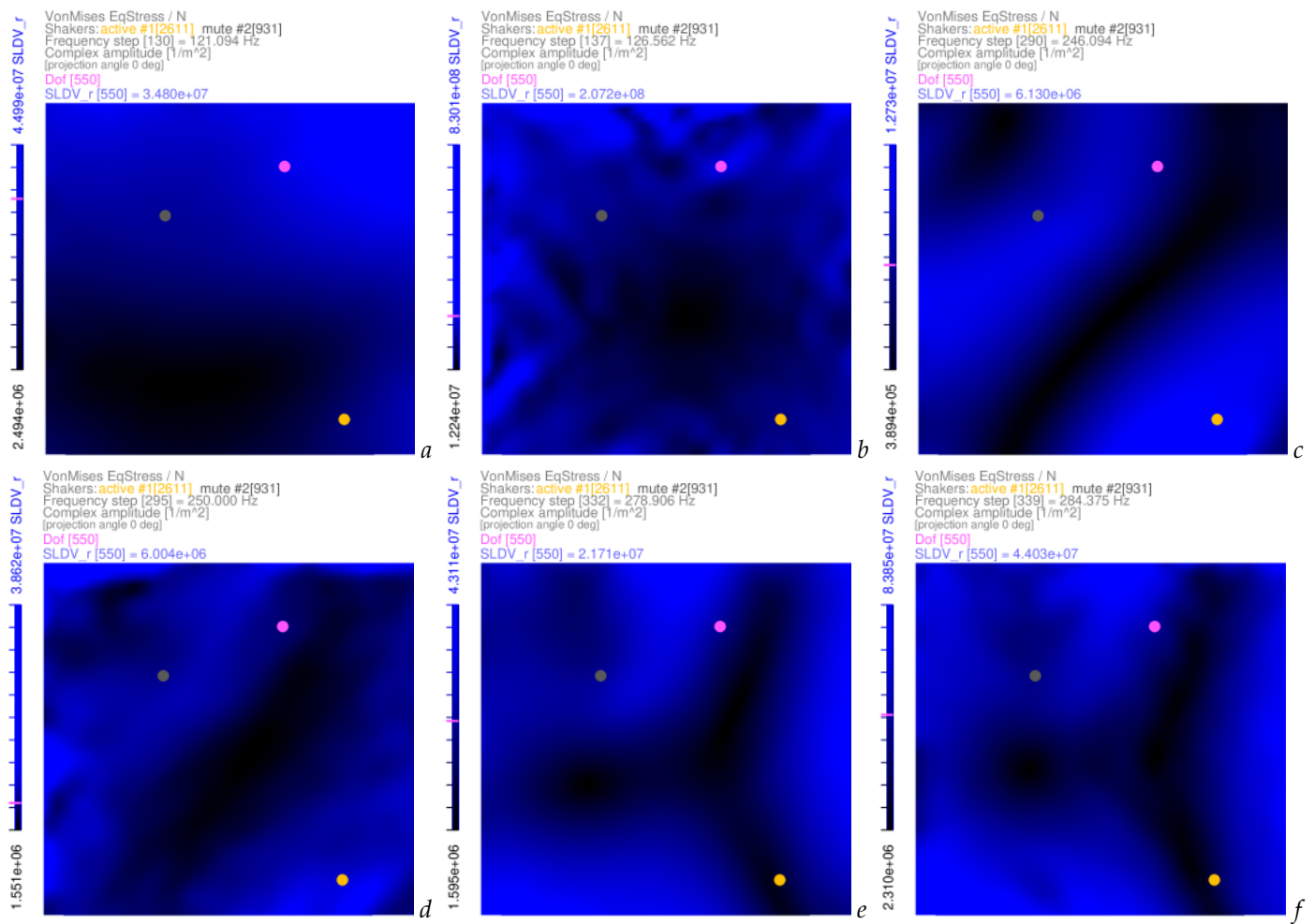


Figure 3. Examples of SLDV-based *von Mises equivalent stress FRF maps* from optical techniques in Equation (7) from shaker 1: at 121 Hz in (a), at 127 Hz in (b), at 246 Hz in (c), at 250 Hz in (d), at 279 Hz in (e), at 284 Hz in (f).

FRF tensor $\mathbf{H}_\sigma(x, y, \omega)$ are expressed in $[1/\text{m}^2]$ and can be here obtained from Equation (B.2) as:

$$\mathbf{H}_\sigma(x, y, \omega) = \mathcal{G}_{cm}(\mathbf{H}_\epsilon(x, y, \omega)) = \mathcal{G}_{cm}(\mathcal{D}_\epsilon(\mathbf{H}_d(x, y, \omega))) \in \mathbb{C}. \quad (5)$$

The same approach can be followed with any specific material (anisotropic and locally linearised included), by means of the deployment of the proper functional relation $\mathcal{G}_{cm}()$.

Once the *experiment-based Stress FRF tensor* $\mathbf{H}_\sigma(x, y, \omega)$ in Equation (5) is diagonalised, the *Principal Stress FRF maps* $\mathbf{H}_{P\sigma}(x, y, \omega)^{14} \in \mathbb{C}$ are obtained, therefore, from the *full-field receptances* $\mathbf{H}_d(x, y, \omega)$, simply as:

$$\mathbf{H}_{P\sigma}(x, y, \omega) = \text{diag}[\mathbf{H}_{P_1\sigma}, \mathbf{H}_{P_2\sigma}, \mathbf{H}_{P_3\sigma}](x, y, \omega) \in \mathbb{C}. \quad (6)$$

By following the general multi-axes von Mises Equivalent Stress¹⁵ definition, a rearrangement of the *Principal Stress FRF maps* $\mathbf{H}_{P\sigma}(x, y, \omega)$ of Equation (6), according to the functional $\mathcal{F}_{vM}()$ defined in Equation (C.2), can bring the *von Mises Equivalent Stress FRFs* $\mathbf{H}_{\sigma_{vM}}(x, y, \omega) \in \mathbb{C}$, which depend upon only the specific constitutive model $\mathcal{G}_{cm}()$ and

¹⁴ $\mathbf{H}_{P_1\sigma}(x, y, \omega)$, $\mathbf{H}_{P_2\sigma}(x, y, \omega)$, $\mathbf{H}_{P_3\sigma}(x, y, \omega)$ are the *First*-, the *Second*-, and the *Third-Principal Stress FRF eigenvalue* maps. In this plane application, $\mathbf{H}_{P_3\sigma}(x, y, \omega)$ was very small compared to the others.

¹⁵ Specifically, see [85–87] and Appendix C for all the details.

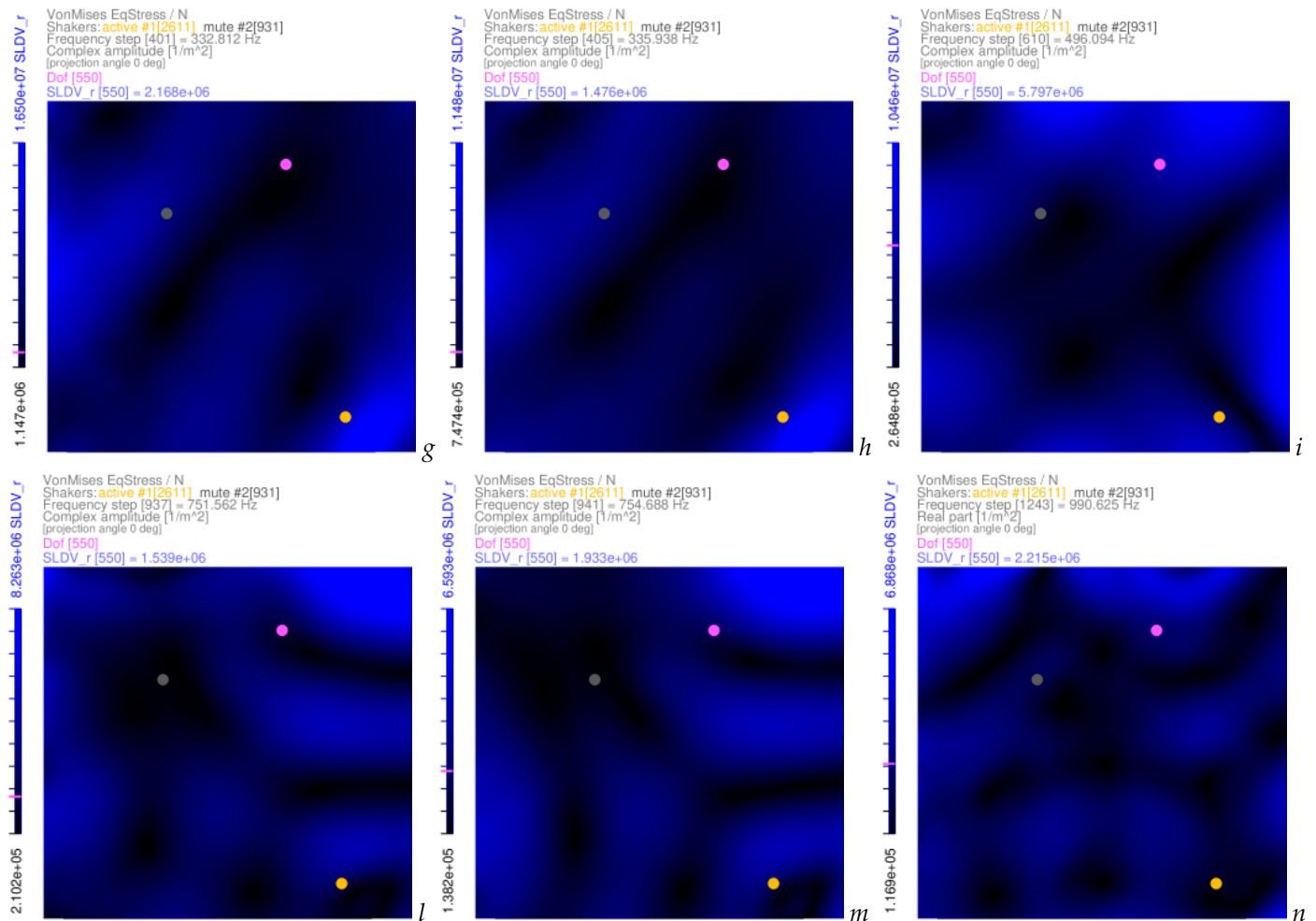


Figure 4. Examples of SLDV-based *von Mises equivalent stress FRF maps* from optical techniques in Equation (7) from shaker 1: at 333 Hz in (g), at 336 Hz in (h), at 496 Hz in (i), at 752 Hz in (l), at 755 Hz in (m), at 991 Hz in (n).

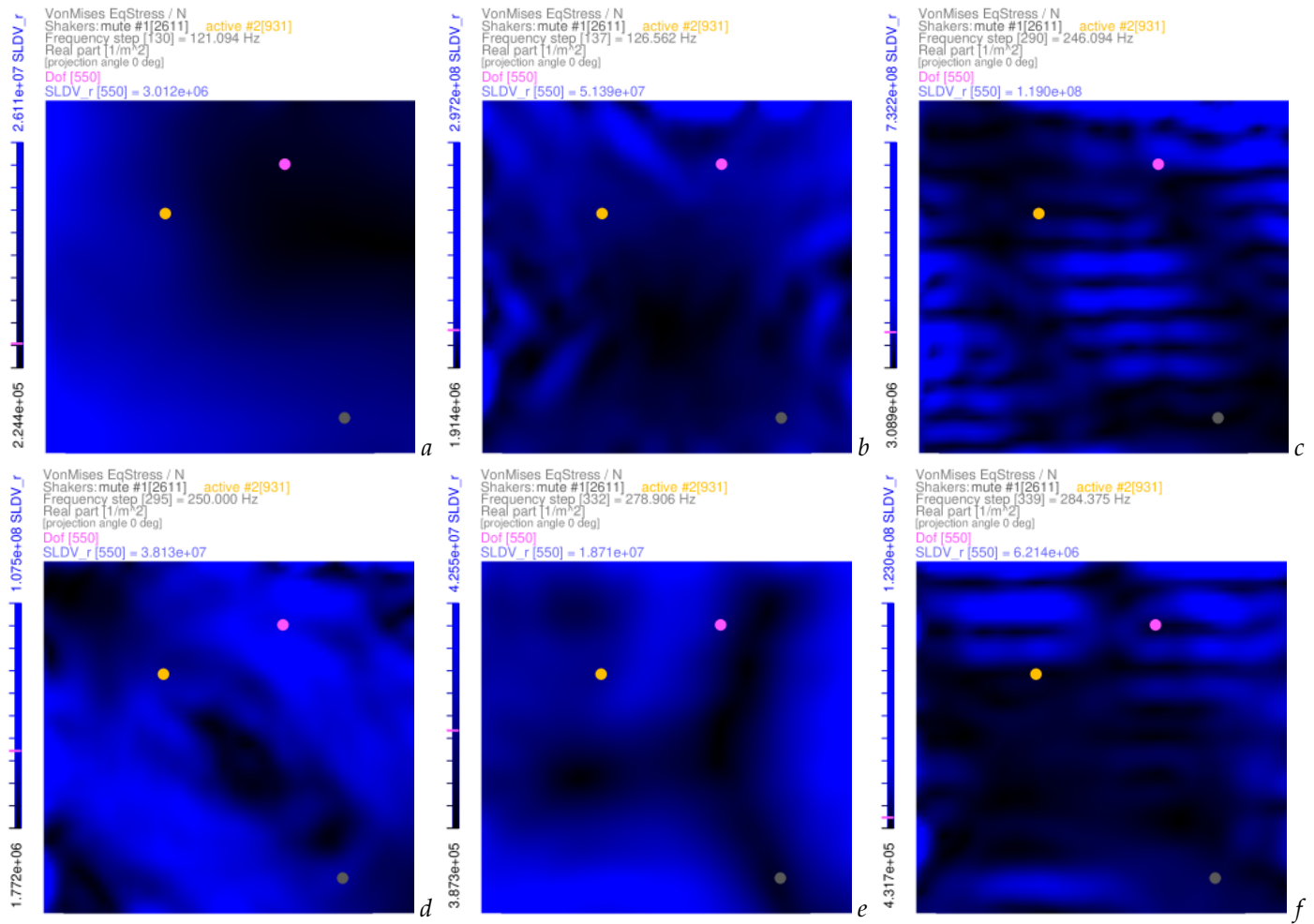


Figure 5. Examples of SLDV-based *von Mises equivalent stress FRF maps* from optical techniques in Equation (7) from shaker 2: at 121 Hz in (a), at 127 Hz in (b), at 246 Hz in (c), at 250 Hz in (d), at 279 Hz in (e), at 284 Hz in (f).

upon the *full-field receptances* $\mathbf{H}_d(x, y, \omega)$, but remain independent from the forcing, while in linearity:

$$\mathbf{H}_{\sigma_{vM}}(x, y, \omega) = \mathcal{F}_{vM}(\mathbf{H}_{p\sigma}(x, y, \omega)). \quad (7)$$

Examples of the *von Mises Equivalent Stress FRF* $\mathbf{H}_{\sigma_{vM}}(x, y, \omega)$ maps in Equation (7) are given in Figures 3,4–5,6, or of single dof graphs in Figure 7, as later recalled in Section 3.1.

2.2.3. Estimating the Dynamic Stresses

By exploiting the *complex-valued FRF* formulation of the *Stress FRF tensor* $\mathbf{H}_\sigma(x, y, \omega)$ as in Equations (5),(B.2), dynamic *Stresses* $\mathbf{\Sigma}(x, y, \omega) \in \mathbb{C}$ are easily computed as:

$$\mathbf{\Sigma}(x, y, \omega) = \mathbf{H}_\sigma(x, y, \omega)F(\omega) \in \mathbb{C}, \quad (8)$$

with components as expressed in $[\text{N}/\text{m}^2]$, but where a $F(\omega) \in \mathbb{C}$ force is used, as in the general modelling of real-life excitations proposed in Section 2.3, under the linearity assumption between structural responses and excitations, grounding Equation (1).

There are two equivalent means – the diagonalisation base being the same – of obtaining the detailed maps of the dynamic *Principal Stresses* $\mathbf{\Sigma}_P(x, y, \omega)^{16} \in \mathbb{C}$: (i) by the

¹⁶ $\mathbf{\Sigma}_{P_1}(x, y, \omega)$, $\mathbf{\Sigma}_{P_2}(x, y, \omega)$, $\mathbf{\Sigma}_{P_3}(x, y, \omega)$ are the *First-*, the *Second-*, and the *Third-Principal Stress eigenvalue* maps, in descending magnitude order. In this plane application, $\mathbf{\Sigma}_{P_3}(x, y, \omega)$ was very small compared to the others.

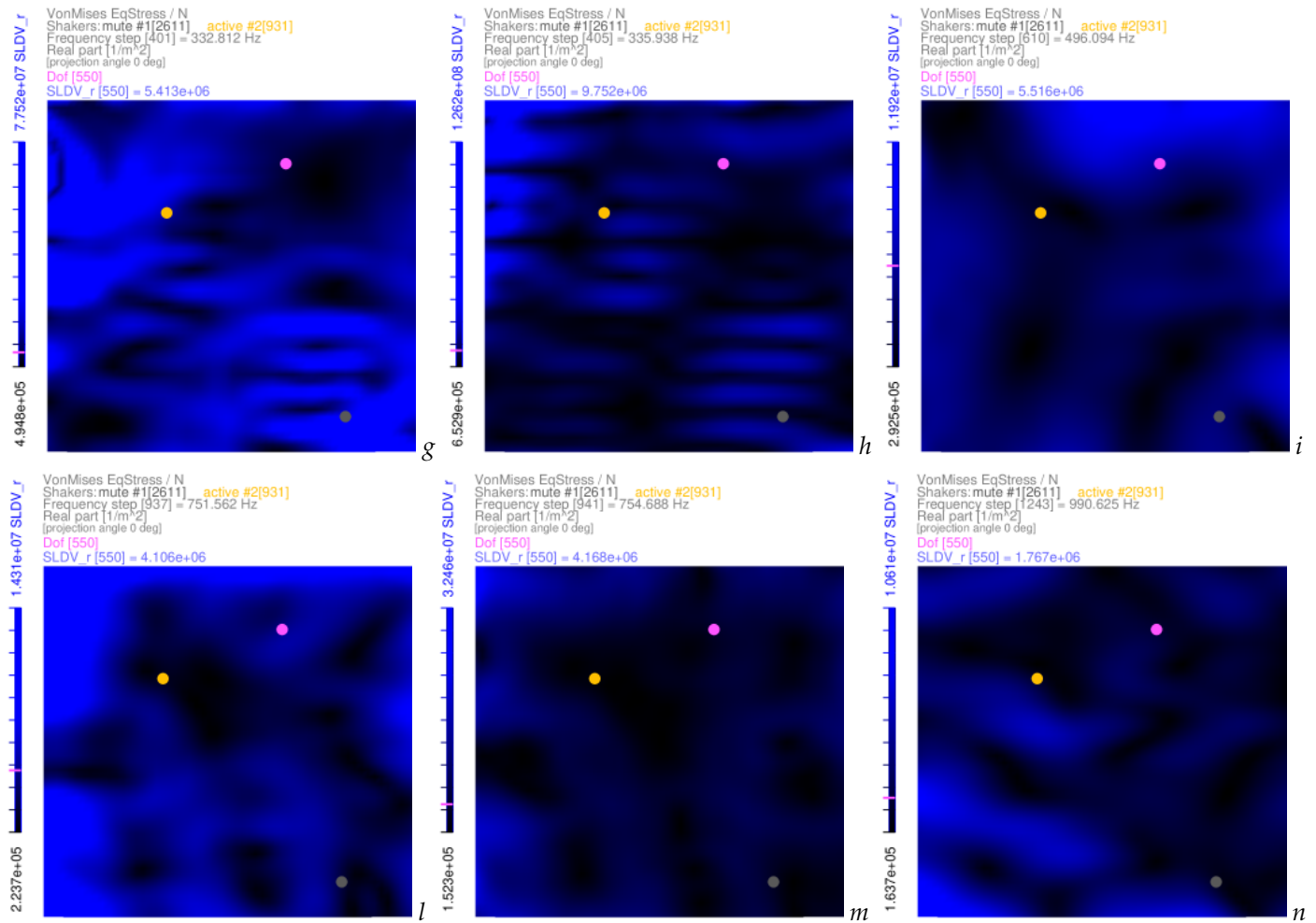


Figure 6. Examples of SLDV-based *von Mises equivalent stress FRF maps* from optical techniques in Equation (7) from shaker 2: at 333 Hz in (g), at 336 Hz in (h), at 496 Hz in (i), at 752 Hz in (l), at 755 Hz in (m), at 991 Hz in (n).

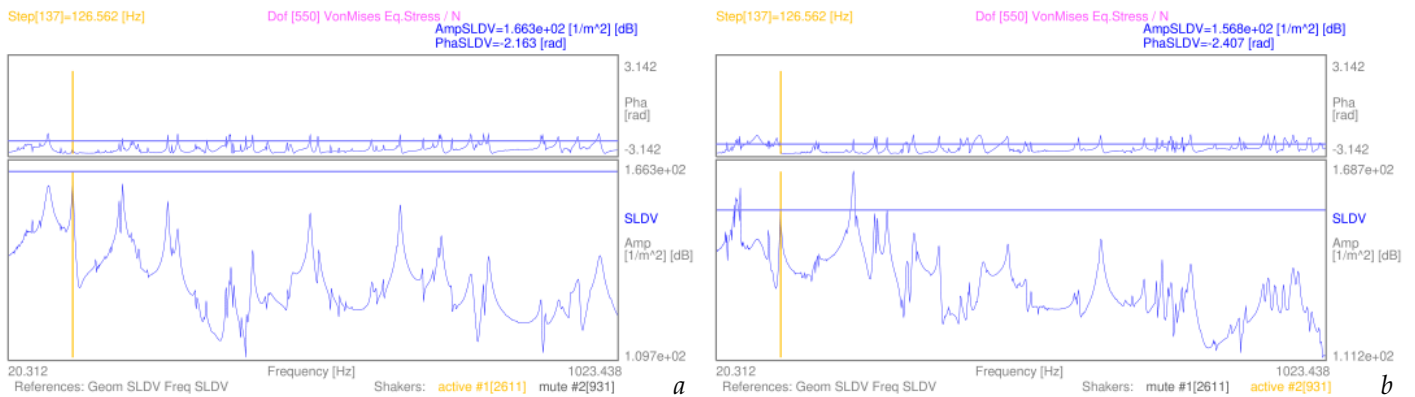


Figure 7. Examples of *von Mises equivalent stress FRF graphs* from SLDV optical technique in Equation (7) in the 20–1023 Hz range: from shaker 1 in (a), from shaker 2 in (b).

diagonalisation of the dynamic *Stresses* $\Sigma(x, y, \omega) \in \mathbb{C}$ in Equation (8); (ii) by exploiting the FRF-based formulation of the *Principal Stress FRF maps* $H_{P\sigma}(x, y, \omega)$ with a proper $F(\omega) \in \mathbb{C}$ excitation. All this means:

$$\begin{aligned} \Sigma_P(x, y, \omega) &= \text{diag}[\Sigma_{P_1}, \Sigma_{P_2}, \Sigma_{P_3}](x, y, \omega) \\ &= H_{P\sigma}(x, y, \omega)F(\omega) = \text{diag}[H_{P_1\sigma}, H_{P_2\sigma}, H_{P_3\sigma}](x, y, \omega)F(\omega) \in \mathbb{C}. \end{aligned} \quad (9)$$

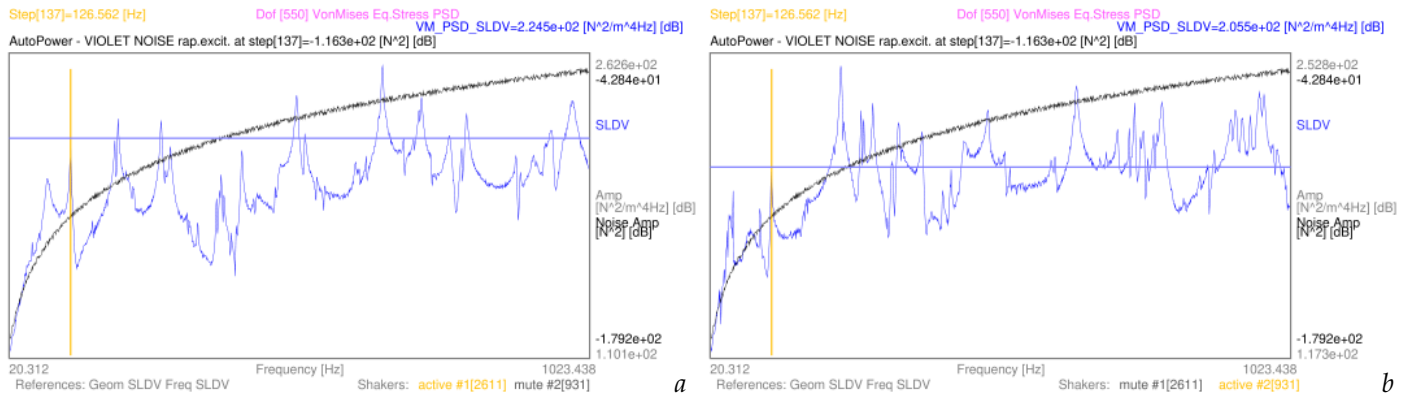


Figure 8. SLDV-based examples of *von Mises equivalent stress PSD* log-amplitude frequency domain graphs of Equation (13), from dof 550 in the 20–1023 Hz range, superimposed to the *Auto Power Spectrum* of the used *violet noise-rap* excitation in Equation (12): from shaker 1 in (a), from shaker 2 in (b).

As above in the FRF formulation of Equation (7), the *Principal Stresses* $\Sigma_P(x, y, \omega)$ coming from Equation (9), version (i), can be re-arranged – according to the functional $\mathcal{F}_{vM}()$ defined in Equation (C.2) for general multi-axes stress definition – in the *von Mises Equivalent Stress* $\Sigma_{vM}(x, y, \omega) \in \mathbb{C}$:

$$\Sigma_{vM}(x, y, \omega) = \mathcal{F}_{vM}(\Sigma_P(x, y, \omega)) \in \mathbb{C}. \quad (10)$$

But here working with the *Principal Stresses* $\Sigma_P(x, y, \omega)$ coming from Equation (9), version (ii), is preferred in obtaining the *von Mises Equivalent Stress* $\Sigma_{vM}(x, y, \omega)$, because the FRF-based approach evaluates $H_{\sigma_{vM}}(x, y, \omega)$ only once, while varying the excitation:

$$\Sigma_{vM}(x, y, \omega) = H_{\sigma_{vM}}(x, y, \omega)F(\omega) \in \mathbb{C}. \quad (11)$$

Albeit both Equations (10),(11) give the same result, Equation (11) is preferred in this approach, or whenever force/input shaping optimisation targets are sought, for the multiple excitations used afterwards in Section 3.

2.3. Excitation Forces: a Simple Formulation of their Spectra

Any *complex-valued* spectrum of the excitation $F(\omega) \in \mathbb{C}$, respecting the *linearity* between forces and structural responses, can be used in this approach, grounded on

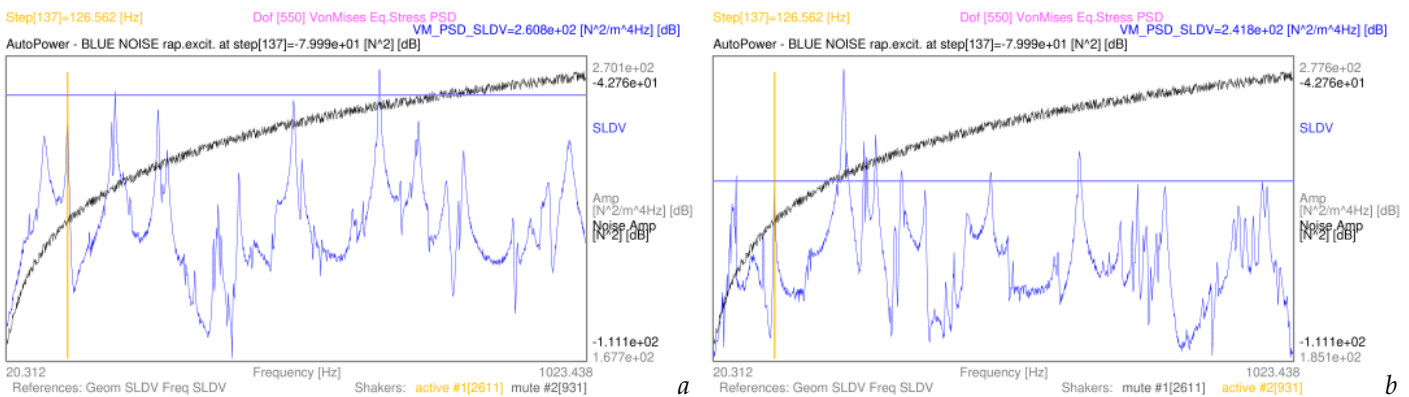


Figure 9. SLDV-based examples of *von Mises equivalent stress PSD* log-amplitude frequency domain graphs of Equation (13), from dof 550 in the 20–1023 Hz range, superimposed to the *Auto Power Spectrum* of the used *blue noise-rap* excitation in Equation (12): from shaker 1 in (a), from shaker 2 in (b).

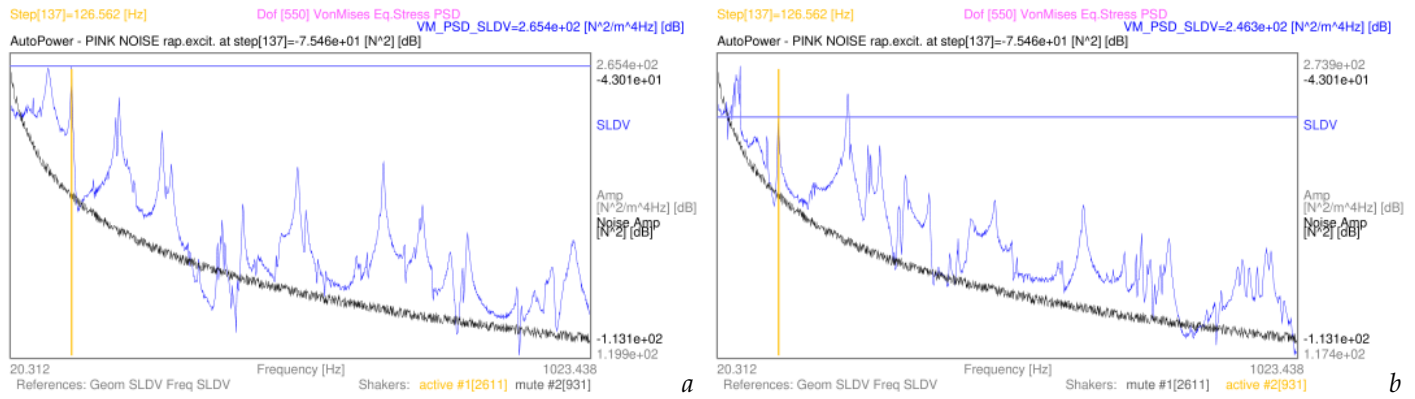


Figure 10. SLDV-based examples of *von Mises equivalent stress PSD* log-amplitude frequency domain graphs of Equation (13), from dof 550 in the 20–1023 Hz range, superimposed to the *Auto Power Spectrum* of the used *pink noise-rap* excitation in Equation (12): from shaker 1 in (a), from shaker 2 in (b).

experiment-based full-field *mobilities* or *receptances*, which are able to retain – in the *complex-valued* representation – the fine details of the real-life structural dynamics. To simulate what can be generally acquired for forces in any real testing, a *complex-valued coloured noise* excitation is adopted, therefore with variable *complex amplitude* and *phase* in a fully populated broad frequency band, potentially with *random variations on the amplitude & phase*.

Complex-Valued Coloured Noise with Random Amplitude & Phase Variations

A general formulation for broad frequency band signals is here proposed for the sake of exemplification: the *complex amplitude* modulation of the excitation spectrum depends on the type of the *coloured noise* adopted, while the *complex phase* is imposed by a specific function. The parameter $\alpha \in [-2, 2]$ defines the *noise colour*: $\alpha = -2$ for *violet noise* as in Figure 8, $\alpha = -1$ for *blue noise* as in Figure 9, $\alpha = 0$ for *white noise* (here not pictured), $\alpha = 1$ for *pink noise* as in Figure 10 and $\alpha = 2$ for *red noise* as in Figure 11. Randomness is added in both the components of the spectrum. Therefore, the signal $F(\omega)$, receiving the ending label of "rap" (for the random variations in the modulated amplitude and sinusoidal phase), can be defined as:

$$F(\omega) = \frac{F_{0r}}{\omega^\alpha} e^{i\theta_r(\omega)} \in \mathbb{C}, \quad (12)$$

with $F_{0r} = F_0(1 + \beta_{F_0}(Rand_{F_0} - 0.5))$, $\theta_r(\omega) = \theta(\omega)(1 + \beta_\theta(Rand_\theta - 0.5))$ with $\theta(\omega)$ as the chosen phase function, contaminated by amplitude & phase random variations. In this

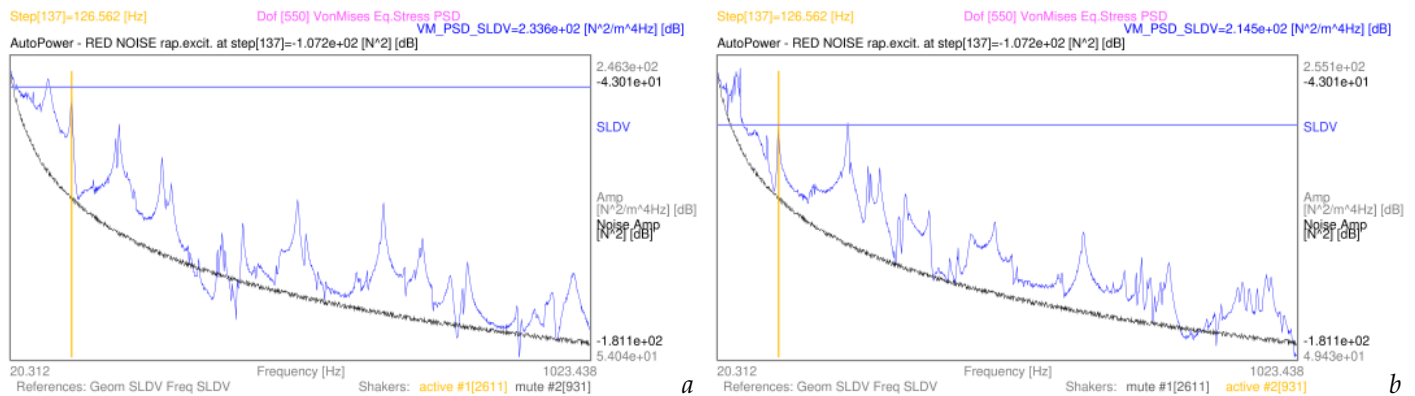


Figure 11. SLDV-based examples of *von Mises equivalent stress PSD* log-amplitude frequency domain graphs of Equation (13), from dof 550 in the 20–1023 Hz range, superimposed to the *Auto Power Spectrum* of the used *red noise-rap* excitation in Equation (12): from shaker 1 in (a), from shaker 2 in (b).

general formulation, the following quantities are defined: $F_0 \in \mathbb{R}$ is the reference amplitude; $\beta_{F_0} \in \mathbb{R}$, being $0 \leq \beta_{F_0} \leq 1$, is the level of randomness in the amplitude; $Rand_{F_0}$, or $Rand_{\theta}$, is a function returning a pseudo-random number in the range 0 to 1, respectively for the amplitudes or for the phases; $\theta(\omega) = S_0 \sin(\pi N_p(\omega - \omega_0)/\Delta\omega + \theta_0)$ is a selected phase function; β_{θ} is the level of randomness in the phase; $S_0 \in \mathbb{R}$ is the sinusoidal phase range multiplier; N_p is the number of half cycles of the phase in the range; ω_0 is the starting frequency; $\Delta\omega$ is the frequency range; θ_0 is a reference phase. For all the examples provided in Section 3, the above mentioned quantities take these values: $F_0 = 0.080$ N, $\beta_{F_0} = 0.15$, $S_0 = 0.5$ rad, $\beta_{\theta} = 0.5$, $N_p = 5.5$, $\theta_0 = \pi/2$ rad.

2.4. Spectral Methods for Cumulative Damage & Fatigue Life Assessment in Brief

The *equivalent range of stress cycles* S_{eq} is targeted by any spectral method for fatigue life estimation, because S_{eq} represents the cumulative damage inferred by the known excitation spectrum on the specific component, due to its involved structural dynamics. The *experiment-based Stress FRF maps* $\mathbf{H}_{\sigma}(x, y, \omega)$ in Equation (5) and the selected *complex-valued spectrum* of $F(\omega)$ offer unprecedented abilities in mapping, in each location (x, y) of the sensed surface, the *equivalent range of stress cycles* $S_{eq}(x, y)$, by means of a full-field extension of the fatigue spectral methods.

Many *spectral methods* (e.g. [46–50,88–98]) are based on punctual evaluations, in each location (x, y) of the spatial domain, of $m_k(x, y) = \int_0^{\infty} f^k PSD_{\Sigma_{vM}}(x, y, \omega) d\omega$, the k -th order moments of the frequency by $PSD_{\Sigma_{vM}}(x, y, \omega)$, the *Power Spectral Density* (PSD) of *von Mises Equivalent Stress*, from which we can obtain other parameters, such as the *effective frequency* $F_{zerocrossing}(x, y) = F_{zc}(x, y) = \sqrt{m_2(x, y)/m_0(x, y)}$, the *expected number of peaks per unit time* $F_{peaks}(x, y) = F_p(x, y) = \sqrt{m_4(x, y)/m_2(x, y)}$, and the *irregularity factor* $\gamma(x, y) = \gamma_2(x, y) = F_{zc}(x, y)/F_p(x, y) = m_2(x, y)/\sqrt{m_0(x, y)m_4(x, y)}$.

The $PSD_{\Sigma_{vM}}(x, y, \omega)$ map can be computed, from the statements for the *von Mises Equivalent Stress* $\Sigma_{vM}(x, y, \omega) \in \mathbb{C}$ in section 2.2.3 and Equation (11), as:

$$PSD_{\Sigma_{vM}}(x, y, \omega) = 2\Sigma_{vM}(x, y, \omega)\Sigma_{vM}^*(x, y, \omega)/\Delta\omega \in \mathbb{R}, \quad (13)$$

where $()^*$ is the complex conjugate operator, $\Delta\omega$ is the frequency domain spectral spacing, and the multiplier 2 is for counting only the positive frequencies in a symmetric domain.

As anticipated above, when adopting varying *complex-valued coloured noises* for $F(\omega)$, the here preferred formulation for evaluating the PSD maps of *von Mises Equivalent Stress* is that in Equation (11), exploiting itself the *von Mises Equivalent Stress FRFs* $\mathbf{H}_{\sigma_{vM}}(x, y, \omega) \in \mathbb{C}$ of Equation (7). For the sake of exemplifications, the latter $\mathbf{H}_{\sigma_{vM}}(x, y, \omega)$ were shown, from both shakers, in the selected maps of the tiles – at single frequencies – in Figures 3,4 – 5,6, and in the whole frequency domain graphs of Figure 7, in the single dof 550. Note that any other¹⁷ spectral method, than what here simulated, can exploit the proposed experiment-based full-field approach. Specifically, all the spectral methods taking advantages from true *complex-valued amplitude & phase relations* – without the limiting multi-axes *von Mises Stress equivalence* – are expected to reach new achievements by means of the *experiment-based full-field Stress FRFs* $\mathbf{H}_{\sigma}(x, y, \omega)$ in Equation (5), by the related *principal FRF components* $\mathbf{H}_{P\sigma}(x, y, \omega)$ in Equation (6), or by the *dynamic Stresses* $\Sigma(x, y, \omega)$ of Equation (8) with their *principal components* $\Sigma_P(x, y, \omega)$ in Equation (9).

Once a specific fatigue spectral method¹⁸ is adopted to evaluate $S_{eq}(x, y)$, it is possible to calculate the *Time-to-Failure spatial distribution* $T_{failure}(x, y)$ ([s] or [h]), or its reciprocal

¹⁷ Suggestions can be found in [50].

¹⁸ Here detailed in Equation (D.2) of Appendix D for the Dirlik's semi-empirical, but widely spread, approach [46].

Frequency-to-Failure spatial distribution $F_{failure}(x, y)$ ([1/s] or [1/h]), evaluated across all the dofs (x, y) of the maps, function of $S_{eq}(x, y)$, of $F_p(x, y)$ and of the K_r fatigue strength coefficient and b exponent¹⁹, as:

$$T_{failure}(x, y) = K_r / \left[F_p(x, y) S_{eq}^b(x, y) \right], \quad (14)$$

$$F_{failure}(x, y) = \left[F_p(x, y) S_{eq}^b(x, y) \right] / K_r. \quad (15)$$

3. Results

This section gives numerical examples of achievements obtainable from the proposed experiment-based full-field approach in Section 2. Section 3.1 recalls the numerical derivative evaluations of the *von Mises Equivalent Stress FRFs*. In Section 3.2 different PSDs of *von Mises Equivalent Stress* are recalled, by changing the amplitude modulation with the coloured noise model and by choosing the energy injection point, thus the underneath experiment-based structural dynamics. The same excitations are further used, with the aid of Dirlik's fatigue spectral method, to simulate the expected distributions of *Time-to-Failure* in Section 3.3 and maps of *Frequency-to-Failure* in Section 3.4.

Some notes on the proposed maps of the figures follow to help the reader in better understanding the information they provide. First, the top-line text describes briefly the sketched function, followed by the indication of the active (in dark yellow text) and mute (in dark grey text) shakers, with corresponding structural dofs, also reported on the map by a big dot. For SLDV datasets, shaker 1 was in dof 2611, shaker 2 in dof 931. All the maps are 3D *complex-valued* fields, but projected with the same geometrical view and phasing angle to keep the comparison viable. Therefore, the third text line says which *complex-valued* part is figured, while the fourth line gives the projecting phase angle. While the (x, y) coordinates correspond to those of the measurement points of the SLDV acquisitions, the (z) ordinate expresses the value reached by the sketched function in the specific dof. Moreover, blue tones are associated to the (z) ordinate, giving brightest blue ($rgb=[0,0,1]$) to the maximal value of the function in the range, while giving darkest blue ($rgb=[0,0,0]$, or pure black) to the lowest value in the range. Also the triangles of the maps, rendered by OpenGL primitives, are coloured accordingly with the colours of the defining (x, y, z) points. It may happen that the OpenGL back-face culling cuts the added big dots in some geometrical 3D views, whereas the top views present no issues. The ranges of the sketched function are drawn at the left side of each map, with the linearly varying blue tones and the extremes of the whole range. On each map, an inquiry dof is highlighted by a magenta big dot; its number (here 550) is written in the magenta text (fifth line) above the map; the value of the function reached in the inquiry dof is reported in blue in the sixth text line, but also by a magenta dash in the side ranges.

Some notes also about all the 2D frequency domain graphs is here given. At the top of each figure, generally three lines of coloured text are annotated. The first contains, on the left side, the frequency step and corresponding value in Hertz, in dark yellow, corresponding to the vertical line in the graph, or cursor; at the centre, in magenta, the dof of inquiry is proposed before the name of the main function of the graph. The second line, on the right side, shows in blue the value of the main function at the cursor's abscissa, with related bracketed quantities; eventually, in the case of *complex amplitude* and *phase*, this information is split also to the third line. The third line can be instead taken, when only the *amplitude* of the functions is shown, by the description of the eventually added function and value at the cursor position, in black with left margin. The sketched functions,

¹⁹ Here $K_r=4.42e+43 \text{ Pa}^b$, $b=4.81$ for the 7075-T6 aluminium alloy in [58].

or *complex-valued* parts, are framed by grey lines, with the main functions' extremes and bracketed dimensions on the right side in grey text, while the frequency range is annotated in grey text at the very bottom of the frame. Another text line, at the very bottom, indicates the geometrical and frequency references (here both reporting SLDV), together with the used shaker in the specific structural dof, marking it with dark yellow, whereas the mute shaker is in dark grey. Inside the frame, the main function is drawn in pure blue, and the measurement technology (here SLDV) is reminded in blue text at the right of the frame. The dark yellow vertical cursor intercepts the main function in a point through which an horizontal blue line is added to show the reached main function value. If another function is superimposed – with its own ordinate quantities, scales and ranges annotated on the right of the frame – the colour of choice is pure black, but the value at the cursor position is only reported in the text above.

3.1. Examples of Von Mises Equivalent Stress FRFs

Figures 3,4 – 5,6 were already referenced in Section 2.2.2 to give direct examples of Equation (7), *von Mises Equivalent Stress FRF* $H_{\sigma_{vM}}(x, y, \omega)$ evaluations from the two shakers; the quantities are expressed in $[1/m^2]$ and give a precise spatial mapping of the *complex amplitude* of $H_{\sigma_{vM}}(x, y, \omega)$, strongly varied in each tile at the specific frequencies highlighted in the captions, due to the underneath ODS, with related measurement errors. In Figures 3,4, the tiles *b, d, f* markedly show the weaknesses of the *mobilities'* estimation from shaker 1, as already underlined in [24,29]. These issues are magnified – in the derivation chain of strains-stresses of Equations (3),(5),(7), despite the advanced numerical tools customised, as in the Appendix of [6] – in the tiles *b, c, d, f, g, h*, but less in *l*, of Figures 5,6, obtained from shaker 2. The latter was the excitation source with more scanning-related problems in the TEFMA project, despite the optimal, or very good, point-wise responses from the SLDV instrumentation during the measurement campaign. Despite the *scanning*-related problems in SLDV-based datasets, it is clear how the complexity of the structural dynamics is well retained from real-life testing, without any simplification, except that of linearity checks. Note how the spatial domain releases clear evidences of compromised measurements, at least in some frequencies: this might be taken into due consideration for refining SHM iterative strategies.

On each tile in Figures 3,4 – 5,6, with a big magenta dot, the structural dof 550 is highlighted for the frequency domain inquiries of the specific function later proposed. Indeed, dof 550 serves to show the *von Mises Equivalent Stress FRF* $H_{\sigma_{vM550}}(\omega)$ in the whole frequency domain, as in Figures 7a,b from both shakers. In the latter figures, the *complex phase* clearly denotes a restraint in the range $[-\pi, -\pi/2]$, due to the specific formulation of Equation (7) and of the functional $\mathcal{F}_{vM}()$ in Equation (C.2). In the single dof graphs the quality of the shown *von Mises Equivalent Stress FRF* $H_{\sigma_{vM550}}(\omega)$ in the whole frequency domain appears (wrongly) less affected by the *scanning*-related problems maps at single frequency lines. As written above, therefore, more attention should be paid to the spatial domain for effective diagnosis of the processing chain in successful SHM.

3.2. Examples of Von Mises Equivalent Stress PSDs from Complex-Valued Coloured Noise Excitations

As already commented in [7–12,17], changing *PSDs* maps in Equation (13) are easily obtained starting from the same *stress FRFs* $H_{\sigma}(x, y, \omega)$ in Equation (11), by introducing specific *excitation signatures* $F(\omega)$, here shaped in the general *complex-valued* and *randomly-noisy* Equation (12) in Section 2.3, and an *energy injection point* (or shaker), therefore a specific impedance-based experimental model of the structural dynamics. Note that the FRFs of the latter change with the excitation point, because dependent on the nodal lines of the *raw receptances*. Instead, in EMA [4,5] or in EFFMA [3,16], the modal eigenshapes are

Table 1. Basic statistics of the Time-to-Failure distributions, according to Equation (14), with the different coloured noises in Equation (12) and excitation location.

Noise colour + shaker	Dof <i>min</i>	<i>min</i> [h]	Dof <i>max</i>	<i>max</i> [h]	<i>Mean</i> [h]	<i>Std.Dev.</i> [h]	<i>Variance</i> [h ²]	<i>Skewness</i> [/]	<i>Kurtosis</i> [/]
violet-rap S1	1	2.462e+00	1107	3.240e+04	9.713e+02	2.719e+03	7.394e+06	6.936e+00	5.943e+01
violet-rap S2	1	2.510e-01	934	5.300e+03	3.790e+02	5.502e+02	3.027e+05	3.372e+00	1.568e+01
blue-rap S1	343	1.661e-01	1687	3.567e+02	3.609e+01	5.053e+01	2.554e+03	2.654e+00	8.724e+00
blue-rap S2	1	5.277e-02	2325	1.139e+02	1.554e+01	1.605e+01	2.576e+02	2.173e+00	5.555e+00
white-rap S1	1	5.549e-04	699	1.803e-01	3.121e-02	2.384e-02	5.684e-04	2.019e+00	6.452e+00
white-rap S2	1	2.773e-04	1452	3.239e-01	3.368e-02	3.753e-02	1.409e-03	3.170e+00	1.364e+01
pink-rap S1	1	3.942e+00	756	1.166e+03	1.815e+02	2.068e+02	4.279e+04	2.155e+00	4.525e+00
pink-rap S2	1	3.405e-01	1277	2.423e+03	1.062e+02	1.755e+02	3.081e+04	4.862e+00	3.297e+01
red-rap S1	2907	1.063e+03	2835	4.354e+05	4.951e+04	5.640e+04	3.180e+09	2.175e+00	6.715e+00
red-rap S2	1	1.728e+01	1277	4.926e+05	1.215e+04	3.486e+04	1.215e+09	6.864e+00	5.735e+01

independent of the excitations, but are obtained as the modal truncated base that fits – with residues – the FRFs, blended by different participation factors.

Here the $PSD_{\Sigma_{vM}}(\omega)$, extracted from the whole maps, are displayed as log-amplitude graphs by blue curves in Figures 8–11, with also a descriptive title string in blue, while the abscissa and ordinate labels are in grey. In the magenta title the dof of inquiry, the same dof 550 as in the previous figures, is highlighted. In the same Figures 8–11, the Auto-Power-spectrum of the specific excitation is superposed in black curves, labels and text. The modulation of the *complex amplitude* of the *coloured-noise* excitation – as the consequence of the specific α – has clearly an influence on the output $PSD_{\Sigma_{vM}}(\omega)$. In particular, the shape of the excitation can give emphasis to the different spectral component of the structural dynamic in the experiment-based full-field FRFs. Furthermore, the randomness in the excitation – either in the *complex amplitude* or in the *complex phase*, even if not fully visualized due to the nature of PSD- and Auto-Power-spectrum-graphs – is properly transferred to the $PSD_{\Sigma_{vM}}(\omega)$, thanks to the *complex-valued* formulation of the approach in Equation (12). Note how the excitation signature in the whole spectrum modulates the *von Mises Equivalent Stress FRFs* in Figure 7. Again, this underlines how real-life excitation spectra can be employed in this approach.

3.3. Examples of Time-to-Failure Distributions with Complex-Valued Coloured Noise Excitations

As stated above in Section 3.2, the choice of the specific coloured noises and the energy injection location brings different *von Mises Equivalent Stress PSD maps*. But the latter are fed to the cumulative damage and fatigue estimation spectral methods in Section 2.4 and Appendix D to obtain the *Equivalent Range of Stress Cycles* $S_{eq}(x, y)$ of Equation (D.2). Therefore, a specific evaluation of the *Time-to-Failure* maps in Equation (14) corresponds to each of the choices. Table 1 gathers the basic statistics of these different choices. Note that also the results from *white-rap noise* are included, although the Auto Power spectrum of that excitation was not depicted, being the amplitude modulation even among the frequency lines, except for the the randomness in the *complex amplitude* and *phase*.

The starting of the crack, or of the failure, is supposed to happen in the dof with the minimal value of each *Time-to-Failure* distribution, whereas the less injured location is the one with the maximum *Time-to-Failure*. As highlighted by the basic statistical descriptors (*min*, *max*, *Mean*, *Standard Deviation*, *Variance*, *Skewness*, *Kurtosis*), each *Time-to-Failure* distribution is totally deterministic, as the whole approach implies, and has no relation to a Gaussian distribution. Therefore, a strong variability in each *Time-to-Failure* map is underlined by Table 1, linked to the excitation type and application point. For the specific specimen tested, most of the crack would start in dof 1, at a corner of the sensed region (but slightly inside the plate borders), where the plate undergoes stronger motions in

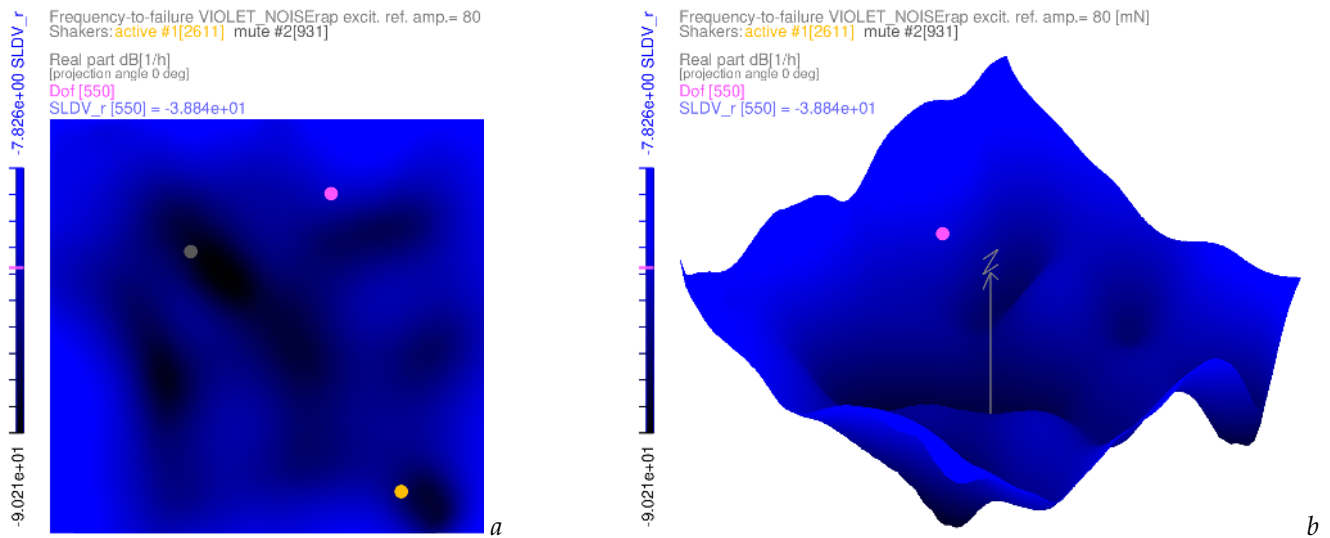


Figure 12. SLDV-based *frequency-to-failure distribution* log-amplitude maps of Equation (15), obtained from the *violet noise-rap* excitation as in Equation (12), from shaker 1: top view in (a), 3D view in (b).

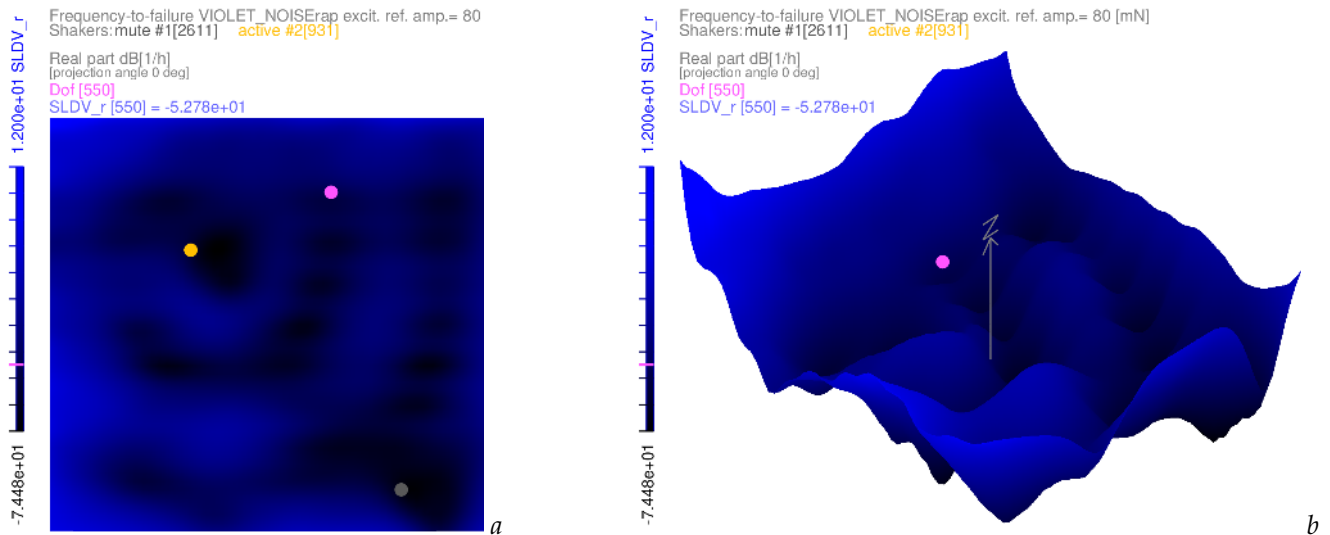


Figure 13. SLDV-based *frequency-to-failure distribution* log-amplitude maps of Equation (15), obtained from the *violet noise-rap* excitation as in Equation (12), from shaker 2: top view in (a), 3D view in (b).

comparison with other positions. Also, looking at the *Mean* value of the *Time-to-Failure* maps, it appears that shaker 2 gives generally a shorter expected life than shaker 1, with relatively minor values of *Standard Deviation* and *Variance*, except for the *white-rap* noise excitation. This can be explained by stronger participations of the bending vibrations when excited from shaker 2, again underlining the relevance of the energy injection location in each specific *receptances*-based formulation. At the same time, it is important to recall the *scanning*-related problems in SLDV-datasets, especially from shaker 2, which surely have a distorting influence on the weight certain frequencies have in the calculations of $S_{eq}(x, y)$. Again, the quality of the *full-field receptances* is crucial for a successful SHM.

3.4. Examples of Frequency-to-Failure Maps with Complex-Valued Coloured Noise Excitations

Similar evaluations as those made in Section 3.3 can be here run, with a focus on the *Frequency-to-Failure* maps, coming from Equation (15). The latter are again obtained by means of the *Equivalent Range of Stress Cycles* $S_{eq}(x, y)$ of Equation (D.2), with the specific choice of the coloured noise, the energy injection location and related *von Mises Equivalent*

Stress PSD map. Therefore, to each of the choices corresponds a specific evaluation of the *Frequency-to-Failure* map in Equation (15). In these evaluations, the starting of the failure should happen in the dof with the maximal value of each *Frequency-to-Failure* distribution, whereas the less injured point is the one with the minimal *Frequency-to-Failure* value. Being Equation (15) the reciprocal of Equation (14), a strong variability is also expected in each *Frequency-to-Failure* map, as can be seen in Figures 12–19, where the results are shown along the log-Z axis (in dB[1/h]), together with brighter blue tones on higher values, to better highlight where the failure should start first. Similar considerations can be made about the stronger solicitation in the corners of the sensed region (under which the restraining cables are attached) and about the structured noise (see [29]), with related influences. However, as for the *Time-to-Failure* distributions in Section 3.3, a marked variability can be found inside the spatial domain, due to the fusion of excitation signature and fully populated experiment-based *full-field receptances*.

The *violet noise-rap* excitation signal (Equation (12), $\alpha = -2$, with randomness in the *complex amplitude* and *phase* as all the *rap* signals) is responsible, through the $PSD_{\Sigma_{vM}}(x, y, \omega)$

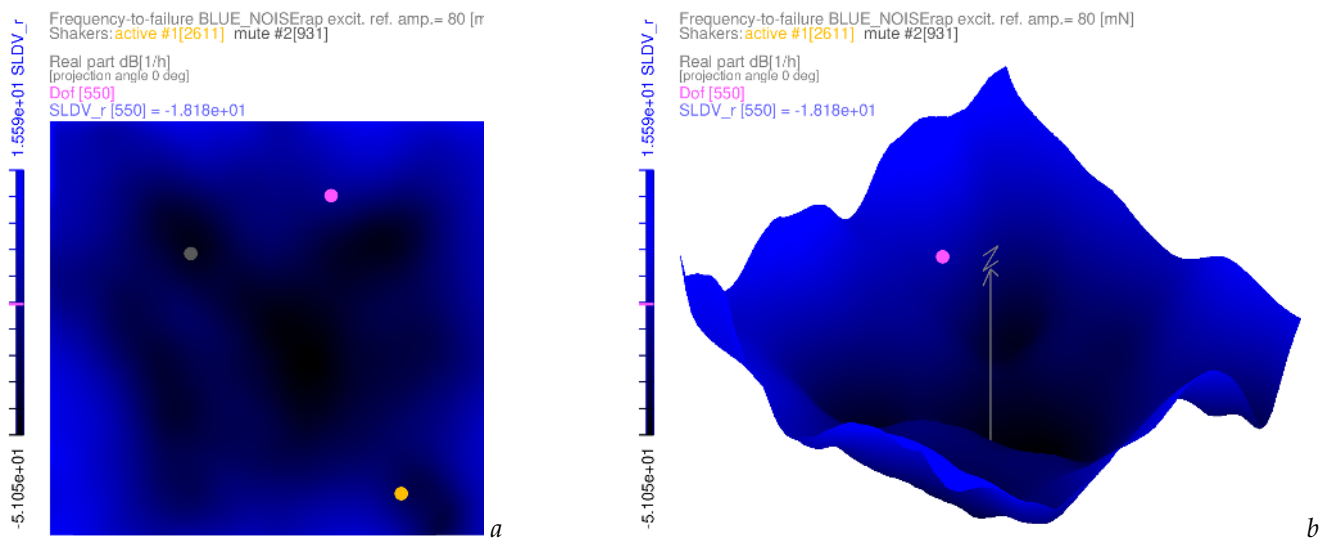


Figure 14. SLDV-based *frequency-to-failure* distribution log-amplitude maps of Equation (15), obtained from the *blue noise-rap* excitation as in Equation (12), from shaker 1: top view in (a), 3D view in (b).

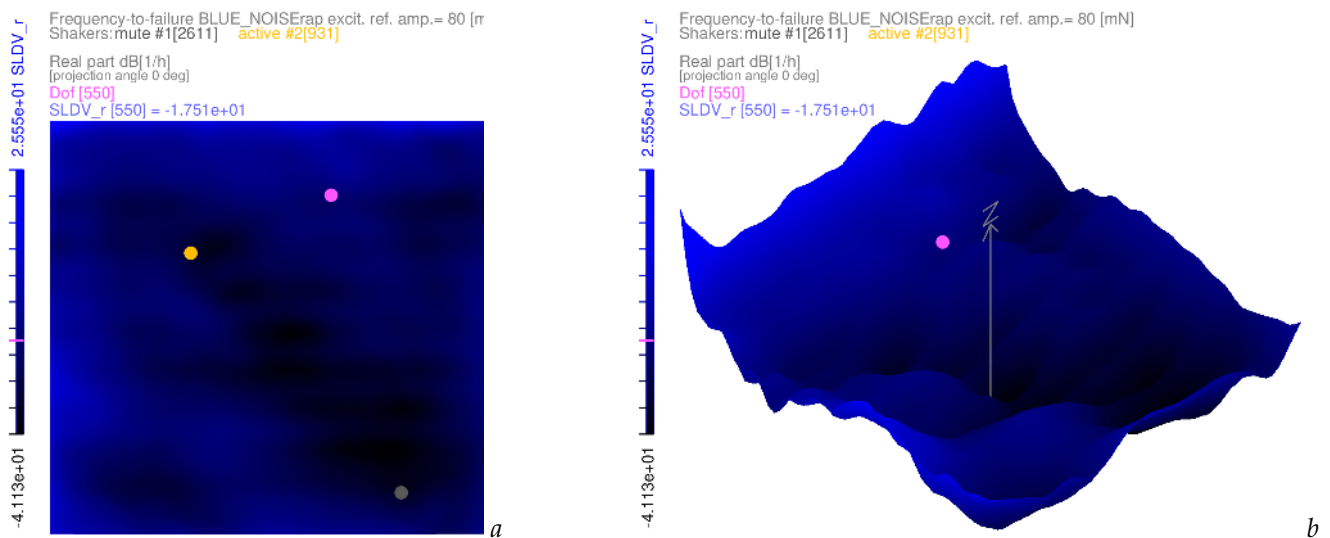


Figure 15. SLDV-based *frequency-to-failure* distribution log-amplitude maps of Equation (15), obtained from the *blue noise-rap* excitation as in Equation (12), from shaker 2: top view in (a), 3D view in (b).

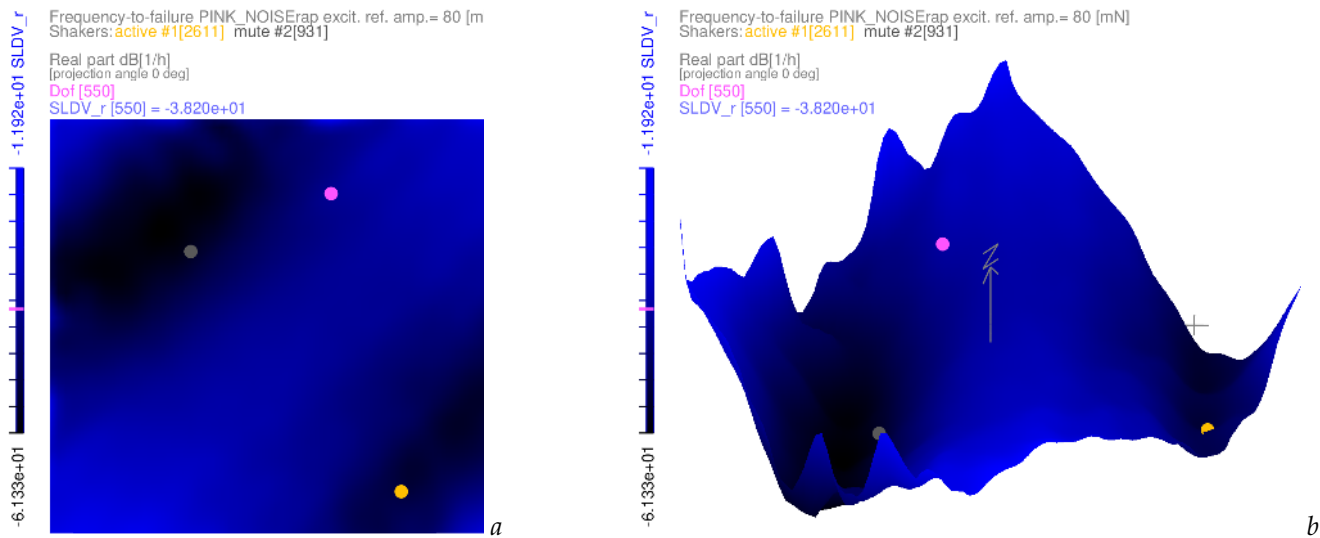


Figure 16. SLDV-based frequency-to-failure distribution log-amplitude maps of Equation (15), obtained from the pink noise-*rap* excitation as in Equation (12), from shaker 1: top view in (a), 3D view in (b).

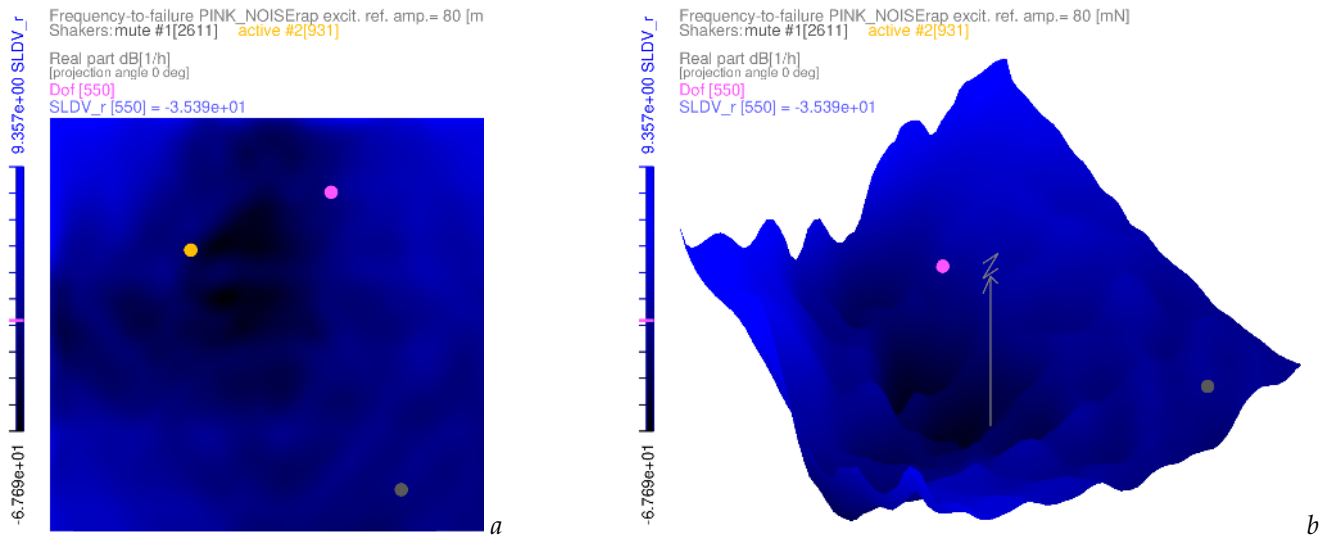


Figure 17. SLDV-based frequency-to-failure distribution log-amplitude maps of Equation (15), obtained from the pink noise-*rap* excitation as in Equation (12), from shaker 2: top view in (a), 3D view in (b).

in Equation (13) for the graphs in Figure 8, of the $F_{failure}(x, y)$ maps of Figures 12–13. Instead, the blue noise-*rap* signal (Equation (12), $\alpha = -1$), thanks to the $PSD_{\Sigma_{vM}}(x, y, \omega)$ shown in Figure 9, generates the $F_{failure}(x, y)$ distributions of Figures 14–15. Instead, by reversing the sign of α , the pink noise-*rap* excitation (Equation (12), $\alpha = 1$) is simulated; this is the source to obtain the $PSD_{\Sigma_{vM}}(x, y, \omega)$ for the graphs in Figure 10 and for $F_{failure}(x, y)$ maps of Figures 16–17. The last simulations here shown come from the selection of the red noise-*rap* excitation (Equation (12), $\alpha = 2$); the red noise-*rap* excitation multiplies the previous $H_{\sigma}(x, y, \omega)$, to obtain the $PSD_{\Sigma_{vM}}(x, y, \omega)$ shown in Figure 11 and used in Equation (15) to evaluate the $F_{failure}(x, y)$ distributions in Figures 18–19.

As in Section 3.3, the change of the structure's task – the couple of the spectrum of the excitation $F(\omega)$ and its energy injection location – brings very different $F_{failure}(x, y)$ maps and expected life scenarios, where the accurate assessment of the dynamic behaviour of the real component, or part, is of uttermost importance for the reliability of any SHM, as in this experiment-based approach no assumptions nor approximations were made to come to any artificial/purely numerical structural model.

4. Discussion

It is important to revisit what proposed in Sections 2–3 from the point of view of the SHM, because the whole methodology has made no assumptions about the structure to simulate the expected life distribution under a specific task. Indeed, this approach is permeated by the experimental assessment of the structural dynamics and of the dynamic loading. But, as demonstrated by the examples of Section 3, the strong variability in the results can be explained only by possessing the whole reliable knowledge of the task-oriented dynamic event. Optical full-field measurement techniques, thanks to their qualities mentioned in Section 1, can extend the sensing to a fully-populated spatial domain, without the need of an a-priori selection of locations for the transducers. Furthermore, the shown results – of a single dof (here 550) in the whole map – demonstrate that, without imposing any simplifying assumption that might deteriorate the understanding of the dynamic responses, the selection of lumped sensors becomes quite complex in changing scenarios, if not impracticable. However, the quality of the structural measurements has an impact on the whole simulation chain, as underlined especially in Section 3.1, where at some

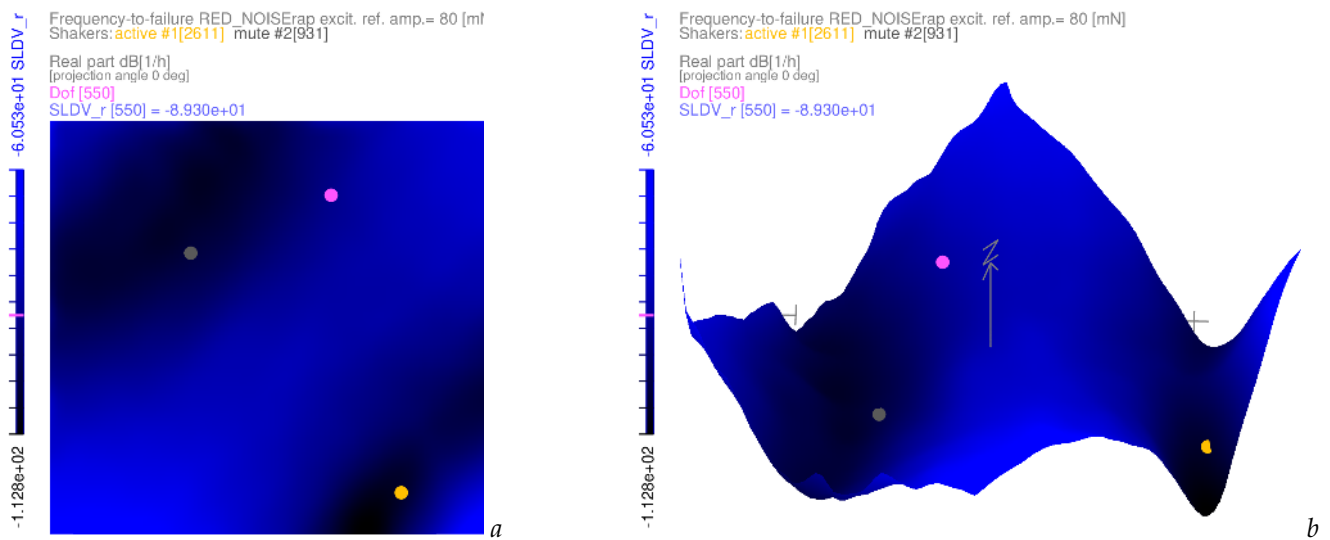


Figure 18. SLDV-based frequency-to-failure distribution log-amplitude maps of Equation (15), obtained from the red noise-rap excitation as in Equation (12), from shaker 1: top view in (a), 3D view in (b).

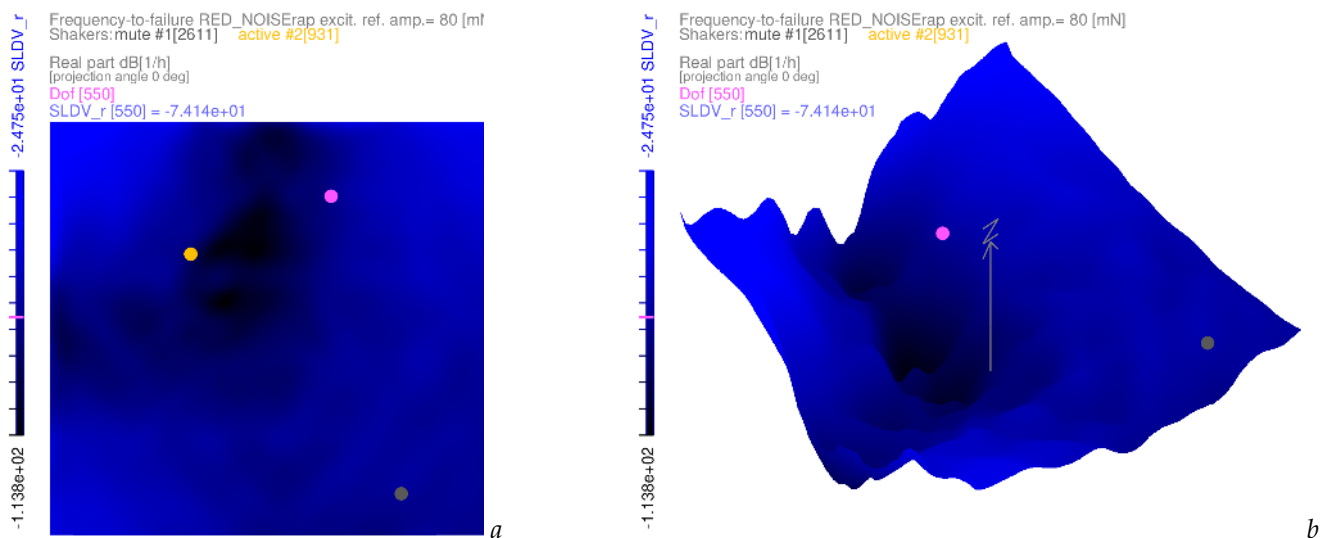


Figure 19. SLDV-based frequency-to-failure distribution log-amplitude maps of Equation (15), obtained from the red noise-rap excitation as in Equation (12), from shaker 2: top view in (a), 3D view in (b).

frequency lines the noisy SLDV *mobilities* gave misinterpretations of the derived quantities due to *scanning*-related issues during the measurement campaign [18,24,29]. Note that the *image*-based *full-field receptances* proved in [6–12] to release much spatially smoother datasets than *scanning*-based techniques, without further noise reduction processing.

As anticipated in Section 1, SHM should extend its assessments of the structural dynamics and loadings during the whole life of the components, especially when ageing, unpredicted accidents or changes happen. These assessments should be run periodically, to change iteratively the expectations on the duration of the parts and on the starting failure location, and to adapt the maintenance programs for them. This is the hard lesson learned from these simulated results: changes to the dynamic behaviour of the structure, as well as on the loadings, result in marked variability of the expected life distribution, therefore of the many managing & processing activities that can follow for a fruitful SHM.

Also manufacturing can get advantages from this approach, specifically for allocating material where needed in mass optimisations and for determining technological aspects for the most solicited areas. Furthermore, quality assessments to find defects during the production of parts can be based on the same achievements, as in [8,12].

Future research activities are needed to assess the validity of the whole approach, e.g. by reaching the failure start and obtain the cumulative damage distribution in simple specimens, like the plate here used, under known excitation and environmental conditions. Once the procedure proves its accuracy on simple test cases, the experiment-based full-field fatigue hybrid predictions and SHM can be extended to more complex parts or assemblies.

5. Conclusions

In this paper SLDV *mobilities* were the experiment-based datasets' sources – instead of other spatially-cleaner *image*-based full-field techniques – for retaining the real dynamic behaviour of the lightweight structure, a simple thin plate, and for giving clear examples from a measurement instrumentation, which is quite widespread among the non-contacting technologies. Also with SLDV *mobilities* the following achievements were possible, with no further modelling assumption nor lumped sensors' selection:

- estimation of *Strain-FRF maps*;
- evaluation of *Stress-FRF maps* by a proper constitutive model of the material;
- evaluation of *von Mises Equivalent Stress-FRF maps*;
- evaluation of *von Mises Equivalent Stress-PSDs*, as dependent on the selected *complex-valued* spectrum and location of the excitation;
- evaluation of high spatial resolution maps of cumulative damage and life expectations by means of fatigue spectral methods and excitation location & spectra.

This paper has shown how an experiment-based updated & reliable knowledge – of the task-oriented responses in a dynamically loaded structure – can improve a proper SHM, especially in changing and ageing scenarios. A complete approach was deployed to exploit the *full-field receptances* of the structure, to describe the actual realisation of the components under analysis without numerical synthetic models, and to use all the components of the *complex-valued spectrum* of the excitations, to be able to properly treat experimental evidences and recordings, all done without specific assumptions, except for linearity.

The paper has shown the strong dependency of the fatigue life distribution on the task the component has to fulfil, together with the relevance of the accuracy of the SLDV *mobilities* in assuring reliable estimation of the actual structural dynamics in the whole approach. Iterative sensing of the component and updated knowledge of the loading become milestones for advanced SHM, in order to better manage maintenance and operations of changing, or ageing, task-oriented structural parts.

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Data Availability Statement: Data are contained within the article.

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Appendix A. Surface Strain FRFs for a Bending Plate

The components of the *full-field generalised strain FRFs* $\mathbf{H}_{\varepsilon_g}(x, y, \omega) \in \mathbb{C}$ are calculated – by the differentiations²⁰ along $j = (x, y, z)$ and $k = (x, y, z)$ directions, applied on the 3D *receptance map* $\mathbf{H}_d(x, y, \omega) \in \mathbb{C}$ – at every map location and frequency:

$$H_{\varepsilon_g}(x, y, \omega)_{jk} = \frac{1}{2} \left(\frac{\partial \mathbf{H}_d(x, y, \omega)_j}{\partial q_k} + \frac{\partial \mathbf{H}_d(x, y, \omega)_k}{\partial q_j} \right) \in \mathbb{C}. \quad (\text{A.1})$$

The *out-of-plane* (z) bending-related displacements, in a plate of thickness s , give their contributions to the following *bending-related strain FRF tensor* $\mathbf{H}_{\varepsilon_b}(x, y, \omega) \in \mathbb{C}$ components, as:

$$\begin{aligned} H_{\varepsilon_b}(x, y, \omega)_{xx} &= -\frac{s}{2} \frac{\partial^2 \mathbf{H}_d(x, y, \omega)_z}{\partial x^2} \in \mathbb{C}, \quad H_{\varepsilon_b}(x, y, \omega)_{yy} = -\frac{s}{2} \frac{\partial^2 \mathbf{H}_d(x, y, \omega)_z}{\partial y^2} \in \mathbb{C}, \\ H_{\gamma_b}(x, y, \omega)_{xy} &= H_{\gamma_b}(x, y, \omega)_{yx} = -s \frac{\partial^2 \mathbf{H}_d(x, y, \omega)_z}{\partial x \partial y} \in \mathbb{C}. \end{aligned} \quad (\text{A.2})$$

The superficial *total strain FRF tensor* $\mathbf{H}_{\varepsilon}(x, y, \omega) \in \mathbb{C}$ is the sum of the above-mentioned contributions:

$$\mathbf{H}_{\varepsilon}(x, y, \omega) = \mathbf{H}_{\varepsilon_g}(x, y, \omega) + \mathbf{H}_{\varepsilon_b}(x, y, \omega) \in \mathbb{C}. \quad (\text{A.3})$$

Note that in the TEFFMA testing only the *out-of-plane* displacements were measurable: Equation (A.1) gives, therefore, a minor contribution, while the major contribution comes from Equation (A.2), taking into account also the different derivative order.

The functional relation $\mathcal{D}_{\varepsilon}()$, incorporating the steps of Equations (A.1)–(A.3), working on the *receptance map* $\mathbf{H}_d(x, y, \omega) \in \mathbb{C}$, releases the *total strain FRF tensor* $\mathbf{H}_{\varepsilon}(x, y, \omega) \in \mathbb{C}$:

$$\mathbf{H}_{\varepsilon}(x, y, \omega) = \mathcal{D}_{\varepsilon}(\mathbf{H}_d(x, y, \omega)) \in \mathbb{C} \quad (\text{A.4})$$

In the TEFFMA project's testing only *out-of-plane* measurements were above the sensitivity threshold, resulting in $\mathbf{H}_d(x, y, \omega)_x = \mathbf{H}_d(x, y, \omega)_y = \mathbf{0}$, while $\mathbf{H}_d(x, y, \omega)_z$ was constant in z ; limited to this work, it happens that $H_{\varepsilon_g}(x, y, \omega)_{xx} = H_{\varepsilon_g}(x, y, \omega)_{yy} = H_{\varepsilon_g}(x, y, \omega)_{xy} = H_{\varepsilon_g}(x, y, \omega)_{yx} = H_{\varepsilon_g}(x, y, \omega)_{zz} = 0$, therefore $H_{\varepsilon_g}(x, y, \omega)_{xz} = H_{\varepsilon_g}(x, y, \omega)_{zx}$ and $H_{\varepsilon_g}(x, y, \omega)_{yz} = H_{\varepsilon_g}(x, y, \omega)_{zy}$ are the only computable contributions. Following these evidences, the *total strain FRF tensor* can be newly arranged in the following matrix as

²⁰ Computed by means of a robust differential operator as in [17,99,100], but with more details in particular shown in the Appendix of [6].

$\hat{H}_\varepsilon(x, y, \omega)$, where one can notice a marked *in-plane* (x, y) magnitude-related relevance of the components:

$$\hat{H}_\varepsilon(x, y, \omega) = \begin{bmatrix} H_{\varepsilon_b}(x, y, \omega)_{xx} & H_{\gamma_b}(x, y, \omega)_{xy} & H_{\varepsilon_g}(x, y, \omega)_{xz} \\ H_{\gamma_b}(x, y, \omega)_{yx} & H_{\varepsilon_b}(x, y, \omega)_{yy} & H_{\varepsilon_g}(x, y, \omega)_{yz} \\ H_{\varepsilon_g}(x, y, \omega)_{zx} & H_{\varepsilon_g}(x, y, \omega)_{zy} & 0 \end{bmatrix} \in \mathbb{C}. \quad (\text{A.5})$$

Appendix B. Linear Constitutive Material Model between Strain & Stress FRFs

Any constitutive material model makes a functional relation between the deformations and the stress in the material. Here, a linear isotropic constitutive model is introduced between the *total Strain FRFs* $H_\varepsilon(x, y, \omega) \in \mathbb{C}$ and the *Stress FRF tensor* $H_\sigma(x, y, \omega) \in \mathbb{C}$, as:

$$H_\sigma(x, y, \omega) = \mathcal{G}_{cm}(H_\varepsilon(x, y, \omega)) \in \mathbb{C}. \quad (\text{B.1})$$

Specifically, the functional relation $\mathcal{G}_{cm}()$ of linear elasticity can be deployed, with the aid of the material parameters (E elastic modulus, ν Poisson ratio, G shear modulus, Λ Lamé constant), as:

$$\begin{aligned} H_\sigma(x, y, \omega)_{kk} &= 2GH_\varepsilon(x, y, \omega)_{kk} + \Lambda(H_\varepsilon(x, y, \omega)_{xx} + H_\varepsilon(x, y, \omega)_{yy}) \in \mathbb{C}; \\ H_\sigma(x, y, \omega)_{kj} &= 2GH_\varepsilon(x, y, \omega)_{kj} \in \mathbb{C}; \\ G &= E/2(1 + \nu) \in \mathbb{R}; \quad \Lambda = E\nu/((1 + \nu)(1 - 2\nu)) \in \mathbb{R}. \end{aligned} \quad (\text{B.2})$$

Appendix C. Von Mises Multi-Axes Equivalence

Be $A(x, y, \omega) \in \mathbb{C}$ a 3×3 tensor defined in the spatial (x, y) and frequency ω domains. Be $\Phi_A(x, y, \omega) \in \mathbb{C}$ the 3×3 matrix of the right [99,101–103] *Principal eigenvectors* of $A(x, y, \omega)$, where $A_{P_1}(x, y, \omega)$ are the *First-*, $A_{P_2}(x, y, \omega)$ are the *Second-*, and $A_{P_3}(x, y, \omega)$ are the *Third-Principal eigenvalues* of $A(x, y, \omega)$. The *Principal maps* $A_P(x, y, \omega) \in \mathbb{C}$ come therefore by means of the diagonalisation of the *tensor* $A(x, y, \omega)$:

$$A_P(x, y, \omega) = \Phi_A^{-1}(x, y, \omega)A(x, y, \omega)\Phi_A(x, y, \omega) = \text{diag}[A_{P_1}, A_{P_2}, A_{P_3}](x, y, \omega) \in \mathbb{C}. \quad (\text{C.1})$$

Let's define the functional $\mathcal{F}_{vM}()$ as the operator to arrange the general multi-axes components of $A(x, y, \omega)$, and its *Principal maps* $A_P(x, y, \omega)$ of Equation (C.1), into the so called *von Mises Equivalent Value* (see [85–87] for further details):

$$A_{vM}(x, y, \omega) = \mathcal{F}_{vM}(A_P(x, y, \omega)) = \sqrt{\frac{[B_1(x, y, \omega)]^2 + [B_2(x, y, \omega)]^2 + [B_3(x, y, \omega)]^2}{2}}, \quad (\text{C.2})$$

with $B_1(x, y, \omega) = A_{P_1}(x, y, \omega) - A_{P_2}(x, y, \omega)$, $B_2(x, y, \omega) = A_{P_2}(x, y, \omega) - A_{P_3}(x, y, \omega)$, $B_3(x, y, \omega) = A_{P_1}(x, y, \omega) - A_{P_3}(x, y, \omega)$.

While it remains quite general and basic due to its discussed (in [86,104–106]) limits and shortcomings, the von Mises approach (see [85]) still finds wide applicability in engineering, thanks to its simplicity. However, note how the power raising of $()^2$ and $()^{\frac{1}{2}}$, coming from the functional $\mathcal{F}_{vM}()$ in Equation (C.2), restrains any *phase* information, with repercussions in Equation (7) and Equations (10)–(11).

Appendix D. Parameters in the Dirlik Semi-Empirical Spectral Method

For the sake of fatigue spectral methods' exemplifications, the *Dirlik semi-empirical spectral method* in [46] was selected, among the many fatigue estimation procedures available

(see [46–50,88–98]), because it gives a sound prediction of the fatigue life for wide-frequency-band spectra of stress responses. The *Dirlik semi-empirical spectral method* was here coded to work for the whole (x, y) ²¹ mapping, combining the maps of factors in Equation (D.1):

$$\begin{aligned}\chi_m &= (m_1/m_0)(m_2/m_4)^{1/2}; & D_1 &= 2(\chi_m - \gamma^2)/(1 + \gamma^2); \\ R &= (\gamma - \chi_m - D_1^2)/(1 - \gamma - D_1 + D_1^2); & D_2 &= (1 - \gamma - D_1 + D_1^2)/(1 - R); \\ D_3 &= 1 - D_1 - D_2; & Q &= 1.25(\gamma - D_3 - D_2R)/D_1.\end{aligned}\quad (\text{D.1})$$

The final output, in each location (x, y) , is the *Equivalent Range of Stress Cycles* S_{eq} , raised to b exponent:

$$S_{eq}^b = D_1(2\sqrt{m_0}Q)^b\Gamma(b+1) + (2^{3/2}\sqrt{m_0})^b\Gamma(1+b/2)[D_2R^b + D_3], \quad (\text{D.2})$$

where $\Gamma()$ is the Gamma function, as used in Equations (14)–(15).

Abbreviations

The following abbreviations are used in this manuscript:

DIC	Digital image correlation
dof(s)	Degree(s) of freedom
EFFMA	Experimental full-field modal analysis
EMA	Experimental modal analysis
ESPI	Electronic speckle pattern interferometry
FEM	Finite element model
FRF	Frequency response function
NDT	Non-destructive testing
NVH	Noise and vibration harshness
ODS	Operative deflection shape
PSD	Power spectral density
SHM	Structural health monitoring
SLDV	Scanning laser Doppler vibrometer
(ω)	Circular frequency dependency
$X(\omega)$	Displacement map
$F(\omega)$	Excitation force
$H_d(\omega)$	Receptance map
$H_v(\omega)$	Mobility map

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²¹ The notation (x, y) can be suppressed for compactness in the spatial extension of the specific spectral method parameters here discussed, which must be intended as maps.

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