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Energy Flows at Earth's Surface

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Two energy flows converge at Earth's surface: one coming from the Sun and the other from Earth's interior. Their intensities are very different, the first one being overwhelming. Solar heat is responsible for all processes involving the outer fluid layers of Earth (atmosphere and oceans). In contrast, processes occurring in Earth's interior, including convection in the core and in the mantle as well as surface manifestations such as seismic and volcanic activity, are unaffected by solar energy, being due only to internal heat. Solar energy only controls the surface temperature of Earth, which is a boundary condition for internal processes. The argument requires use of basic concepts in thermodynamics, such as the radiance and the spectral radiance of a blackbody. The equation of heat conduction in spherical symmetry is derived and solved for a simple model, yielding an approximate temperature profile within Earth.

Energy from the Sun

With a good approximation, the Sun can be considered as a blackbody, i.e., an ideal body that absorbs all incident radiation and emits thermal radiation in the whole frequency range. Radiance q of a blackbody is given by Stefan's law, which can be derived from thermodynamics and classical electromagnetism[1]:

$$q = \sigma T^4, \tag{1}$$

where T is the absolute temperature of the surface and σ is the Stefan constant (5.67×10^{-8} W · m^{-2} · K^{-4}).

The energy flow coming from the Sun can be easily calculated from the values of a few quantities: radius r_S and surface temperature T_S of the Sun, average Earth–Sun distance R, and radius r_E and albedo α of Earth. According to Eq. (1), the radiance of the Sun is

$$q_S = \sigma T_S^4. \tag{2}$$

As distance r from the center of the Sun increases, radiant energy is diluted over spherical surfaces, with areas increasing proportionally to r^2. Therefore, at distance R, energy flow density is

$$q_R = \frac{r_S^2}{R^2} q_S. \tag{3}$$

With $T_S = 5800$ K, $r_S = 6.96 \times 10^8$ m, and $R = 1.5 \times 10^{11}$ m, we obtain $q_R \approx 1380$ W/m^2. This quantity is known as the solar constant.

It is not really a constant. Depending on Earth–Sun distance, it is greater when Earth is at perihelion and smaller when it is at aphelion, with a difference of a few percent. In addition, q_R depends on surface temperature T_S of the Sun. Models of solar evolution indicate that, when the Sun was young, T_S was 10% lower than the present value, so that q_R was about 30% lower.[2]

To calculate the energy flow absorbed by Earth, we must take into account that Earth's surface is spherical and that part of the incident flow is reflected into space. Then, if $\hat{\mathbf{q}}$ is a unit vector in the flow direction and $\hat{\mathbf{n}}$ is the unit normal to the surface, the absorbed flow is

$$Q_{\text{in}} = (1-\alpha) q_R \int_S \hat{\mathbf{q}} \cdot \hat{\mathbf{n}}\, dS, \tag{4}$$

where S is the hemisphere exposed to solar radiation. The integral can be easily calculated and yields

$$Q_{\text{in}} = (1-\alpha)\pi r_E^2\, q_R. \tag{5}$$

With $r_E = 6371$ km, this gives $Q_{\text{in}} = 1.2 \times 10^{17}$ W. Due to Earth's rotation, this energy is distributed over the whole surface, so that the power absorbed per unit area is

$$q_{\text{in}} = \frac{1-\alpha}{4} q_R. \tag{6}$$

Since $\alpha = 0.3$ for Earth,[3] the flow density is $q_{\text{in}} \approx 240$ W/m^2.

Surface temperature

Surface temperature of Earth is controlled by the energy flow coming from the Sun, because endogenous heat flow gives a negligible contribution, as is shown in the "Endogenous flow" section. Therefore, in studying thermal evolution of Earth's interior, Earth can be considered as a body with a fixed surface temperature T_E, determined by solar energy. T_E can be calculated under the assumption of steady-state conditions, in which Earth radiates into space as much energy as it receives from the Sun. The calculation can be made for all planets.[4]

Rigorously, Earth is not a blackbody, because it does not absorb all the incident radiation. However, given its surface temperature T_E, radiance can be still calculated from Eq. (1), so that the outgoing flow is

$$Q_{\text{out}} = 4\pi r_E^2\, \sigma T_E^4. \tag{7}$$

This must be equal to ingoing flow [Eq. (5)], from which we get

$$T_E = \left(\frac{1-\alpha}{4\sigma} q_R \right)^{1/4}. \tag{8}$$

Introducing values of α, σ, and q_R, we obtain $T_E = 255$ K $= -18$ °C, which is called the effective radiation temperature. In fact, the average surface temperature of Earth is higher: $T_E = 288$ K $= 15$ °C. The difference is mostly due to the presence of the atmosphere, producing the greenhouse effect, which ensures the existence of liquid water and a biosphere on Earth.

When one considers models of Earth's interior, T_E is a boundary condition that is controlled by solar heat flow. However, T_E is very small if compared to inner temperatures, so that its value is not relevant for processes in Earth's interior. Transient changes in T_E decay exponentially with depth,

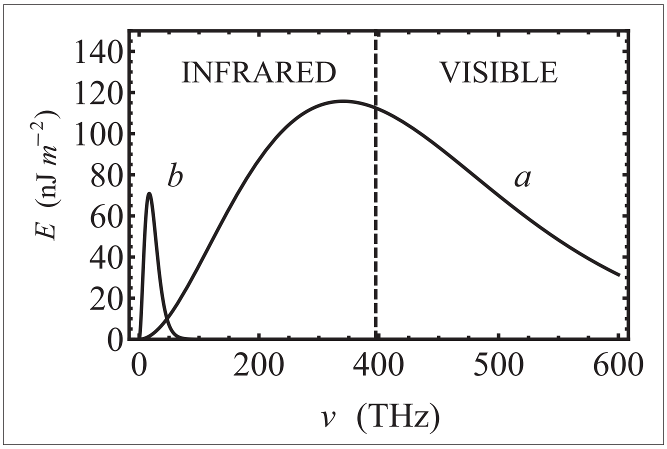

Fig. 1. Spectral radiance *E* of the Sun (*a*) and Earth (*b*) as functions of frequency *ν*, calculated from Eq. (9) with T_S = 5800 K and T_E = 288 K. To show the two curves in the same graph, curve *b* has been multiplied by a factor 5000. The boundary between infrared and visible regions is at 395 THz.

and their penetration is negligible due to the low thermal diffusivity of rocks.[5] Even long-term changes, such as those associated with climatic changes, do not influence inner dynamics.[6]

Spectral radiance and entropy flow

While the two flows Q_{in} and Q_{out} have the same value in steady-state conditions, they strongly differ in frequency content. Spectral radiance E of a blackbody at temperature T is given by Planck's formula[1]

$$E(\nu) = \frac{2\pi}{c^2} \frac{h\nu^3}{e^{\frac{h\nu}{kT}} - 1}, \tag{9}$$

where ν is frequency, h is the Planck constant, k is the Boltzmann constant, and c is the velocity of light. Integration of Eq. (9) over the whole frequency range yields Stefan's law [Eq. (1)]. Function E(ν) has its maximum at frequency

$$\nu_0 = 2.82 \frac{kT}{h}. \tag{10}$$

Due to the high value of T_S, a large part of the spectrum of solar radiation is in the visible region, with ν_0 = 340 THz, while radiation emitted from Earth is in the infrared region, with ν_0 = 17 THz (Fig. 1).

From Eq. (10), the maximum energy $h\nu_0$ of photons is proportional to temperature. Since $T_S/T_E \approx 20$, a photon emitted from the Sun typically has an energy 20 times greater than that of a photon emitted from Earth. If $Q_{in} = Q_{out}$, for each high-energy photon received from the Sun, Earth emits 20 low-energy photons.

The system made of Earth, the Sun, and the surrounding space is far from thermal equilibrium, because Earth can cool down by emitting energy into external space. Ingoing and outgoing entropy flows are respectively

$$\Phi_{in} = \frac{Q_{in}}{T_S}, \quad \Phi_{out} = \frac{Q_{out}}{T_E}. \tag{11}$$

In steady-state conditions, $\Phi_{out} \approx 20\Phi_{in}$, meaning that Earth receives low-entropy energy and emits high-entropy energy. The former is used in irreversible processes involving atmosphere, oceans, and biosphere.[7]

Endogenous flow

Temperature in Earth's interior cannot be measured directly and must be estimated from several pieces of information, such as the primeval thermal state of Earth, chemical composition and state of aggregation, thermal properties of materials, mechanisms of heat production and transfer, and heat flow at Earth's surface. For this reason, estimates of inner temperatures are still subject to a greater uncertainty.[3]

Average heat flow density at Earth's surface ranges from 65 mW/m^2 in continental areas to 101 mW/m^2 in oceanic areas,[5,6] with an average global value of 87 mW/m^2. Multiplication of the latter value times the area of Earth's surface yields an endogenous flow Q_e = 44 TW, which is less than 4% of flow Q_{out} given in Eq. (7).

An approximate picture of the temperature profile in Earth's interior can be obtained by a simple model considering Earth as a solid sphere made of two layers representing the core and the mantle, respectively. Two-layer models are often used for didactic purposes.[8,9]

Let r_1 be the radius of the core. We assume steady-state conditions, with uniform heat production at rate θ per unit volume and thermal conductivity

$$\kappa(r) = \begin{cases} \kappa_1, & 0 \le r < r_1 \\ \kappa_2, & r_1 < r \le r_E \end{cases}. \tag{12}$$

Of course, Earth's interior is not a simply conductive medium. Heat is mainly transferred by convection in the core and in the mantle, and heat production is not uniform. So κ must be considered as an effective conductivity taking into account convection.

Heat conduction can be described by an energy balance equation.[5] We assume spherical symmetry, so that heat flow and temperature depend only on the distance r from the center of Earth. We consider a thin spherical shell of inner radius r and thickness dr. If f is the radial heat flow density, flow into the shell at its inner surface is

$$F_{in} = 4\pi r^2 f(r), \tag{13}$$

and flow out of the shell through its outer surface is

$$F_{out} = 4\pi(r + dr)^2 f(r + dr), \tag{14}$$

so that the net flow out of the shell is

$$dF = F_{out} - F_{in}. \tag{15}$$

Since dr is infinitesimal, we can expand $f(r + dr)$ in a Taylor series and keep only the first-order term:

$$f(r + dr) = f(r) + \frac{df}{dr} dr. \tag{16}$$

Replacing Eqs. (13) and (14) in Eq. (15) and neglecting terms

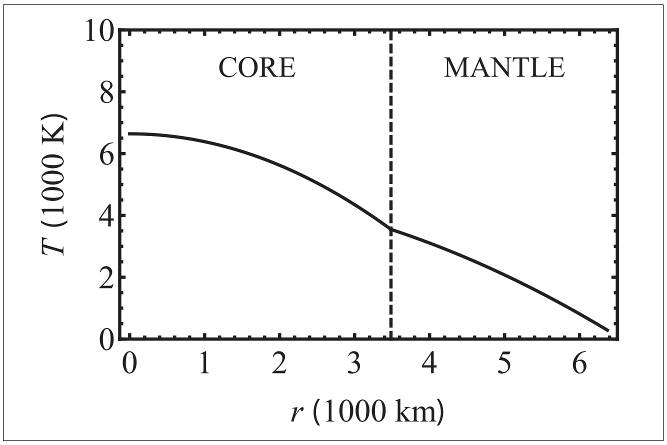

Fig. 2. Temperature profile in Earth's interior calculated from a two-layer model, with uniform heat production and steady-state heat conduction (T_E = 288 K, θ = 40 nW/m^3, κ_1 = 30 W · m^{-1} · K^{-1}, κ_2 = 60 W · m^{-1} · K^{-1}).

in dr^2, we obtain

$$dF = 4\pi r^2 \left(\frac{df}{dr} + \frac{2}{r} f \right) dr. \tag{17}$$

If Θ is the volumetric heat production, the rate at which heat is produced in the shell is

$$d\Theta = 4\pi r^2 \theta \, dr. \tag{18}$$

By equating $d\Theta$ to dF, we get

$$\frac{df}{dr} + \frac{2}{r} f = \theta. \tag{19}$$

This can be converted into an equation for temperature T by Fourier's law:

$$f = -\kappa \frac{dT}{dr}, \tag{20}$$

so that the final equation is

$$\kappa \left(\frac{d^2T}{dr^2} + \frac{2}{r} \frac{dT}{dr} \right) + \theta = 0. \tag{21}$$

Boundary conditions are the following: Earth's surface $r = r_E$ has temperature T_E, there is continuity of temperature and heat flow at the core–mantle boundary ($r = r_1$), and heat flow vanishes at the center of Earth ($r = 0$). The solution of Eq. (21) is

$$T(r) = T_E + \frac{\theta}{6\kappa_2} \left(r_E^2 - r^2 \right), \quad r_1 \le r \le r_E \tag{22}$$

in the mantle and

$$T(r) = T_1 + \frac{\theta}{6\kappa_1} \left(r_1^2 - r^2 \right), \quad 0 \le r \le r_1 \tag{23}$$

in the core, where

$$T_1 = T_E + \frac{\theta}{6\kappa_2} \left(r_E^2 - r_1^2 \right). \tag{24}$$

A graph is shown in Fig. 2, where we use for θ the average value of thermal productivity due to radioactivity.[5] An upper limit to this value can be derived by attributing the entire endogenous flow to radiogenic heat, so that

$$\theta = \frac{Q_e}{V}, \tag{25}$$

where V is the volume of Earth, yielding θ = 40 nW/m^3. The graph yields temperature values at the center of Earth and at the core–mantle boundary within uncertainty ranges. Even important changes in surface temperature T_E (such as during ice ages) have negligible effect on the inner temperature, since $T_E \ll \theta/(6\kappa)$.

According to Eq. (20) with Eqs. (12) and (22), endogenous surface flow density is

$$q_e = \frac{\theta}{3} r_E, \tag{26}$$

yielding the observed average value. Comparing this value with heat flow q_{in} from the Sun given in Eq. (6), we see that the latter is greater by a factor of 2760. For completeness, we mention heat produced by volcanic activity, which adds to q_e, but does not appreciably change the ratio with solar energy. Volcanic activity contributes with about 0.4 TW.[10] This quantity is just 1% of flow Q_e. However, volcanism, in its extreme manifestations, may affect surface temperature by changing Earth's albedo. Mass extinctions of living species have been attributed to such episodes.[11]

Conclusions

Solar energy flow is about 2800 times greater than endogenous flow and controls all thermal processes involving the atmosphere, the oceans, and the biosphere. It also determines the surface temperature of the planet, but has no effect on internal processes.

On the other hand, endogenous heat is responsible for all processes concerning Earth's interior, from convection in the core and mantle to surface geodynamics. However, it is irrelevant for surface temperature and processes.

This neat separation occurs on Earth, but is not true in general. In planets with a much higher surface temperature, like Venus, thermal evolution is strongly affected by interaction between inner and outer heat flows.[12,13] Models indicate that atmospheric warming controls the surface tectonics of Venus by inducing high thermal stresses in the lithosphere and producing characteristic fracture patterns that are observed in volcanic plains.[14]

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